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Structural damage identification using test static data based on grey system theory*

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Abstract: A new structural damage identification method using limited test static displacement based on grey system theory is proposed in this paper. The grey relation coefficient of displacement curvature is defined and used to locate damage in the structure, and an iterative estimation scheme for solving nonlinear optimization programming problems based on the quadratic programming technique is used to identify the damage magnitude. A numerical example of a cantilever beam with single or multiple damages is used to examine the capability of the proposed grey-theory-based method to localize and identify damages. The factors of measurement noise and incomplete test data are also discussed. The numerical results showed that the damage in the structure can be localized correctly through using the grey-related coefficient of displacement curvature, and the damage magnitude can be identified with a high degree of accuracy, regardless of the number of measured displacement nodes. This proposed method only requires limited static test data, which is easily available in practice, and has wide applications in structural damage detection.

Key words: Damage identification, Grey relation coefficient, Static test data
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INTRODUCTION

Structural damage detection technique addresses the problem of how to locate and detect damage that occurred in a structure by using the observed changes of its dynamic and static characteristics. In recent years, damage assessment of structure has drawn wide attention from various engineering fields. Generally, the existing approaches proposed in this area can be clarified into two major categories: the dynamic identification methods using dynamic test data and the static identification methods using static test data (static displacement, static strain, etc.) (Gu *et al*., 1999). Since the static equilibrium equation is solely related to the structural stiffness, and accurate static displacement and strain data can be obtained rapidly and cheaply, the static damage identification methods have attracted relatively more attention in recent years (Hejelmstad and Shin, 1997; Cui *et al*., 2000; Wang *et al*., 2001). Hejelmstad and Shin (1997) proposed an adaptive parameter-grouping algorithm to localize damage in a structural system for which the measured data are sparse. The algorithm can evaluate the sensitivity of each member parameter simultaneously with the process of damage detection, but requires much computation due to the number of perturbation trials involved in the algorithm. Cui *et al*.(2000) developed a damage detection algorithm based on static displacement and static strain. The identified result is ideal, but this method requires sufficient measurement information and load cases. Wang *et al*.(2001) presented a damage identification algorithm using both static test data and natural frequencies, which is very effective in the case of single damage, but only the location important to the structural deformation can be identified in the case of

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multiple damages. Xiang *et al.*(2003) discussed the detection of damage to prestressed concrete continuous beam on the basis of the static response. When the finite element model was used to update the damage variables continually, the theoretically obtained damage development occurred with the tested behaviors. But the many optimal variables for large-scale simulation will lead to reduction of efficiency. Although the static damage identification methods have many advantages, some difficulties still exist. The main problems in the static identification methods are due to: First, the information used in the static damage identification methods is less than that in the dynamic identification, which makes it more difficult to get the ideal identification result. For example, the angular displacement or rotational freedom is difficult to determine. Second, the effects of the damage may be concealed due to the limited load paths. Lastly, the static data provide only the local structural damage information, and the measured static data are very limited.

Deng (1982)'s grey system theory, which uses relatively small data sets and does not demand strict compliance with certain statistical laws, is a truly multidisciplinary and generic method for dealing with systems characterized by poor information and/or for which information is lacking. Grey relation analysis (GRA), an essential part of the grey systems theory, deals with poor or incomplete data, or uncertain problems of some systems. It was successfully applied to evaluate the effect of environment factors on corrosion failure of oil tubes and in fields such as hydro-electricity (Liang, 1999), electricity demand forecasting (Albert *et al.*, 2003), information processing (Liua *et al.*, 2004) and chemistry (Fu and Zheng, 2001). The grey relation coefficient essentially indicates the approaching degree of two geometric curves: the larger the relational coefficient is, the nearer the geometry curves are. When the structure is damaged, the geometric shape of the structure will change. Especially, the geometric shape of the damaged elements changes more largely than that of the other undamaged elements. According to the grey system theory, the changes of the geometric shape can be evaluated by the grey relation coefficient which can, thus, be expected to identify the damaged elements.

In this paper, the feasibility of using the grey system theory to identify the damages of structure based on static test data is exploited. A two-stage damage identification algorithm is presented by using the changes of measured static displacement curvature which is relatively sensitive. The damages are localized firstly by using the node grey relation coefficient, and then, a based on the quadratic programming technique iterative procedure for solving nonlinear programming problems is used to identify the extent of the damage. A cantilever beam with single and multiple damages was used to verify the effectiveness of the proposed method, and the factors of measurement noise and incomplete test data are also considered.

DAMAGE DETECTION BASED ON THE GREY THERORY

Grey relation analysis

Now, we write the reference sequence X_0 and test sequence X_i in the form $X_0=(x_0(1), x_0(2), \ldots, x_0(k))$, $X_i=(x_i(1), x_i(2), \ldots, x_i(k))$; the essential expression of the grey relation coefficient (GRC) $\xi_i(k)$ is given in Deng (1982)

$$\xi_i(k) = \frac{\min_i \min_k X + \alpha \max_i \max_k X}{X + \alpha \max_i \max_k X} \tag{1}$$

where $X=|x_0(k)-x_i(k)|$; $\alpha\in[0, 1]$ is the distinguishable coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\alpha=1$, the comparison environment is unaltered; when $\alpha=0$, the comparison environment disappears. In cases where data variation is large, α usually ranges from 0.1 to 0.5 for reducing the influence of extremely large $\max_i \max_k X$. The grey relation coefficient $\xi_i(k)$, usually ranges from 0 to 1, evaluates the point-relation degree at the ith point of the test sequence $X_i(k)$ and reference sequence $X_0(k)$. Generally, $\xi_i(k)>0.9$ indicates that the reference point and the test point are related completely; $0.8<\xi_i(k)<0.9$ indicates the good relation of the two points; $0.6<\xi_i(k)<0.8$ indicates that the two points are relative or irrespective possibly; $\xi_i(k)<0.6$ represents that the two points are almost irrelative (Fu, and Zheng, 2001).

Damage localization

The displacements of undamaged and damaged structure are, respectively

$$u^{(u)} = \{u_1^{(u)}, u_2^{(u)}, \cdots, u_i^{(u)}\} \quad (2)$$

$$u^{(d)} = \{u_1^{(d)}, u_2^{(d)}, \cdots, u_i^{(d)}\} \quad (3)$$

where i means the node of structure.

According to the central difference approximation, the node displacement curvatures of the ith node of the structure before and after damage are expressed as, respectively

$$\rho_i^{(u)} = \frac{u_{i-1}^{(u)} + u_{i+1}^{(u)} - 2u_i^{(u)}}{l_{i-1} l_{i+1}} \quad (4)$$

$$\rho_i^{(d)} = \frac{u_{i-1}^{(d)} + u_{i+1}^{(d)} - 2u_i^{(d)}}{l_{i-1} l_{i+1}} \quad (5)$$

where l_{i-1} is the distance between the (i−1)th node and the ith node; l_{i+1} is the distance between the ith node and the (i+1)th node.

For every measured node, GRC $\xi_i(k)$ of the ith node can be described as:

$$\xi_i(k) = \frac{\min_i \min_i \rho + \alpha \max_i \max_i \rho}{\rho + \alpha \max_i \max_i \rho} \quad (6)$$

where $\rho = |\rho_i^{(d)} - \rho_i^{(u)}|$; k denotes the number of load case, and α=0.5 (Deng, 1982).

For a load case, damage in some elements was sensitive while damage in the other elements was not. As we know, only sensitive damages can be detected. Therefore, it is impossible to identify all the damages under one load case. The more load cases there are, the better detection results are. Actually the load case is not unlimited. Thus, in the following sections, one of the objectives is to determine whether applying limited load cases can detect correctly all damages of the structure.

GRC in different node under the different load case can be used to localize the damage. GRC is between 0 and 1; for undamaged structure, it is equal to 1 in all nodes; for a damaged structure, the smaller GRC in some nodes is, the bigger the probability of damage in these nodes is.

Damage magnitude detection

For the ith load case, the governing equation of static equilibrium is

$$\boldsymbol{K}u_i = \boldsymbol{F}_i \quad (7)$$

where $\boldsymbol{F}_i$ is the vector of applied forces for the ith load case; and u_i is the corresponding static displacement response. When the damage in the jth element occurs, the element stiffness matrix can be written as:

$$[\boldsymbol{K}_j]^d = \beta_j [\boldsymbol{K}_j]^u \quad (8)$$

where β_j is the coefficient of element stiffness which represents the magnitude of damage in the element. For an undamaged element β_j=1; and β_j=0 represents the complete damage in the jth element. $[\boldsymbol{K}_j]^d$ and $[\boldsymbol{K}_j]^u$ are the element stiffness matrices of the jth element in damaged and undamaged states, respectively. Then the global stiffness matrix of damaged structure can be described as

$$[\boldsymbol{K}]^d = \sum_{j=1}^{n} \beta_j [\boldsymbol{K}]^u \quad (9)$$

where n is the total number of elements in the structure.

From Eqs.(7) and (9), the computed displacement, u_i^a, after damage can be obtained as

$$u_i^a = \boldsymbol{K}^{d^{-1}}(\beta_j)\boldsymbol{F}_i \quad (10)$$

If the structural stiffness matrix captures the properties of the system, and if the measured responses were free from error, the governing equation would be exactly satisfied. In general, such perfection is impossible. The output error between computed and measured displacement for the ith load case is defined as

$$e_i(\beta_j) = u_i^a - u_i^m \quad (11)$$

where u_i^m is the measured displacement in the ith load case.

The element stiffness coefficient β_j can be obtained by minimizing the output error vector defined by Eq.(11). However, here, a constrained nonlinear optimization problem is solved for the optimal parameters by minimizing the following objective function using the quadratic programming technique (Hao and Xia, 2002).

$$\boldsymbol{J} = e_i(\boldsymbol{\beta}) \cdot e_i(\boldsymbol{\beta})^{\mathrm{T}} \quad \text{subject to} \quad 0 \le \beta_j \le 1 \tag{12}$$

From the value of the element stiffness coefficient, the damage extent of the element can be known. When β_j approaches 1, the damage extent in the jth element is small, contrarily, the damage extent is large.

The magnitude of damage in the jth element can be described as

$$\alpha_j = (1 - \beta_j) \times 100\% \tag{13}$$

NUMERICAL EXAMPLE

In this section a cantilever beam is used to demonstrate the applicability of the proposed method above to identify the damage. The factors of incomplete test data, limited load cases and measurement noise are also considered. The length of the beam is 300 mm and square cross-section is 20 mm×20 mm. The elastic module is 2.0E+11 Pa, the density of material is ρ=7800 kg/m^3. The value of the loads are P_1=1 kN, P_2=2 kN and P_3=1 kN. The static test data such as displacements are obtained from the FEM simulation. A group of random numbers with normal distribution (μ=0, σ=0) is added to the data from FEM analysis to simulate the measured error.

Ten elements

The finite element model with 10 elements is shown in Fig.1. Three load cases are shown in Table 1. Four damage cases with moderate damage are considered. Cases A and B denote single damage of different magnitude, the flexural stiffness of E5 is reduced 10% in Case A, and 7% reduction of E8 in Case B. Case C denotes multiple damages in separated elements where 7% reduction is in flexural stiffness of E4 and 10% reduction is in E7. In Case D representing multiple damages in adjacent elements 7% reduction in flexural stiffness is in E4 and E5.

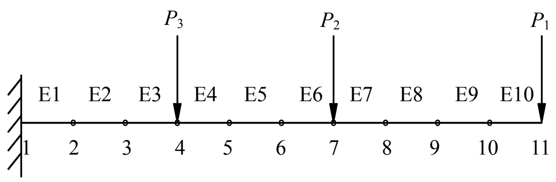

Fig.1 Cantilever beam of 10 finite elements with 3 loads

Table 1 Load cases for the 10-element cantilever beam

Load case	Load location	Load
1	Node 11	P_1
2	Nodes 11 and 7	P_1, P_2
3	Nodes 11 and 4	P_1, P_3

The values of GRC $\xi_i(k)$ for the 4 cases calculated from Eq.(6) are shown in Fig.2. Curve a shows that the values at the two nodes of the 5th element are 0.3333 and 0.3472, respectively, and that the values at the nodes of the other elements are above 0.95, which indicates that the correlation of nodes 5 and 6 is very poor, thus, it is showed that the element between these two nodes is the damaged element. Curve b clearly shows that the GRC value sharply decreases from 1.0 at node 7 to 0.3333 at node 8, and remains unchanged from node 8 to node 9, then greatly increases from 0.3333 at node 9 to 1.0 at node 10, thus forming a valley. This indicates that the 8th element between nodes 8 and 9 is the damaged element. The results in Curves a and b show that the GRC-based approach is very effective and exact in pinpointing the single damage. Curves c and d show the identified results for multiple damages in Cases C and D. Curve c clearly tells us that the GRC values at the two nodes of the 4th and 8th elements are far smaller than those of the other nodes, thus indicating that the 4th and 8th elements are damaged elements. In Curve d, there is a sharp angle between node 4 and node 6, the GRC value sharply decreases from 1.0 at node 3 to 0.4643 at node 4, decreases to the minimum 0.3333 at node 5, and then increases to 0.5287 at node 6 and then 0.9583 at node 7. According to the rule mentioned above to localize damages, elements between the 4th and the 6th nodes are damaged. The results from Curves c and d show that the proposed approach also very effective for localizing multiple damages in adjacent or separated elements.

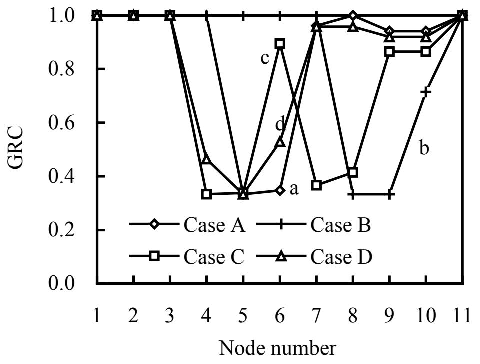

Fig.2 Damage localization in 10-element model

Once the damage can be correctly localized, solving Eq.(12) within the potential damaged elements can identify the magnitude of damage. The comparison between the preset and the identified magnitudes of damage are given in Fig.3, showing that the preset single or multiple damages can be identified with a relatively high degree of accuracy. In this method, 3 loads at most are applied and the measurement noise is also considered, so the proposed method is robust.

Six elements

The finite element model with 6 elements is shown in Fig.4. Three load cases are shown in Table 2, four slight damage cases are considered. Cases A and B denote single damage of different damage magnitudes, the flexural stiffness of E2 is reduced 1% in Case A, and 3% reduction of E4 in Case B. Case C denotes multiple damages in separated elements where 1% reduction is in flexural stiffness of E2 and 3% reduction is in E5. In Case D representing multiple damages in adjacent elements 1% reduction in flexural stiffness is in E3 and E4.

The values of GRC $\xi_i(k)$ for the 4 damage cases of the 6-element cantilever beam calculated from Eq.(6) are shown in Fig.5. Curve a shows that GRC values in the two nodes of the 2nd element are 0.3333

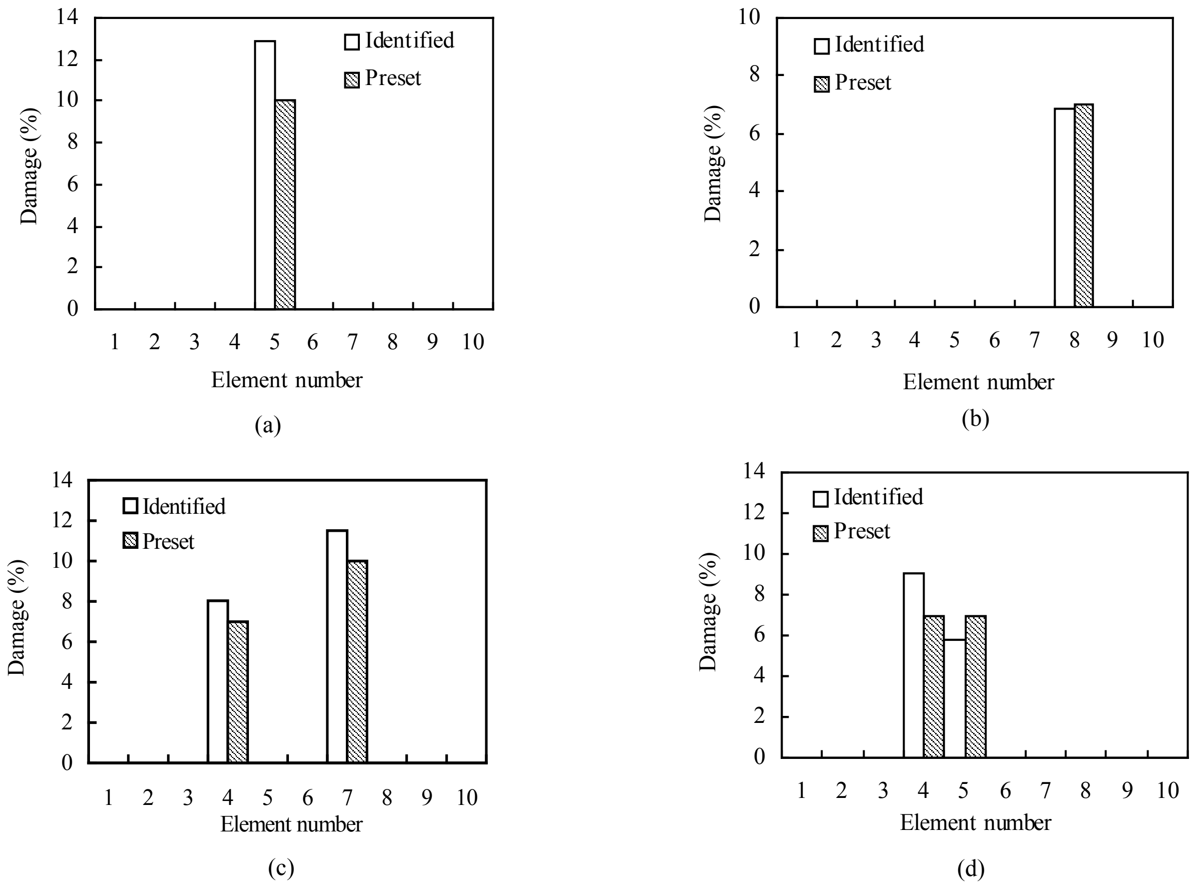

Fig.3 Identification of damage extent in (a) Case A; (b) Case B; (c) Case C; (d) Case D

Table 2 Load cases for the 6-elements cantilever beam

Load case	Load location	Load
1	Node 7	P_1
2	Nodes 4, 7	P_2, P_1
3	Nodes 3, 7	P_3, P_1

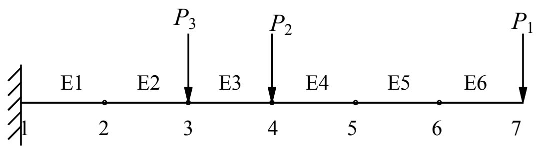

Fig.4 Cantilever beam of 6 finite elements with 3 loads

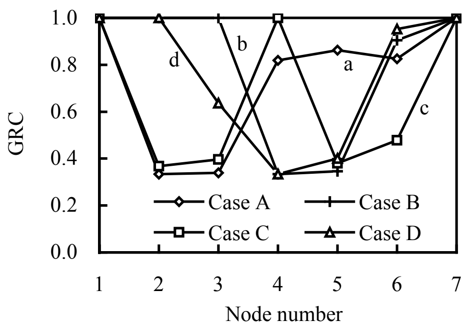

Fig.5 Damage localization in 6-element model

and 0.3398, respectively, which are relatively smaller than those in the other nodes, seemingly indicate that the correlation of these two nodes is very poor, means the element (E2) between the two nodes is damaged. Curve b shows that GRC values in the 4th and 5th nodes are 0.3333 and 0.3455, respectively, which are further smaller than those in the other nodes, so it can be judged that E4 is the damaged element. The results in Curves a and b in Fig.5 show the proposed method is very effective for localizing small magnitude the single damage, even 1%.

Curves c and d in Fig.5 show the identified results of multiple damages. Curve c shows that the GRC values at nodes 2, 3, 5 and 6 are obviously smaller than those at the other nodes, and form two valleys, so the 2nd element and the 4th element are easily judged as the damaged elements. Curve d shows that the GRC values at nodes 3, 4 and 5 are further smaller than those at the other nodes, and form a valley, that is, the 3rd and 4th elements are localized as the damaged elements. The results in Curves c and d indicate that the proposed approach is also very effective for localizing separated and adjacent multi-

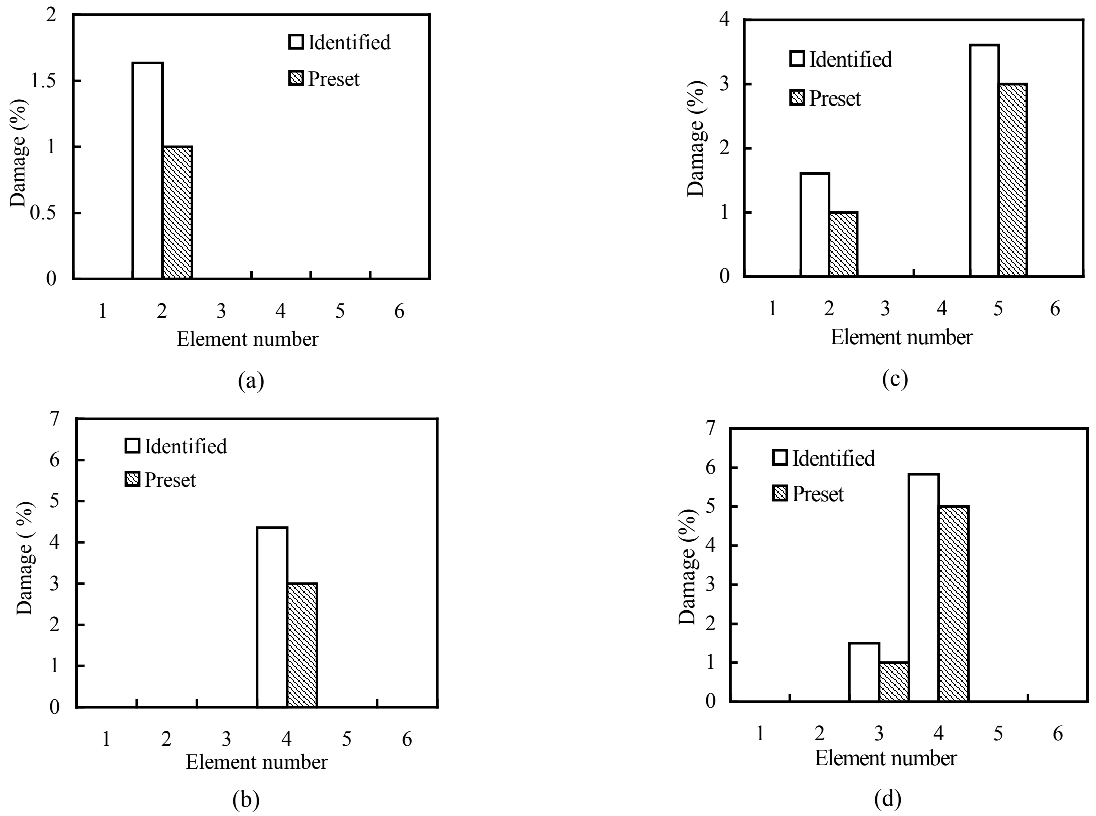

Fig.6 Identification of damage extent in (a) Case A; (b) Case B; (c) Case C; (d) Case D

ple slight damages of even 1%.

As mentioned in the 10-element beam, solving Eq.(12) for the localized damage elements can yield the magnitudes of damage. The preset and identified magnitudes of damage are compared in Fig.6, showing that the preset slight single or multiple damages can be identified with a relatively high degree of accuracy.

CONCLUSION

A two-stage damage detection approach is proposed in this paper. The grey relation coefficient is firstly employed to localize the damage location by only using the structural measured static displacement. Then, an iterative estimation scheme for solving a constrained nonlinear optimization-programming problem based on the quadratic programming technique is used to identify the damage magnitude. The effectiveness of the proposed technique was verified by the numerical example with 10 finite-element and 6 finite-element cantilever beams. The influence of limited test data and measurement noise on the damage identification was considered. The numerical results showed that the grey-relation-analysis-based method can localize accurately slight or moderate, single or multiple damages, and identify the damage magnitude with satisfactory accuracy. The present study is only at the first stage to apply the grey-relation-analysis-based method to identify structure damage; the further study will focus on the application in large-scale and complex structures with limited measured static data.

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