Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Analysis of a functionally graded piezothermoelastic hollow cylinder*

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Received Feb. 3, 2005; revision accepted May 8, 2005

Abstract: A long thick-walled hollow cylinder of piezothermoelastic materials was studied in this work. The gradient property of the piezoelectric parameter g_{31} was taken into account. The theory of elasticity was applied to obtain the exact solutions of the cylinder subjected simultaneously to thermal and electric loadings. As an application, these solutions have been successfully used to study the inverse problems of the material. For comparison, numerical results have been carried out for both graded and double-layered cylinders.

Key words: FGM, Piezothermoelastic materials, Thick-walled hollow cylinder, Elastic analysis, Inverse problem, Parameter identification
doi:10.1631/jzus.2005.A0956 **Document code:** A **CLC number:** TU432

INTRODUCTION

Piezoelectric symmetric structures always play an important role in the use of piezoelectric materials (Olesiak and Pyryev, 1995; Ding *et al*., 2003; Wu *et al*., 2003). To improve the durability of this kind of piezoelectric structures, functionally gradient piezoelectric materials (FGPM) have been developed and used to produce these devices such as sensors and actuators (Hauke *et al*., 2000; Zhu *et al*., 1995). For the case when a rectangular plate made of functionally gradient piezothermoelectric materials subjected to different loadings, the analytical solutions were obtained by Zhong and Shang (2003). It has been realized that sometimes temperature loading is so far as to be the predominant reason of failure of smart structures (Birman, 1996; Tian and Shen, 2003). The fundamental solutions for a kind of density functionally gradient piezoelectric cantilever were investigated by Shi (2002) and Shi and Chen (2004).

In the present paper, a long thick-walled hollow cylinder made of piezothermoelastic materials is analyzed. And the nonlinear property of the piezoelectric parameter g_{31} is considered. Based on the theory of elasticity and by the use of the mixed solving method, the exact solutions for the cylinder subjected simultaneously to thermal and electric loadings were obtained. As an application, these solutions have been successfully used to study the inverse problem of the materials, i.e. to identify the pyroelectric constant q_3. At the end of the present work, some numerical results were carried out.

BASIC EQUATIONS UNDER PLANE STRAIN CONDITION

Let us consider a long cylinder (Fig.1) subjected to symmetric loading on any cross section, which can be considered under plane strain condition. The polar coordinate system (r, θ) is introduced in the present analysis. Let ε_{ij}, σ_{ij}, D_i and E_i denote the components of strain, stress, induction and the elec-

* Project supported by the National Natural Science Foundation of China (No. 50272003) and the Teaching and Research Award Fund for Outstanding Young Teachers in Higher Education Institutions of MOE, China

tric field strength of the piezoelectric media, respectively. The constitutive equations of the piezothermoelastic material under plane strain condition can be expressed in the common formation (Wu *et al.*, 2003).

$$\begin{cases}\varepsilon_\theta = s_{11}\sigma_\theta + s_{13}\sigma_r + g_{31}D_r - \mu_{11}T \\ \varepsilon_r = s_{13}\sigma_\theta + s_{33}\sigma_r + g_{33}D_r - \mu_{33}T \\ \gamma_{r\theta} = s_{44}\tau_{r\theta} + g_{15}D_\theta\end{cases} \tag{1.1}$$

$$\begin{cases}E_\theta = -g_{15}\tau_{r\theta} + \zeta_{11}D_\theta \\ E_r = -g_{31}\sigma_\theta - g_{33}\sigma_r + \zeta_{33}D_r - q_3T\end{cases} \tag{1.2}$$

where s_{ij}, g_{ij} and ζ_{ij} are the coefficients of the effective elastic compliance, piezoelectric and dielectric impermeability, respectively; T is the temperature rise; μ_{ii} and q_3 are the thermal strain and pyroelectric coefficients of the material, respectively. Without consideration of body force and body charge, the equilibrium equations can be given as

$$\begin{cases}\dfrac{\partial\sigma_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial\tau_{r\theta}}{\partial\theta} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0 \\ \dfrac{\partial\tau_{r\theta}}{\partial r} + \dfrac{1}{r}\dfrac{\partial\sigma_\theta}{\partial\theta} + \dfrac{2\tau_{r\theta}}{r} = 0\end{cases} \tag{2.1}$$

$$\frac{1}{r}\frac{\partial D_\theta}{\partial\theta} + \frac{\partial D_r}{\partial r} + \frac{D_r}{r} = 0 \tag{2.2}$$

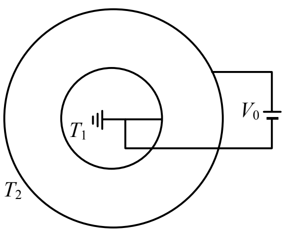


Fig.1 The graded hollow cylinder subjected to thermal and electric loading

The temperature field is governed by Fourier's heat conduction equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2} = 0 \tag{3}$$

The components of strain and electric field strength are related to the displacement (u, w) and electrical potential ϕ by the following equations

$$\begin{cases}\varepsilon_\theta = \dfrac{u_r}{r} + \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial\theta} \\ \varepsilon_r = \dfrac{\partial u_r}{\partial r} \\ \gamma_{r\theta} = \dfrac{1}{r}\dfrac{\partial u_r}{\partial\theta} + \dfrac{\partial u_\theta}{\partial r} - \dfrac{u_\theta}{r}\end{cases} \tag{4.1}$$

$$\begin{cases}E_\theta = -\dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta} \\ E_r = -\dfrac{\partial\phi}{\partial r}\end{cases} \tag{4.2}$$

Based on the theory of elasticity, the compatibility equation expressed by the components of strain is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)\varepsilon_\theta + \left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)\varepsilon_r = \left(\frac{1}{r^2}\frac{\partial}{\partial\theta} + \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\right)\gamma_{r\theta} \tag{5}$$

ELASTIC SOLUTIONS OF A FUNCTIONALLY GRADED HOLLOW CYLINDER UNDER THE COUPLED LOADINGS

It is known that the piezoelectric parameter g_{31} plays an important role in the judgment on the behavior of piezoelectric materials and the performance of piezoelectric products. In this section, an elastic analysis of a functionally graded thick-walled hollow cylinder with the inner radius R_1 and outer radius R_2 as shown in Fig.1 will be studied. In the following analysis the piezoelectric parameter g_{31} is assumed varying quadratically in radial direction while the other material parameters are assumed to be constants, i.e.

$$g_{31} = m_2r^2 + m_1r + m_0 \tag{6}$$

where m_i (i=0,1,2) are material constants.

For the functionally graded thick-walled hollow cylinder subjected to uniform and symmetrically coupled thermal and electric loadings as shown in Fig.1, the exact solutions can be found by using the theory of elasticity. The Airy stress function method is used to find the mechanical field of the piezoelectric hollow cylinder, and the stress function φ_i and the electric potential ϕ_i are introduced for layer i. For symmetry, the components of stress and electric field strength can be expressed as

$$\sigma_\theta = \varphi''(r),\ \sigma_r = \frac{1}{r}\varphi'(r),\ \tau_{r\theta} = 0 \tag{7}$$

$$E_\theta = 0,\ E_r = -\phi'(r) \tag{8}$$

Use of Eq.(1) yields

$$D_\theta = 0,\ \gamma_{r\theta} = 0 \tag{9}$$

It is easily found from Eq.(3) that the temperature field is related to temperature change only. So the temperature field should be found first. Supposing the piezoelectric hollow cylinder is homogeneously heated at the surface, and the temperature rise at the inner and outer surfaces keeps constant T_1 and T_2, respectively. That means we have the following thermal boundary conditions

$$T(r=R_1)=T_1,\quad T(r=R_2)=T_2 \tag{10}$$

For symmetry, the heat conduction Eq.(3) can be simplified as

$$T''(r)+T'(r)/r=0 \tag{11}$$

The solution of the above equation is

$$T(r)=t_1\ln r+t_0 \tag{12}$$

where

$$\begin{cases} t_0 = \dfrac{T_1\ln R_2 - T_2\ln R_1}{\ln R_2 - \ln R_1} \\ t_1 = \dfrac{T_2 - T_1}{\ln R_2 - \ln R_1} \end{cases} \tag{13}$$

Having the distribution of temperature, we will try to find the mechanical and electrical fields. Substituting Eqs.(7) and (8) into Eq.(2), it can be found that Eq.(2.1) is automatically satisfied and Eq.(2.2) becomes

$$\frac{\mathrm{d}D_r}{\mathrm{d}r}+\frac{D_r}{r}=0 \tag{14}$$

The general solution of the above equation is

$$D_r = a_0/r \tag{15}$$

where a_0 is an unknown constant to be determined. Keeping Eq.(1) in mind, the compatibility Eq.(5) can be rewritten as

$$\begin{gathered} s_{11}\varphi^{(4)}(r)r^3 + 2s_{11}\varphi'''(r)r^2 - s_{33}\varphi''(r)r + s_{33}\varphi'(r) \\ + 2a_0m_2r^2 + (\mu_{33}-\mu_{11})t_1r + g_{33}a_0 = 0 \end{gathered} \tag{16}$$

Integration of the above equation yields

$$\varphi'(r) = a_2r^2 + a_1r + br\ln r + c_1r^s + c_2r^{-s} + c \tag{17}$$

where

$$\begin{cases} a_2 = \dfrac{2m_2}{s_{33}-4s_{11}}a_0 \\ b = \dfrac{t_1(\mu_{33}-\mu_{11})}{s_{33}-s_{11}} \\ c = -\dfrac{g_{33}}{s_{33}}a_0 \end{cases} \tag{18}$$

in which a_1, c_1, c_2, c are unknown constants to be determined. After substituting Eq.(17) into Eq.(7), the stress components can be expressed as

$$\begin{cases} \sigma_\theta = 2a_2r + a_1 + b\ln r + b + c_1sr^{s-1} - c_2sr^{-s-1} \\ \sigma_r = a_2r + a_1 + b\ln r + c_1r^{s-1} + c_2r^{-s-1} + cr^{-1} \\ \tau_{r\theta} = 0 \end{cases} \tag{19}$$

And from Eq.(1.1), the strain components can be expressed as

$$\begin{cases} \varepsilon_\theta = (2s_{11}a_2 + s_{13}a_2 + m_2a_0)r + [(s_{11}+s_{13})a_1 + s_{11}b + m_1a_0 \\ \quad -\mu_{11}t_0] + [(s_{11}+s_{13})b - \mu_{11}t_1]\ln r + (s_{13}c + m_0a_0)r^{-1} \\ \quad + (s_{11}s + s_{13})c_1r^{s-1} - (s_{11}s - s_{13})c_2r^{-s-1} \\ \varepsilon_r = (2s_{13}+s_{33})a_2r + (s_{13}s+s_{33})c_1r^{s-1} - (s_{13}s - s_{33})c_2r^{-s-1} \\ \quad + [(s_{13}+s_{33})a_1 + s_{13}b - \mu_{33}t_0] + [(s_{13}+s_{33})b - \mu_{33}t_1]\ln r \\ \gamma_{r\theta} = 0 \end{cases} \tag{20}$$

Further, the displacement components can be obtained by the use of Eq.(1) as

$$\begin{cases} u_r = \frac{1}{2}(2s_{13}+s_{33})a_2r^2+[(s_{13}+s_{33})a_1+s_{13}b-\mu_{33}t_0]r \\ \quad +[(s_{13}+s_{33})b-\mu_{33}t_1](\ln r-1)r+\frac{s_{13}s+s_{33}}{s}c_1r^s \\ \quad +\frac{s_{13}s-s_{33}}{s}c_2r^{-s}+(s_{13}c+m_0a_0)+A_1\sin\theta+A_2\cos\theta \\ u_\theta = A_1\cos\theta-A_2\sin\theta+Br \end{cases} \tag{21}$$

where A_1, A_2 and B are unknown constants. Taking the condition of single-valued displacement into account, we can obtain

$$a_1=\frac{m_1a_0+(s_{11}+s_{33})b+(\mu_{33}-\mu_{11})t_0-\mu_{33}t_1}{s_{33}-s_{11}} \tag{22}$$

To find the electric potential Eq.(1.2) is expressed as

$$\begin{cases} E_\theta=0 \\ E_r=-\sum_{i=1}^{4}K_ir^{i-1}-\sum_{i=0}^{2}\Big[b(m_i+d_i)r^i\ln r \\ \quad +c_1(m_is+d_i)r^{s+i-1}-c_2(m_is-d_i)r^{-s+i-1}\Big] \\ \quad -(g_{33}c-\varsigma_{33}a_0)r^{-1}-q_3(t_1\ln r+t_0) \end{cases} \tag{23}$$

where

$$\begin{cases} K_i=2a_2m_{i-2}+(a_1+b)m_{i-1}+g_{33}a_i \\ d_0=g_{33},\ d_1=d_2=d_3=0 \\ m_{-1}=0,\ a_3=a_4=0 \\ i=1,2,3,4 \end{cases} \tag{24}$$

By the use of Eq.(4), the electric potential can be obtained as follows:

$$\begin{aligned} \phi=&\sum_{i=1}^{4}\frac{1}{i}K_ir^i++q_3t_1(\ln r-1)r+q_3t_0r \\ &\times\sum_{i=0}^{2}\left\{b\left[\frac{\ln r}{i+1}-\frac{1}{(i+1)^2}\right](m_i+d_i)r^{i+1}+c_1\frac{m_is+d_i}{s+i}r^{s+i}\right. \\ &\left.-c_2\frac{m_is-d_i}{-s+i}r^{-s+i}\right\}+(g_{33}c-\varsigma_{33}a_0)\ln r+F \end{aligned} \tag{25}$$

where F is another unknown constant.

It is obvious that once the unknown parameters a_0, a_1, c_1, c_2, c, A_1, A_2, B and F are determined by using some suitable boundary conditions, the distributions of all the mechanical and electrical fields in the cylinder can be found.

For the loading case shown in Fig.1, the electric boundary conditions can be expressed as follows:

$$\begin{cases} \phi_2(r=R_2)=V_0 \\ \phi_1(r=R_1)=0 \end{cases} \tag{26}$$

The mechanical boundary conditions are

$$\begin{cases} \sigma_r(r=R_1)=\sigma_r(r=R_2)=0 \\ \tau_{r\theta}(r=R_1)=\tau_{r\theta}(r=R_2)=0 \end{cases} \tag{27}$$

Then all the above unknown constants can be determined as follows

$$\begin{cases} c_1=\frac{H_1}{H},\ c_2=\frac{H_2}{H},\ a_0=\frac{H_3}{H}, \\ c=\alpha a_0,\ a_2=\alpha_2a_0,\ a_1=\alpha_1a_0+\alpha_0 \\ F=-\sum_{i=1}^{4}\frac{K_i}{i}R_1^i-(g_{33}c-\varsigma_{33}a_0)\ln R_1 \\ \quad -\sum_{i=0}^{2}\left\{b\left[\frac{\ln R_1}{i+1}-\frac{1}{(i+1)^2}\right](m_i+d_i)R_1^{i+1}\right. \\ \quad \left.+c_1\frac{m_is+d_i}{s+i}R_1^{s+i}-c_2\frac{m_is-d_i}{-s+i}R_1^{-s+i}\right\} \\ \quad -q_3t_1(\ln R_1-1)R_1-q_3t_0R_1 \end{cases} \tag{28}$$

where

$$\begin{cases} H=\begin{vmatrix} R_1^{s-1} & R_1^{-s-1} & Y_1 \\ R_2^{s-1} & R_2^{-s-1} & Y_2 \\ W_1 & W_2 & W_3 \end{vmatrix} \\ H_1=\begin{vmatrix} G_1 & R_1^{-s-1} & Y_1 \\ G_2 & R_2^{-s-1} & Y_2 \\ G_3 & W_2 & W_3 \end{vmatrix} \\ H_2=\begin{vmatrix} R_1^{s-1} & G_1 & Y_1 \\ R_2^{s-1} & G_2 & Y_2 \\ W_1 & G_3 & W_3 \end{vmatrix} \\ H_3=\begin{vmatrix} R_1^{s-1} & R_1^{-s-1} & G_1 \\ R_2^{s-1} & R_2^{-s-1} & G_2 \\ W_1 & W_2 & G_3 \end{vmatrix} \end{cases} \tag{29}$$

In the above expressions, besides introducing the denotations $\Delta[f(R)]_{ij}=f(R_i)-f(R_j)$ such as $\Delta[R^i]_{12}=R_1^i-R_2^i$, the following symbols are also used

$$\alpha_2=\frac{2m_2}{s_{33}-4s_{11}},\quad \alpha_1=\frac{m_1}{s_{33}-s_{11}},\quad \alpha=-\frac{g_{33}}{s_{33}}$$
$$\alpha_0=\frac{(s_{11}+s_{33})b+(\mu_{33}-\mu_{11})t_0-\mu_{33}t_1}{s_{33}-s_{11}}$$
$$Y_i=\alpha_2R_i+\alpha_1+\alpha R_i^{-1}\quad (i=1,2)$$
$$W_1=\sum_{i=0}^{2}\frac{m_is+d_i}{s+i}\Delta[R^{s+i}]_{21}$$
$$W_2=-\sum_{i=0}^{2}\frac{m_is-d_i}{-s+i}\Delta[R^{-s+i}]_{21}$$
$$W_3=-(\alpha g_{33}+\varsigma_{33})\Delta[\ln R]_{21}+\sum_{i=1}^{4}\frac{1}{i}[2(m_{i-2}+d_{i-2})\alpha_2+(m_{i-1}+d_{i-1})\alpha_1]\Delta[R^i]_{21}$$
$$G_i=-\alpha_0-b\ln R_i\quad (i=1,2)$$
$$G_3=\sum_{i=1}^{3}\left[\frac{(m_{i-1}+d_{i-1})}{i}\alpha_0+m_{i-1}b\right]\Delta[R^i]_{12}\times\sum_{i=0}^{2}\Delta\left\{b\left[\frac{\ln R}{i+1}-\frac{1}{(i+1)^2}\right](m_i+d_i)R^{i+1}\right\}_{12}+q_3t_1\Delta[(\ln R-1)R]_{12}+q_3t_0\Delta[R]_{12}+V_0 \tag{30}$$

Now, all the unknown constants when a functionally graded thick-walled hollow cylinder is subjected to coupled thermal and electric loadings are determined. Therefore all the distributions of stress, strain, displacement and the electric potential of the functionally graded hollow cylinder can be found by the use of Eqs.(19), (20), (21) and (25), respectively.

PARAMETER IDENTIFICATION

Parameter identification as a kind of inverse problem plays an important role in precisely describing the internal behavior of materials. How to identify a non-homogeneous material has received more and more attention (Fang, 1999). In this section, the method to identify the pyroelectric constant q_3 will be discussed.

For the case the cylinder loaded by a temperature rise only at the inner and outer surfaces, the electric potential at the outer surface can be given based on the results obtained in the last section as

$$\phi_2=\phi(r=R_2)=P+\lambda q_3 \tag{31}$$

where

$$\lambda=t_1\Delta[(\ln R-1)R]_{21}+t_0\Delta[R]_{21}$$
$$P=\sum_{i=1}^{4}\frac{1}{i}K_i\Delta[R^i]_{21}+g_{33}c\Delta[\ln R]_{21}+\sum_{i=0}^{2}b\Delta\left[\left[\frac{\ln R}{i+1}-\frac{1}{(i+1)^2}\right](m_i+d_i)R^{i+1}\right]_{21}+\sum_{i=0}^{2}\left(c_1\frac{m_is+d_i}{s+i}\Delta[R^{s+i}]_{21}+c_2\frac{m_is-d_i}{-s+i}\Delta[R^{-s+i}]_{21}\right) \tag{32}$$

So we have

$$q_3=(\phi_2-P)/\lambda \tag{33}$$

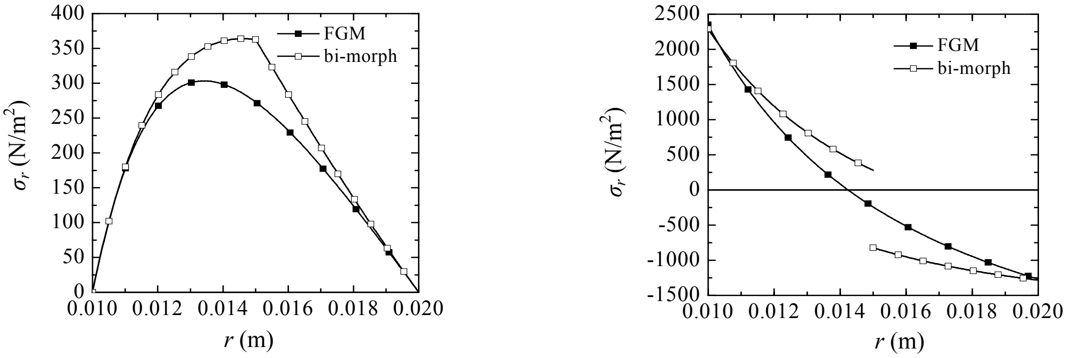
The above formula shows that once the electric potential at the outer surface of the cylinder is obtained from a test, the pyroelectric constant q_3 can be identified.

NUMERICAL RESULTS AND DISCUSSIONS

To give a clear explanation, numerical results have been carried out in this section. For comparison, two kinds of cylinders made of cadmium selenide will be considered. One is a functionally graded cylinder, another is a double-layered cylinder. It is assumed that both cylinders have the same geometric sizes. The radius of the inner and outer surfaces of the cylinders is taken as 10 mm and 20 mm, respectively. For simplicity, the piezoelectric parameter g_{31} of the functionally graded hollow cylinder is assumed to be varying linearly as $g_{31}=m_1r+m_0$. The piezoelectric parameters g_{31} of the inner and outer layers of the doubled-layered cylinder are taken as -41.66×10^{-3} m^2/C and -70.00×10^{-3} m^2/C, respectively. So we can get $m_1=-28.33\times10^{-1}$ and $m_0=-13.33\times10^{-3}$. Moreover, the other material parameters of the cylinder are listed in Table 1. For both cylinders subjected to an electric potential V=100 V, the distributions of normal stresses σ_r and σ_θ are plotted in Fig.2. These figures show that the internal stresses are drastically reduced in the materials and devices with functionally graded properties.

Table 1 Some material parameters of cadmium selenide (Kapuria *et al.*, 1996)

Elastic constant ($\times10^{-12}$ m^2/N)				Piezoelectric constant ($\times10^{-3}$ m^2/C)		Dielectric impermeability constant ($\times10^{9}$ m/F)		Thermal strain constant ($\times10^{-7}$ 1/K)		Pyroelectric coefficient ($\times10^{3}$ N/(K·C))
s_{11}	s_{13}	s_{33}	s_{44}	g_{33}	g_{15}	ζ_{11}	ζ_{33}	μ_{11}	μ_{33}	q_3
23.20	−5.38	16.68	74.62	83.25	−12.48	11.91	10.62	−42.50	−27.49	−37.10



(a) (b)

Fig.2 The normal stress σ_r (a) and σ_θ (b) of the cylinder in different analytical models at V=100 V

CONCLUSION

Based on the theory of elasticity, the present analysis provides some exact solutions for the functionally graded piezothermoelastic hollow cylinder under coupled thermal and electric loadings. As a kind of inverse problems, the solutions obtained in the present paper can be used to determine the pyroelectric constant q_3. Numerical results showed that the stress mismatch can be avoided in FGM materials and structures.

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