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Equivalent thickness of materials of fused silica and stainless steel in the flow of microtubes*

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Abstract: The deviation of flow characteristics from the predictions of the conventional theory for microtubes was attributed to the change of fluid viscosity resulted from the interactions between the molecules on solid wall and in fluid. The degree of this departure is dependent on the microtubes materials. A concept of equivalent thickness with which conventional theory can be used to predict the flow in microtubes without modifying the fluid viscosity was put forward. The values of equivalent thickness for fused silica and stainless steel materials were determined as 1.8 μm and 1.5 μm, respectively, by repeated numerical simulation.

Key words: Microtubes, Flow, Fused silica, Stainless steel, Equivalent thickness
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Over the past decade, significant attention has been paid to flow in microtubes due to the development in micro fluidic devices and systems. The understanding of flow characteristics such as pressure drop is important in the design and process control of microfluidic devices.

However, experimental evidence indicates significant departure of flow characteristics from the predictions of conventional theory for smaller diameter microtubes. Pfahler *et al.*(1991) observed that as the channel size decreased, the apparent viscosity began to decrease from the theoretical value for a given pressure drop. Qu *et al.*(2000) found that pressure gradient and flow friction in microchannels were higher than those predicted by conventional theory. Mala and Li (1999) indicated that for smaller microtubes, the pressure gradients were up to 35% higher than those predicted by conventional theory.

The flow behavior in microtubes is dependent on the microtubes materials. Mala and Li (1999) found that a fused silica microtube requires a higher pressure gradient than a stainless steel microtube for the same flow rate and the same diameter.

We attribute the departure of flow characteristics from the predictions of conventional theory and the material dependence of the flow behavior in microtubes to the interactions between the molecules on solid wall and in fluid. As the size of microtube decreases, the intermolecular force becomes stronger than that in macrotube, which leads to the change of fluid viscosity. Israelachvili (1986) and Gee *et al.*(1990) showed that the viscosity of liquids within a very thin film adjacent to a solid wall can be much different from that in bulk region. The viscosity in this near-wall layer can be higher or lower than the viscosity in the bulk region, depending on the property of the fluid and the wall materials. Li *et al.*(2000) showed that the change of thermal conductivity of fluid in 'wall-adjacent layer' would have significant influence on the heat transfer if the passage is small

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enough.

Generally speaking, the intermolecular force acts over a short range, for example, the action range of two molecules is about 0.3~0.5 nm which is approximately 2~3 molecular layers of water. However, there are billions of molecules on solid wall, and the cumulative effects of these intermolecular forces can extend over a rather long range. Therefore, the effect of intermolecular force in microtubes cannot be neglected.

The above analyses shows that intermolecular force should be introduced to modify the viscosity of fluid when the flow in microtubes is predicted using conventional theory. But the difficulty in this approach lies in relating the intermolecular force to the fluid viscosity. Therefore, the aim of this letter is to put forward a concept of equivalent thickness with which conventional theory can be used to predict the flow in microtubes without modifying the fluid viscosity.

Intermolecular force causes the velocity distribution in microtubes to be different from that predicted by conventional theory as shown in Fig.1 in which the solid line represents the parabolic velocity distribution predicted by conventional theory, and the dash dot line represents the velocity distribution in microtubes for the hydrophilic material. It can be seen that the velocity in microtubes near the solid wall is much smaller than that predicted by conventional theory. This means that the effective diameter of microtube is smaller than its real one. Based on this phenomenon we introduce an equivalent thickness δ to denote the reduction of tube diameter as shown in Fig.1 where the effective tube diameter d is defined as $d=D-2\delta$ with D being the real tube diameter. The pressure drop when fluid flows through a microtube with diameter D, in which the intermolecular force is taken into consideration, is the same as that through a microtube with diameter d without considering the intermolecular force. The mass fluxes in these two situations are the same. Therefore, for predicting the pressure drop in a flow of microtube with diameter D, we can predict the pressure drop in a flow of microtube with effective diameter d by the conventional theory.

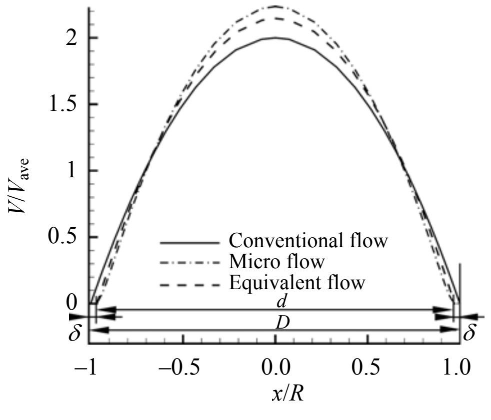


Fig.1 Sketch map of equivalent thickness

The effective diameter d is dependent on the equivalent thickness δ which is related to the interaction between the molecules on the solid wall and in the fluid. The stronger the molecular interaction is, the larger is the value of equivalent thickness δ. The value of δ for hydrophilic material is larger than that for lyophobic material. Microtubes made from the same material have the same δ value. The determined δ for a definite material can be used to predict the pressure drop in microtubes with same materials and for any diameters, using conventional theory, which makes the prediction of pressure drop in microtubes convenient.

In this paper, the δ values of fused silica and stainless steel materials are determined by repeated numerical simulation. For incompressible and steady flow, the conventional continuity and momentum equations are:

$$\partial u_j / \partial x_j = 0, \tag{1}$$

$$u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right). \tag{2}$$

where u is the velocity, p is the pressure, μ is the viscosity of the fluid. The SIMPLE algorithm is used to solve Eqs.(1) and (2). The grid independence of the results and code verifications were examined before further calculations were carried out. It was found that 52×8 grids as shown in Fig.2 provide very good accuracy, with velocity error of less than 0.01%. It can be noted that more grid lines distributed near the wall, which enable capture of the expected rapid change or much steeper velocity gradient. The present calculation was performed for fully developed flow, with periodic flow field along the flow direction. Hence it is sufficient to use eight nodes along the flow direction.

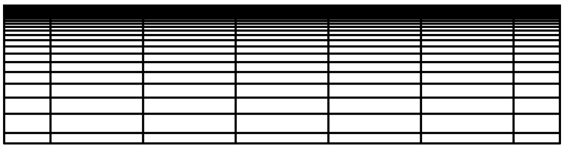


Fig.2 Grid system of 52×8 in the flow

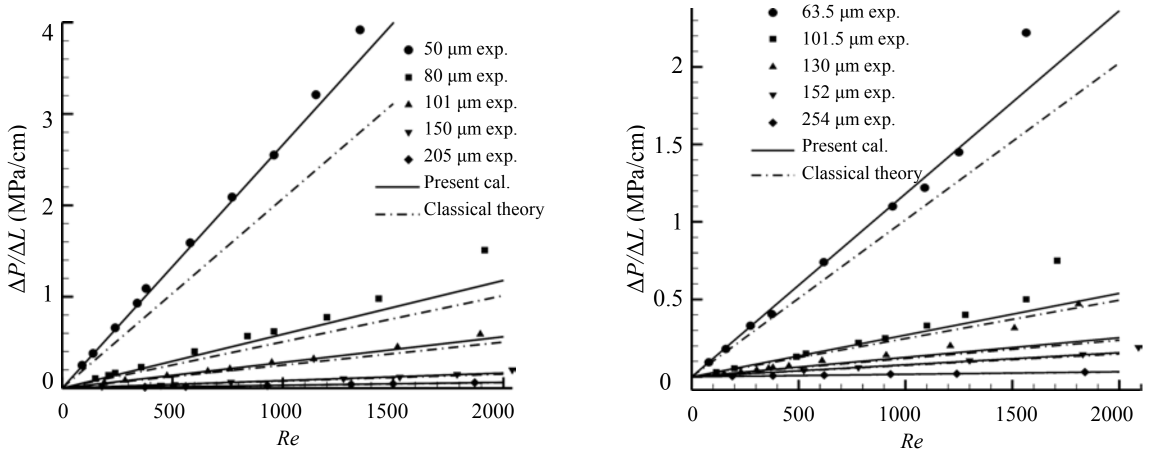
Periodical boundary condition is applied at the inlet and outlet sections. A non-slip velocity boundary condition is used on the wall, and a symmetric boundary condition is applied in the centre line.

Finally the equivalent thickness δ of material was determined as 1.8 μm for fused silica and 1.5 μm for stainless steel, respectively, according to the materials and experimental data (Mala and Li, 1999).

Taking the value of δ as 1.8 μm and 1.5 μm for fused silica and stainless steel, we compute the pressure drop in fused silica microtubes with 50 μm to 205 μm diameter and in stainless steel microtubes with 63.5 μm to 254 μm diameter, respectively. The results are shown in Fig.3 in which the experimental data (Mala and Li, 1999) are also given.

The figures show that the calculated pressure drops agree well with the experimental ones for microtubes with different diameters. Besides, Fig.3 shows that for the flow in microtubes with relative large diameter (for example, D>150 μm), the differences of the pressure drop predicted with and without considering the equivalent thickness are insignificant. It indicates that the effect of intermolecular force can be neglected for the tubes with relative large diameter because the equivalent thickness is small compared to the tube diameter.

In conclusion, it is very convenient to calculate the pressure drop with good accuracy in microtubes based on the concept of equivalent thickness. The value of equivalent thickness is 1.8 μm and 1.5 μm for fused silica and stainless steel materials, respectively.



(a) (b)

Fig.3 Comparison of calculated results and experimental data for fused silica tube (a) and for stainless steel tube (b)

References

Gee, M.L., McGuiggan, P.M., Israelachvili, J.N., Homola, A.M., 1990. Liquid to solidlike transition of molecularly thin films under shear. *J. Chem. Phys.*, **93**(3):1895-1906.

Israelachvili, J.N., 1986. Measurement of the viscosity of liquids in very thin films. *J. Colloid and Interface Science*, **110**(1):263-271.

Li, J.M., Wang, B.X., Peng, X.F., 2000. 'Wall-adjacent layer' analysis for developed-flow laminar heat transfer of gases in microchannels. *Int. J. Heat Mass Transfer*, **43**:839-847.

Mala, G.M., Li, D.Q., 1999. Flow characteristics of water in microtubes. *Int. J. Heat Fluid Flow*, **20**:142-148.

Pfahler, J., Harley, J., Bau, H., Zemel, J., 1991. Gas and liquid flow in small channels. *ASME DSC*, **32**:49-60.

Qu, W.L., Mala, G.M., Li, D.Q., 2000. Pressure-driven water flows in trapezoidal silicon microchannels. *Int. J. Heat Mass Transfer*, **43**:353-364.