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Dynamic responses of a multilayer piezoelectric hollow cylinder under electric potential excitation*

WANG Hui-ming (王惠明)†, CHEN Yun-min (陈云敏), DING Hao-jiang (丁皓江)

(*Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China*)

†E-mail: wanghuiming@zju.edu.cn

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Abstract: The dynamic responses of a multilayer piezoelectric infinite hollow cylinder under electric potential excitation were obtained. The method of superposition was used to divide the solution into two parts, the part satisfying the mechanical boundary conditions and continuity conditions was first obtained by solving a system of linear equations; the other part was obtained by the separation of variables method. The present method is suitable for a multilayer piezoelectric infinite hollow cylinder consisting of arbitrary layers and subjected to arbitrary axisymmetric electric excitation. Dynamic responses of stress and electric potential are finally presented and analyzed.

Key words: Axisymmetric, Dynamic response, Multilayer, Hollow cylinder, Piezoelectric
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INTRODUCTION

Recently, more and more investigations have been conducted on multilayer piezoelectric media. Kharouf and Heyliger (1994) and Heyliger and Ramirez (2000) dealt with the free vibration problems of laminated piezoelectric cylinders and discs, respectively. Chen (2000) considered the free vibration of (multilayer) nonhomogeneous piezoceramic hollow spheres by employing a separation formulation for displacements. Chen (2001) developed a state-space method for free vibration analysis of laminated piezoelectric hollow spheres. Li *et al.*(2001) studied the free vibration of a piezoelectric laminated cylindrical shell under hydrostatic pressure. The transient plane strain responses of a multilayer piezoelectric hollow cylinder under electric potential excitation have not been reported yet.

In this paper, the combination of the superposition method and analysis method for transient responses of homogeneous (single layer) piezoelectric hollow cylinders (Ding *et al.*, 2003a) was successfully applied to solve the dynamic problems of multilayer hollow cylinders. The key of the method is that the dynamic solution is divided into two parts by the superposition method and then treated separately.

BASIC EQUATIONS

Consider an infinite piezoelectric hollow cylinder composed of n layers with inner radius $r_0=a$ and outer radius $r_n=b$. The first layer is the innermost and the nth layer is the outermost. The inner and outer radii of the ith layer are denoted as r_{i-1} and r_i, respectively. In the cylindrical coordinate system (r,θ,z) and if only the axisymmetric plane strain case is considered, we have $u_\theta^{(i)}=u_z^{(i)}=0$, $u_r^{(i)}=u_r^{(i)}(r,t)$ for the components of displacement, and $\Phi^{(i)}=\Phi^{(i)}(r,t)$ for the component electric potential in the ith layer ($r_{i-1}\le r\le r_i$). We first introduce the non-dimensional quantities as Eq.(1).

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$$
\begin{aligned}
&c_{jmP}^{(i)} = c_{jm}^{(i)}\big/c_{33}^{(1)} \quad (j,m=1,2,3), \quad \varepsilon_3^{(i)} = \varepsilon_{33}^{(i)}\big/\varepsilon_{33}^{(1)} \\
&e_j^{(i)} = e_{3j}^{(i)}\Big/\sqrt{c_{33}^{(1)}\varepsilon_{33}^{(1)}} \quad (j=1,2,3), \quad \overline{\rho}^{(i)} = \rho^{(i)}\big/\rho^{(1)} \\
&\sigma_j^{(i)} = \sigma_{jj}^{(i)}\big/c_{33}^{(1)} \quad (j=r,\theta,z), \quad u^{(i)} = u_r^{(i)}\big/b \\
&D_r^{(i)} = D_{rr}^{(i)}\Big/\sqrt{c_{33}^{(1)}\varepsilon_{33}^{(1)}}, \quad \varphi_c = b\sqrt{c_{33}^{(1)}\big/\varepsilon_{33}^{(1)}} \\
&\phi^{(i)} = \varphi^{(i)}\big/\varphi_c, \quad \phi_0 = \varphi_0\big/\varphi_c, \quad \phi_n = \varphi_n\big/\varphi_c \\
&u_0^{(i)} = U_0^{(i)}\big/b, \quad v_0^{(i)} = V_0^{(i)}\big/c_v, \quad c_v = \sqrt{c_{33}^{(1)}\big/\rho^{(1)}} \\
&p_0 = q_0\big/c_{33}^{(1)}, \quad p_n = q_n\big/c_{33}^{(1)}, \quad \tau = c_v t\big/b \\
&\xi = r\big/b, \quad \xi_i = r_i\big/b \quad (i=0,1,\cdots,n)
\end{aligned} \tag{1}
$$

in which $\sigma_{jj}^{(i)}$ and $D_{rr}^{(i)}$ are the components of stress and radial electric displacement; $c_{jm}^{(i)}$, $e_{3j}^{(i)}$ and $\varepsilon_{33}^{(i)}$ are the elastic, piezoelectric and dielectric constants of the ith layer; $\rho^{(i)}$ is the mass density of the ith layer; φ_0 and φ_n are known electric potentials applied on the inner and outer surfaces, respectively; $U_0^{(i)}$ and $V_0^{(i)}$ are known functions of the radial coordinate r. If each layer characterizes material orthotropy, the constitutive relation, equation of motion and the charge equation of electrostatics can be written as

$$
\begin{aligned}
&\sigma_\theta^{(i)} = c_{11P}^{(i)}\, u^{(i)}\big/\xi + c_{13P}^{(i)} u_{,\xi}^{(i)} + e_1^{(i)}\phi_{,\xi}^{(i)} \\
&\sigma_z^{(i)} = c_{12P}^{(i)}\, u^{(i)}\big/\xi + c_{23P}^{(i)} u_{,\xi}^{(i)} + e_2^{(i)}\phi_{,\xi}^{(i)} \\
&\sigma_r^{(i)} = c_{13P}^{(i)}\, u^{(i)}\big/\xi + c_{33P}^{(i)} u_{,\xi}^{(i)} + e_3^{(i)}\phi_{,\xi}^{(i)} \\
&D_r^{(i)} = e_1^{(i)}\, u^{(i)}\big/\xi + e_3^{(i)} u_{,\xi}^{(i)} - \varepsilon_3^{(i)}\phi_{,\xi}^{(i)}
\end{aligned} \tag{2}
$$

$$\sigma_{r,\xi}^{(i)} + (\sigma_r^{(i)} - \sigma_\theta^{(i)})\big/\xi = \overline{\rho}^{(i)} u_{,\tau\tau}^{(i)} \tag{3}$$

$$D_{r,\xi}^{(i)} + D_r^{(i)}\big/\xi = 0 \tag{4}$$

where a subscript comma denotes partial differentiation. The boundary conditions, the continuity conditions, and the initial condition can be denoted as

$$\sigma_r^{(1)}(\xi_0,\tau) = 0, \quad \sigma_r^{(n)}(\xi_n,\tau) = 0 \tag{5a}$$

$$\phi^{(1)}(\xi_0,\tau) = \phi_0(\tau), \quad \phi^{(n)}(\xi_n,\tau) = \phi_n(\tau) \tag{5b}$$

$$
\left.\begin{aligned}
&\sigma_r^{(i+1)}(\xi_i,\tau) = \sigma_r^{(i)}(\xi_i,\tau) \\
&u^{(i+1)}(\xi_i,\tau) = u^{(i)}(\xi_i,\tau)
\end{aligned}\right. \quad (i=1,2,\cdots,n-1) \tag{6a}
$$

$$
\left.\begin{aligned}
&\phi^{(i+1)}(\xi_i,\tau) = \phi^{(i)}(\xi_i,\tau) \\
&D_r^{(i+1)}(\xi_i,\tau) = D_r^{(i)}(\xi_i,\tau)
\end{aligned}\right. \quad (i=1,2,\cdots,n-1) \tag{6b}
$$

$$
\left.\begin{aligned}
&u^{(i)}(\xi,0) = u_0^{(i)}(\xi) \\
&u_{,\tau}^{(i)}(\xi,0) = v_0^{(i)}(\xi)
\end{aligned}\right. \quad (i=1,2,\cdots,n) \tag{7}
$$

SOLUTION TECHNIQUE

From Eq.(4) and by virtue of the second equation in Eq.(6b), we have

$$D_r^{(i)}(\xi,\tau) = \eta(\tau)\big/\xi \tag{8}$$

Then we can obtain the following equation from the fourth equation in Eq.(2)

$$\phi_{,\xi}^{(i)} = [e_1^{(i)}\, u^{(i)}\big/\xi + e_3^{(i)} u_{,\xi}^{(i)} - \eta(\tau)\big/\xi]\big/\varepsilon_3^{(i)} \tag{9}$$

The substitution of Eq.(9) into the first three equations in Eq.(2) derives

$$
\begin{aligned}
&\sigma_\theta^{(i)} = c_{11D}^{(i)}\, u^{(i)}\big/\xi + c_{13D}^{(i)} u_{,\xi}^{(i)} - e_{1D}^{(i)}\,\eta(\tau)\big/\xi \\
&\sigma_z^{(i)} = c_{12D}^{(i)}\, u^{(i)}\big/\xi + c_{23D}^{(i)} u_{,\xi}^{(i)} - e_{2D}^{(i)}\,\eta(\tau)\big/\xi \\
&\sigma_r^{(i)} = c_{13D}^{(i)}\, u^{(i)}\big/\xi + c_{33D}^{(i)} u_{,\xi}^{(i)} - e_{3D}^{(i)}\,\eta(\tau)\big/\xi
\end{aligned} \tag{10}
$$

where

$$
\begin{aligned}
&c_{jmD}^{(i)} = c_{jmP}^{(i)} + e_j^{(i)} e_m^{(i)}\big/\varepsilon_3^{(i)} \ (j,m=1,2,3) \\
&e_{jD}^{(i)} = e_j^{(i)}\big/\varepsilon_3^{(i)} \ (j=1,2,3)
\end{aligned} \tag{11}
$$

By the method of superposition, the displacement and the radial stress can be assumed as

$$u^{(i)} = u_s^{(i)} + u_d^{(i)}, \quad \sigma_r^{(i)} = \sigma_{rs}^{(i)} + \sigma_{rd}^{(i)} \tag{12a}$$

$$
\begin{aligned}
&\sigma_{rs}^{(i)} = c_{13D}^{(i)}\, u_s^{(i)}\big/\xi + c_{33D}^{(i)} u_{s,\xi}^{(i)} - e_{3D}^{(i)}\,\eta(\tau)\big/\xi \\
&\sigma_{rd}^{(i)} = c_{13D}^{(i)}\, u_d^{(i)}\big/\xi + c_{33D}^{(i)} u_{d,\xi}^{(i)}
\end{aligned} \tag{12b}
$$

where $u_s^{(i)}$ and $\sigma_{rs}^{(i)}$ are the parts satisfying the corresponding boundary conditions Eq.(13) and the continuity conditions Eq.(14).

$$\sigma_{rs}^{(1)}(\xi_0,\tau) = 0, \quad \sigma_{rs}^{(n)}(\xi_n,\tau) = 0 \tag{13}$$

$$
\left.\begin{aligned}
&\sigma_{rs}^{(i+1)}(\xi_i,\tau) = \sigma_{rs}^{(i)}(\xi_i,\tau) \\
&u_s^{(i+1)}(\xi_i,\tau) = u_s^{(i)}(\xi_i,\tau)
\end{aligned}\right. \quad (i=1,2,\cdots,n-1) \tag{14}
$$

We assume

$$u_s^{(i)} = (A^{(i)}\xi + B^{(i)})\eta(\tau) \tag{15}$$

where $A^{(i)}$ and $B^{(i)}$ are unknown constants. Subsequently we have

$$\sigma_{rs}^{(i)}(\xi,\tau) = [A^{(i)}(c_{13D}^{(i)} + c_{33D}^{(i)}) + (B^{(i)}c_{13D}^{(i)} - e_{3D}^{(i)})\big/\xi]\eta(\tau) \tag{16}$$

Then by using Eqs.(13) and (14) and utilizing Eqs.(15) and (16), $A^{(i)}$ and $B^{(i)}$ can be easily determined by solving a system of linear equations only.

Then substituting the first and third equations in Eq.(10) into Eq.(3), and utilizing the first equation in Eq.(12), we obtain

$$\begin{aligned} & u_{d,\xi\xi}^{(i)} + u_{d,\xi}^{(i)}/\xi - \mu_i^2 u_d^{(i)}/\xi^2 \\ & = u_{d,\tau\tau}^{(i)}/c_i^2 + f_1^{(i)}(\xi)\eta_{,\tau\tau}(\tau)/c_i^2 + f_2^{(i)}(\xi)\eta(\tau) \end{aligned} \tag{17}$$

where

$$\begin{aligned} & \mu_i = \sqrt{c_{11D}^{(i)}/c_{33D}^{(i)}}, \quad c_i = \sqrt{c_{33D}^{(i)}/\overline{\rho}^{(i)}} \\ & f_1^{(i)}(\xi) = A^{(i)}\xi + B^{(i)} \\ & f_2^{(i)}(\xi) = \mu_i^2 \xi^{-2} f_1^{(i)}(\xi) - \xi^{-1} f_{1,\xi}^{(i)}(\xi) \\ & \qquad - f_{1,\xi\xi}^{(i)}(\xi) - \xi^{-2} e_{1D}^{(i)}/c_{33D}^{(i)} \end{aligned} \tag{18}$$

By means of separation of variables method, $u_d^{(i)}(\xi,\tau)$ can be assumed as

$$u_d^{(i)}(\xi,\tau) = \sum_{m=1}^{\infty} R_m^{(i)}(\xi)\Omega_m(\tau) \tag{19}$$

where $\Omega_m(\tau)$ is an undetermined function and $R_m^{(i)}(\xi)$ is a known function which can be determined by means of initial parameter method (Wang and Ding, 2004), here we only present the final expressions

$$R_m^{(i)}(\xi) = M_{11}^{(i-1)} S_{11}^{(i)}(k_m^i\xi) + M_{21}^{(i-1)} S_{12}^{(i)}(k_m^i\xi) \tag{20}$$

where

$$\begin{aligned} & [M^{(i)}] = \prod_{j=i}^{1} [S^{(j)}(k_m^j, \xi_j)] \\ & \left[S^{(i)}(k_m^i,\xi)\right] = \begin{bmatrix} S_{11}^{(i)}(k_m^i,\xi) & S_{12}^{(i)}(k_m^i,\xi) \\ S_{21}^{(i)}(k_m^i,\xi) & S_{22}^{(i)}(k_m^i,\xi) \end{bmatrix} \end{aligned} \tag{21a}$$

$$\begin{aligned} & S_{11}^{(i)}(k_m^i,\xi) = \left[P_Y(i,k_m^i,\xi_{i-1}) J_{\mu_i}(k_m^i\xi) \right. \\ & \qquad \left. - P_J(i,k_m^i,\xi_{i-1}) Y_{\mu_i}(k_m^i\xi)\right]/\Lambda_i \\ & S_{12}^{(i)}(k_m^i,\xi) = [J_{\mu_i}(k_m^i\xi_{i-1}) Y_{\mu_i}(k_m^i\xi) \\ & \qquad - Y_{\mu_i}(k_m^i\xi_{i-1}) J_{\mu_i}(k_m^i\xi)]/\Lambda_i \\ & S_{21}^{(i)}(k_m^i,\xi) = [P_Y(i,k_m^i,\xi_{i-1}) P_J(i,k_m^i,\xi) \\ & \qquad - P_J(i,k_m^i,\xi_{i-1}) P_Y(i,k_m^i,\xi)]/\Lambda_i \\ & S_{22}^{(i)}(k_m^i,\xi) = [J_{\mu_i}(k_m^i\xi_{i-1}) P_Y(i,k_m^i,\xi) \\ & \qquad - Y_{\mu_i}(k_m^i\xi_{i-1}) P_J(i,k_m^i,\xi)]/\Lambda_i \end{aligned} \tag{21b}$$

$$\begin{aligned} & P_J(i,k_m^i,\xi) = c_{13D}^{(i)} J_{\mu_i}(k_m^i\xi)/\xi + c_{33D}^{(i)} J_{\mu_i,\xi}(k_m^i\xi) \\ & P_Y(i,k_m^i,\xi) = c_{13D}^{(i)} Y_{\mu_i}(k_m^i\xi)/\xi + c_{33D}^{(i)} Y_{\mu_i,\xi}(k_m^i\xi) \\ & \Lambda_i = P_Y(i,k_m^i,\xi_{i-1}) J_{\mu_i}(k_m^i\xi_{i-1}) - P_J(i,k_m^i,\xi_{i-1}) Y_{\mu_i}(k_m^i\xi_{i-1}) \end{aligned} \tag{21c}$$

and

$$k_m^i = \omega_m / c_i \tag{22}$$

in which ω_m (m=1,2,…,∞) is a series of positive real roots of the following transcendental equation

$$M_{21}^{(n)} = 0 \tag{23}$$

Then substituting Eq.(19) into Eq.(17) and by virtue of the orthogonal properties of Bessel functions (Yin and Yue, 2002), we obtain

$$\Omega_{m,\tau\tau}(\tau) + \omega_m^2 \Omega_m(\tau) = q_m(\tau) \quad (m = 1,2,\cdots,\infty) \tag{24}$$

where

$$\begin{aligned} & q_m(\tau) = I_{1m}(\tau)\ddot{\eta}(\tau) + I_{2m}\eta(\tau) \\ & I_{1m} = -\sum_{i=1}^{n} \overline{\rho}^{(i)} \int_{\xi_{i-1}}^{\xi_i} \xi f_1^{(i)}(\xi) R_m^{(i)}(\xi) \mathrm{d}\xi \Big/ J_m \\ & I_{2m} = -c_i^2 \sum_{i=1}^{n} \overline{\rho}^{(i)} \int_{\xi_{i-1}}^{\xi_i} \xi f_2^{(i)}(\xi) R_m^{(i)}(\xi) \mathrm{d}\xi \Big/ J_m \\ & J_m = \frac{1}{2} \sum_{i=1}^{n} \{ c_{33D}^{(i)} \omega_m^{-2} [\xi R_{m,\xi}^{(i)}(\xi)]^2 \\ & \qquad - \mu_i^2 c_{33D}^{(i)} \omega_m^{-2} [R_m^{(i)}(\xi)]^2 + \overline{\rho}^{(i)} [\xi R_m^{(i)}(\xi)] \} \Big|_{\xi_{i-1}}^{\xi_i} \end{aligned} \tag{25}$$

The solution of Eq.(24) is

$$\begin{aligned} \Omega_m(\tau) = {} & \Omega_m(0)\cos\omega_m\tau + \Omega_{m,\tau}(0)\sin\omega_m\tau/\omega_m \\ & + \int_0^\tau q_m(p)\sin\omega_m(\tau - p)\mathrm{d}p \Big/ \omega_m \end{aligned} \tag{26}$$

Utilizing the initial conditions Eq.(7), Eq.(15) and the first equation in Eq.(12a) and by virtue of the orthogonal properties of Bessel functions, we can determine $\Omega_m(0)$ and $\Omega_{m,\tau}(0)$ as

$$\begin{aligned} & \Omega_m(0) = I_{1m}\eta(0) + I_{3m} \\ & \Omega_{m,\tau}(0) = I_{1m}\eta_{,\tau}(0) + I_{4m} \end{aligned} \tag{27}$$

where

$$\begin{aligned} & I_{3m} = \sum_{i=1}^{n} \overline{\rho}^{(i)} \int_{\xi_{i-1}}^{\xi_i} \xi u_0^{(i)}(\xi) R_m^{(i)}(\xi) \mathrm{d}\xi \Big/ J_m \\ & I_{4m} = \sum_{i=1}^{n} \overline{\rho}^{(i)} \int_{\xi_{i-1}}^{\xi_i} \xi v_0^{(i)}(\xi) R_m^{(i)}(\xi) \mathrm{d}\xi \Big/ J_m \end{aligned} \tag{28}$$

Considering $\eta(0)$, $\eta_{,\tau}(0)$ and $\eta(\tau)$ in Eqs.(26) and (27) are still unknown, we will determine them by means of the electric boundary condition Eq.(5b) and the electric potential continuity condition Eq.(6b). The substitution of Eqs.(15) and (19) into the first equation in Eq.(12a) yields

$$u^{(i)}(\xi,\tau)=\sum_{m=1}^{\infty}R_m^{(i)}(\xi)\Omega_m(\tau)+f_1^{(i)}(\xi)\eta(\tau) \tag{29}$$

Substituting Eq.(29) into Eq.(9) and then integrating it over the space interval $[\xi_{i-1}, \xi_i]$ (i=1, 2, …, n), we can derive Eq.(30) by summing the n equations and utilizing Eqs.(5b) and (6b).

$$\psi_1(\tau)=K_1\eta(\tau)+\sum_{m=1}^{\infty}K_{2m}\Omega_m(\tau) \tag{30}$$

where

$$\psi_1(\tau)=\phi_n(\tau)-\phi_0(\tau)$$

$$K_1=\frac{1}{\varepsilon_3^{(i)}}\sum_{i=1}^{n}\Big\{e_1^{(i)}\int_{\xi_{i-1}}^{\xi_i}\xi^{-1}f_1^{(i)}(\xi)\mathrm{d}\xi$$
$$+e_3^{(i)}[f_1^{(i)}(\xi_i)-f_1^{(i)}(\xi_{i-1})]-\ln(\xi_i/\xi_{i-1})\Big\} \tag{31}$$

$$K_{2m}=\frac{1}{\varepsilon_3^{(i)}}\sum_{i=1}^{n}\Big\{e_1^{(i)}\int_{\xi_{i-1}}^{\xi_i}\xi^{-1}R_m^{(i)}(\xi)\mathrm{d}\xi$$
$$+e_3^{(i)}[R_m^{(i)}(\xi_i)-R_m^{(i)}(\xi_{i-1})]\Big\}$$

From Eq.(30), we have

$$\psi_{1,\tau}(\tau)=K_1\eta_{,\tau}(\tau)+\sum_{m=1}^{\infty}K_{2m}\Omega_{m,\tau}(\tau) \tag{32}$$

Setting τ=0 in Eqs.(30) and (32) and utilizing Eq.(27), we can determine $\eta(0)$ and $\eta_{,\tau}(0)$. Then $\Omega_m(0)$ and $\Omega_{m,\tau}(0)$ become known. Applying the integration-by-parts method to integrate the term involving $\eta_{,\tau\tau}(p)$ in Eq.(26) and then substituting it into Eq.(30), we obtain

$$\psi(\tau)=N_1\eta(\tau)+\sum_{m=1}^{\infty}N_{2m}\int_0^{\tau}\eta(p)\sin\omega_m(\tau-p)\mathrm{d}p \tag{33}$$

where

$$\psi(\tau)=\psi_1(\tau)-\sum_{m=1}^{\infty}K_{2m}\Omega_{1m}(\tau)$$

$$N_1=K_1+\sum_{m=1}^{\infty}K_{2m}I_{1m},\quad N_{2m}=-\omega_m K_{2m}I_{1m} \tag{34}$$

$$\Omega_{1m}(\tau)=\Omega_m(0)\cos\omega_m\tau+\Omega_{m,\tau}(0)\sin\omega_m\tau/\omega_m$$
$$-I_{1m}[\eta_{,\tau}(0)\sin\omega_m\tau+\eta(0)\omega_m\cos\omega_m\tau]/\omega_m$$

It is noted that Eq.(33) is a Volterra integral equation of the second kind (Kress, 1989). Ding *et al.*(2003b) developed an efficient recursive formula for solving this equation. After $\eta(\tau)$ is obtained, the displacement, electric potential and stresses can then be completely determined.

NUMERICAL RESULTS AND ANALYSIS

Now we consider a three layer (n=3) piezoelectric infinite hollow cylinder. The material is taken as PZT-4 and PZT-5H. The non-dimensional inner radius, the radii of the interfaces as well as the outer radius are ξ_0=0.6, ξ_1=0.7, ξ_2=0.9, ξ_3=1.0. The hollow cylinder, which is formed as PZT-4/PZT-5H/PZT-4 and is at rest at τ=0, i.e. $u_0^{(i)}(\xi)=0$, $v_0^{(i)}(\xi)=0$ (i=1, 2,3) is subjected to a sudden constant electric potential at the external surface. That is

$$\phi_0(\tau)=0,\quad \phi_3(\tau)=H(\tau) \tag{35}$$

where $H()$ denotes the Heaviside function.

The time histories of the radial stress σ_r at each interface are shown in Fig.1. From the curves, we find that the non-dimensional radial stress peaks periodically and that the maximum tensile radial stress at the inner interface (ξ=0.7) is always larger than that at the outer interface (ξ=0.9). Fig.2 depicts the time histories of the hoop stress σ_θ at the inner surfaces of each layer. We can find that the maximum tensile hoop stress appears at the inner surface.

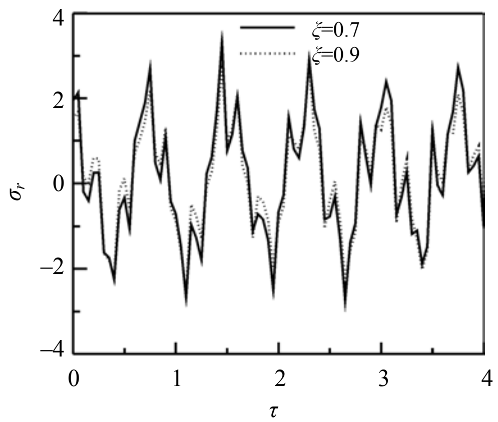

Fig.1 Histories of σ_r at the each interface

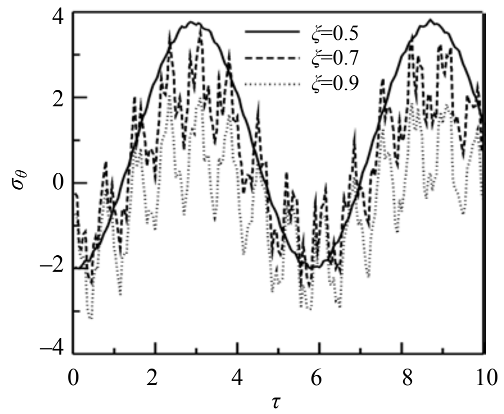

Fig.2 Histories of σ_θ at the inner surface of each layer

The distributions of the non-dimensional electric potential are shown in Fig.3. From the curves, we find that the calculated electric potentials at the inner surfaces are zero and those at the outer surfaces are 1, which are the right electric boundary conditions. The correctness of the numerical results is thus clarified in this respect.

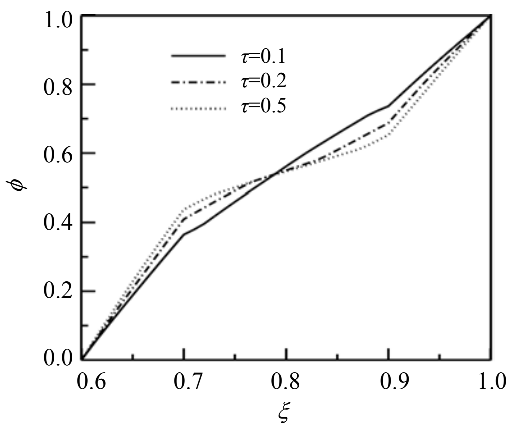

Fig.3 Distributions of ϕ at different times

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