Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Decomposition method for solving parabolic equations in finite domains

INC Mustafa
(*Department of Mathematics, Firat University, Elazig 23119, Turkey*)
E-mail: minc@firat.edu.tr

Received Feb. 30, 2005; revision accepted May 25, 2005

Abstract: This paper presents a comparison among Adomian decomposition method (ADM), Wavelet-Galerkin method (WGM), the fully explicit (1,7) finite difference technique (FTCS), the fully implicit (7,1) finite difference method (BTCS), (7,7) Crank-Nicholson type finite difference formula (C-N), the fully explicit method (1,13) and 9-point finite difference method, for solving parabolic differential equations with arbitrary boundary conditions and based on weak form functionals in finite domains. The problem is solved rapidly, easily and elegantly by ADM. The numerical results on a 2D transient heat conducting problem and 3D diffusion problem are used to validate the proposed ADM as an effective numerical method for solving finite domain parabolic equations. The numerical results showed that our present method is less time consuming and is easier to use than other methods. In addition, we prove the convergence of this method when it is applied to the nonlinear parabolic equation.

Key words: Adomian decomposition method (ADM), Adomian polynomials, Parabolic differential equations
doi:10.1631/jzus.2005.A1058 **Document code:** A **CLC number:** TP391

INTRODUCTION

The decomposition method was introduced by Adomian (1989; 1994) in the 1980's for solving linear and nonlinear functional equations (algebraic, differential, partial differential equations (PDEs) and systems, integral, differential-delay, integro-differential equations, etc.) (Adomian, 1989; 1994; Guellal and Cherruault, 1995; Adomian *et al*., 1996; Laffez and Abbaoui, 1996; Ndour *et al*., 1996; Guellal *et al*., 1997; Abbaoui and Cherruault, 1999; Adjedj, 1999). This method leads to computable, accurate, approximately convergent solutions to linear and nonlinear deterministic and stochastic operator equations. The solution can be verified to any degree of approximation.

Over the last twenty years, the Adomian decomposition approach has been applied to obtain formal solutions to a wide class of both deterministic and stochastic PDEs. These works are below. Guellal and Cherruault (1995) used Adomian's technique for solving an elliptical boundary value problem with an auxiliary condition. Adomian *et al*.(1996) solved mathematical models of the dynamic interaction of immune response with a population of bacteria, viruses, antigens or tumor cells which had been modeled as systems of nonlinear differential equations or delay-differential equations by the ADM. Laffez and Abbaoui (1996) studied a model of thermic exchanges in a drilling well which was solved with the decomposition method. Ndour *et al*.(1996) presented an example of an interaction model between two species. Guellal *et al*.(1997) used the decomposition method for solving differential systems in physics. They gave a comparison between the Runge-Kutta method and the decomposition method. Abbaoui and Cherruault (1999) used the decomposition method for solving the Cauchy problem without using the canonical form of Adomian. They also gave a proof of convergence by using a new formulation of the Adomian polynomials and compared the ADM with the Picard method. Adjedj (1999) used Adomian's scheme for solving differential systems for modeling the HIV immune dynamics. Sanchez *et al*.(2000) investigated the weaknesses of the thin-sheet approximation method and proposed a higher-order development allowing increase in the range of convergence and preserving the nonlinear dependence of the variables.

The convergence of this method was investigated by Cherruault and co-workers. Cherruault (1989) proposed a new definition of the method and insisted that it is possible to prove the convergence of the decomposition method. Cherruault and Adomian (1993) proposed a new convergence proof of Adomian's method based on properties of convergent series. Abbaoui *et al*.(2001) obtained a new approach to the decomposition method in a relatively more natural way than that in the classical presentation. Lesnic (2002a) investigated the convergence of Adomian's method to periodic temperature fields in heat conductors.

The model considered here is a parabolic equation in a 2D region Ω enclosed by boundary Γ (Ho and Yang, 2001)

$$c\frac{\partial u}{\partial t}-\frac{\partial u}{\partial x}\left(a_{11}\frac{\partial u}{\partial x}\right)-\frac{\partial u}{\partial y}\left(a_{22}\frac{\partial u}{\partial y}\right)+a_0u=f(x,y,t) \quad (1)$$

with the boundary conditions being

$$\begin{aligned} &\hat{u}\big|_{\Gamma_1}=u_0 \qquad (t\ge 0),\\ &\hat{q}_n\big|_{\Gamma_2}=q_n \qquad (t\ge 0),\\ &\Gamma=\Gamma_1\cup\Gamma_2, \end{aligned} \quad (2)$$

where

$$\hat{q}_n=a_{11}\frac{\partial u}{\partial x}n_x+\frac{\partial u}{\partial y}n_y, \quad (3)$$

and initial condition is

$$u(x,y,0)=u_0(x,y), \quad (4)$$

where t denotes time, and $c, a_{11}, a_{22}, a_0, \hat{u}, u_0, f$ and $\hat{q}_n$ are given functions of position and time.

Ho and Yang (2001) employed Wavelet-Galerkin method to obtain a numerical solution to the parabolic equations given by Eq.(1). Problems of this type include chemical diffusion, thermo elasticity, heat conduction processes, population dynamics, control theory, medical science, biochemistry and certain biological processes (Camon and Hoek, 1986; Camon *et al*., 1990; Camon and Matheson, 1993; Capso and Kunisch, 1988; Wang, 1990).

This problem was solved by Dehghan (2003b) by various methods which were parallel techniques, the fully explicit (1, 7) finite difference technique, the fully implicit (7, 1) finite difference method, the (7, 7) Crank-Nicholson type finite difference method (Deghan, 2002; 2003a). In these studies, the calculations were difficult and time consuming. In addition, when the unknown u was determined, approximate solutions are tried to find by using only the initial condition with the decomposition method, without using any transformation in less calculation steps.

In this work, we will describe and adapt Adomian's decomposition method to obtain an approximate solution for Eq.(1). As we will see, the method converges rapidly. The balance of this paper is as follows: In Section 2, we will give an analysis of ADM for the problem; in Section 3 we will present test 2D and 3D parabolic differential equations in finite domains to compare the numerical results of our method with those of other methods; in Section 4 we will prove the convergence of this method applied to the nonlinear parabolic equation and in the last Section we give some conclusions.

ANALYSIS

We first consider Eq.(1) in operator form

$$L_t u=L_x u+L_y u+u+f(x,y,t) \quad (5)$$

where L_t, L_x and L_y are the linear differential operators which we define in the following form

$$L_t=\frac{\partial}{\partial t},\ L_x=\frac{\partial^2}{\partial x^2},\ L_y=\frac{\partial^2}{\partial y^2}. \quad (6)$$

Assuming that the inverse of the operator L_t^{-1} exists and can conveniently be taken as the one-fold definite integral with respect to t from 0 to t. That is

$$L_t^{-1}(\cdot)=\int_0^t(\cdot)\,\mathrm{d}t. \quad (7)$$

Application of the inverse operator L_t^{-1} to Eq.(5) yields

$$L_t^{-1}L_t u = L_t^{-1}L_x u + L_t^{-1}L_y u + L_t^{-1}u + L_t^{-1}[f(x,y,t)] \quad (8)$$

and

$$L_t^{-1}L_t u = u(x,y,0). \quad (9)$$

Therefore, it follows that

$$u(x,y,t) = u(x,y,0) + L_t^{-1}(L_x u + L_y u + u) + L_t^{-1}[f(x,y,t)] \quad (10)$$

We obtain the zero-th component as

$$u_0(x,y,t) = u(x,y,0) + L_t^{-1}[f(x,y,t)] \quad (11)$$

which is defined by a term that arises from the initial condition. The unknown function $u_n(x,y,t)$, $n \geq 1$, is decomposed into a sum of components defined by the decomposition series

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t). \quad (12)$$

With the zeroth component as defined above, the remaining components $u_n(x,y,t)$, $n \geq 1$, can be completely determined in such a way that each term is computed by using the previous term. Since u_0 is known,

$$\begin{aligned} &u_1(x,y,t) = L_t^{-1}(L_x u_0 + L_y u_0 + u_0),\\ &u_2(x,y,t) = L_t^{-1}(L_x u_1 + L_y u_1 + u_1),\\ &\vdots\\ &u_{n+1}(x,y,t) = L_t^{-1}(L_x u_n + L_y u_n + u_n),\ n \geq 0. \end{aligned} \quad (13)$$

It is noteworthy that the recursive relationship is constructed on the basis that the zeroth component $u_0(x,y,t)$ defined by a term that arises from the initial condition. The remaining components $u_n(x,y,t)$, $n \geq 0$ can be completely determined; each term is computed by using the previous term. As a result, the components u_0, u_1, u_2,… are identified and the series of solutions are thus entirely determined. However, in many cases the exact solution in a closed form may be obtained.

The n-term approximant ϕ_n defined by

$$\phi_n = \sum_{k=0}^{n-1} u_k(x,y,t), \quad (14)$$

can be used for numerical approximations.

NUMERICAL ILLUSTRATION

To give a clear overview of our study and to illustrate the above discussed technique, we use the following examples.

Example 1 The approximate solutions of the same example were presented by Ho and Yang (2001). We present them here mainly aimed at using the algorithm given in Section 2. As a first example, consider Eq.(15):

$$\frac{\partial T}{\partial t} - \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = 1, \quad (15)$$

subject to the boundary conditions that for $t \geq 0$,

$$\begin{aligned} &T(1,y,t) = 0,\ T(x,1,t) = 0,\\ &\frac{\partial T}{\partial x}(0,y,t) = 0,\ \frac{\partial T}{\partial y}(x,0,t) = 0, \end{aligned} \quad (16)$$

Comparison of the WGM and the ADM solutions with the exact ones (Ho and Yang, 2001) is given in Table 1 where ADM and WGM values are presented which correspond to the various values of x,y. As seen in these tables, the values we got and the values obtained by Ho and Yang (2001) are very close. It should be noted that only 10(ϕ_{10}) iterations were needed to obtain the approximately accurate solution. The overall errors can be made even much smaller by adding new terms of the decomposition. The convergence is rapid.

The numerical solutions showed that the ADM is a very convenient method for such heat conduction problem. With this method, it is possible to obtain more precise results than the traditional methods, with less calculation and less time. As shown in this example, the ADM is simple and easy to use. In addition, our method minimizes the computational cal-

Table 1 Comparison of the WGM and the ADM solutions with the exact solution of the heat conducting problem

(x, y)	Exact solution	WGM (steady)	WGM at t=0.5 s	ADM
(0,0)	0.2927	0.2969	0.2689	0.2925
(0,0.25)	0.2789	0.2793	0.2536	0.2785
(0,0.5)	0.2293	0.2293	0.2097	0.2292
(0,0.75)	0.1397	0.1396	0.1290	0.1397
(0,1)	0.0000	0.0000	0.0000	0.0000
(0.25,0.25)	0.2642	0.2642	0.2406	0.2642
(0.25,0.5)	0.2178	0.2176	0.1995	0.2177
(0.25,0.75)	0.1333	0.1331	0.1234	0.1332
(0.25,1)	0.0000	0.0018	0.0014	0.0003
(0.5,0.5)	0.1811	0.1808	0.1670	0.1810
(0.5,0.75)	0.1127	0.1125	0.1049	0.1127
(0.5,1)	0.0000	0.0000	0.0000	0.0000
(0.75,0.75)	0.0728	0.0724	0.0684	0.0726
(0.75,1)	0.0000	0.0020	0.0016	0.0002
(1,1)	0.0000	0.0000	0.0000	0.0000

culus and gives quantitatively reliable results. The WGM has a complicated computational calculus and is not easy to use. El-Sayed and Abdel-Aziz (2003) compared the ADM and the WGM for solving nonlinear integro-differential equations. In this study, they showed that the WGM is more expansive and boring than the ADM.

Example 2 We consider a 3D time-dependent diffusion equation in the following form:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}, \tag{17}$$

with initial conditions

$$u(x,y,z,0)=\exp(x+y+z),\ 0\le x,y,z\le 1 \tag{18}$$

and boundary conditions

$$u(1,y,z,t)=u_x(1,y,z,t)=\exp(1+y+z+3t),\quad 0<t\le 1,\ 0\le x,z\le 1 \tag{19}$$

$$u(x,1,z,t)=u_y(x,1,z,t)=\exp(1+x+z+3t),\quad 0<t\le 1,\ 0\le x,z\le 1 \tag{20}$$

$$u(x,y,1,t)=u_z(x,y,1,t)=\exp(1+x+y+3t),\quad 0<t\le 1,\ 0\le x,y\le 1 \tag{21}$$

Proceeding as before, we find

$$\begin{aligned}u_0(x,y,z,t)=\frac{1}{3}\{&u(1,y,z,t)+u(x,1,z,t)+u(x,y,1,t)\\&+(x-1)u_x(1,y,z,t)+(y-1)u_y(x,1,z,t)\\&+(z-1)u_z(x,y,1,t)\}.\end{aligned} \tag{22}$$

and

$$\begin{aligned}u_{n+1}(x,y,z,t)=\frac{1}{3}\{&L_{xx}{}^{-1}L_t+L_{yy}{}^{-1}L_t+L_{zz}{}^{-1}L_t-L_{xx}{}^{-1}L_{yy}\\&-L_{yy}{}^{-1}L_{xx}-L_{yy}{}^{-1}L_{zz}-L_{zz}{}^{-1}L_{yy}-L_{zz}{}^{-1}L_{xx}\}u_n,\end{aligned} \tag{23}$$

where

$$L_{xx}^{-1}=\int_1^x \mathrm{d}x'\int_1^{x'}\mathrm{d}x'',\ L_{yy}^{-1}=\int_1^y \mathrm{d}y'\int_1^{y'}\mathrm{d}y'',\ L_{zz}^{-1}=\int_1^z \mathrm{d}z'\int_1^{z'}\mathrm{d}z'', \tag{24}$$

Table 2 presents the errors of ADM, FTCS, BTCS and C-N which correspond to the various values of x, y and z. As seen in this table, the values we got were better than the results obtained by Dehghan (2003a). It should be noted that only 7(ϕ_7) iterations were needed to obtain the approximately accurate solutions. The overall errors can be made even much smaller by adding new terms of the decomposition. The convergence is rapid. In addition, we can find the exact solution by using only the initial condition.

Table 2 A comparison of errors of the FTCS, the BTCS, the C-N and the ADM

x	y	z	Exact solution	(1, 7) FTCS	(7, 1) BTCS	(7, 7) C-N	ADM
0.1	0.1	0.1	27.112638	3.1×10^{-4}	2.5×10^{-4}	1.5×10^{-4}	4.7×10^{-8}
0.2	0.2	0.2	36.598234	3.2×10^{-4}	2.6×10^{-4}	1.5×10^{-4}	6.4×10^{-8}
0.3	0.3	0.3	49.402449	3.4×10^{-4}	2.6×10^{-4}	1.6×10^{-4}	7.3×10^{-8}
0.4	0.4	0.4	66.686331	3.4×10^{-4}	2.7×10^{-4}	1.7×10^{-4}	8.5×10^{-8}
0.5	0.5	0.5	90.017131	3.3×10^{-4}	2.8×10^{-4}	1.8×10^{-4}	9.8×10^{-8}
0.6	0.6	0.6	121.510417	3.5×10^{-4}	2.7×10^{-4}	1.9×10^{-4}	2.3×10^{-7}
0.7	0.7	0.7	164.021907	3.6×10^{-4}	2.9×10^{-4}	1.8×10^{-4}	4.6×10^{-7}
0.8	0.8	0.8	221.406416	3.7×10^{-4}	2.8×10^{-4}	1.7×10^{-4}	6.6×10^{-7}
0.9	0.9	0.9	298.867401	3.8×10^{-4}	2.6×10^{-4}	1.6×10^{-4}	8.1×10^{-7}

Example 3 We finally consider a 2D time-dependent diffusion equation in the following form:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \tag{25}$$

with initial conditions

$$u(x,y,0)=(1-y)\exp(t),\ 0\le t\le T,\ 0\le y\le 1, \tag{26}$$

and boundary conditions

$$u(0,y,t)=(1-y)\exp(t),\ 0<t\le T,\ 0\le y\le 1 \tag{27}$$

$$u(1,y,t)=u_x(1,y,t)=(1-y)\exp(1+t),\quad 0<t\le T,\ 0\le y\le 1 \tag{28}$$

$$u(x,0,t)=u_y(x,1,t)=0,\ 0<t\le T,\ 0\le x\le 1 \tag{29}$$

$$u(x,0,t)=\exp(x+t),\ 0<t\le T,\ 0\le x\le 1 \tag{30}$$

Proceeding as before, we find

$$u_0(x,y,t)=\frac{1}{2}\{u(1,y,t)+u(x,1,t)+(x-1)u_x(1,y,t)+(y-1)u_y(x,1,t)\} \tag{31}$$

and

$$u_{n+1}(x,y,z,t)=\frac{1}{3}\left\{L_{xx}^{-1}L_t+L_{yy}^{-1}L_t-L_{xx}^{-1}L_{yy}-L_{yy}^{-1}L_{xx}\right\}u_n, \tag{32}$$

where

$$L_{xx}^{-1}=\int_1^x \mathrm{d}x'\int_1^{x'}\mathrm{d}x'',\ L_{yy}^{-1}=\int_1^y \mathrm{d}y'\int_1^{y'}\mathrm{d}y'',\quad L_{zz}^{-1}=\int_1^z \mathrm{d}z'\int_1^{z'}\mathrm{d}z'', \tag{33}$$

Table 3 presents the errors obtained by the standard FTCS, the 9-point, the (1,13) and the ADM which correspond to the various values of x,y. As seen in this table, the values we got were better than the results obtained by Dehgnan (2003a). It should be noted that only 10(ϕ_{10}) iterations were needed to obtain the approximately accurate solutions. The overall errors can be made even much smaller by adding new terms of the decomposition.

CONVERGENCE ANALYSIS

Here, we will study the convergence analysis as same manner in (Ngarhasta *et al*., 2002) of the decomposition method to the nonlinear 2D time-dependent diffusion equation. Let us consider the Hilbert space H which may be defined as $H=L^2((\alpha,\beta)\times[0,T])$:

$$u:(\alpha,\beta)\times[0,T]\to R$$

with $$\int_{(\alpha,\beta)\times[0,T]}u^2(x,s)\mathrm{d}s\mathrm{d}\tau<+\infty. \tag{34}$$

We consider the nonlinear 2D time-dependent diffusion equation in light of the above assumptions. The operator of the 2D time-dependent diffusion equation is

$$L_t u=\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+f(u), \tag{35}$$

where $f(u)$ is a nonlinear term. The ADM is convergent if the following two hypotheses are satisfied (Ngarhasta, 2002):

(H_1) $(L_t(u)-L_t(v),\ u-v)\ge k\|u-v\|^2,\ k>0,\ \forall u, v\in H$

(H_2) For whatever $M>0$ there exists a constant $C(M)>0$ such that for $u,v\in H$ with $\|u\|\le M$, $\|v\|\le M$ we

Table 3 Comparison of errors of the standard FTCS, the 9-point, the (1,13) and the ADM

x	y	Exact solution	Standard FTCS	9-point	(1,13)	ADM
0.1	0.1	9.025013	2.2×10^{-4}	5.4×10^{-5}	1.6×10^{-8}	1.8×10^{-11}
0.2	0.2	11.023176	3.1×10^{-4}	1.0×10^{-5}	1.9×10^{-8}	3.5×10^{-11}
0.3	0.3	13.463738	3.5×10^{-4}	9.2×10^{-5}	2.1×10^{-8}	5.3×10^{-11}
0.4	0.4	16.444647	6.7×10^{-4}	1.7×10^{-4}	2.3×10^{-8}	7.5×10^{-11}
0.5	0.5	20.905243	9.2×10^{-4}	2.2×10^{-4}	2.5×10^{-8}	8.7×10^{-11}
0.6	0.6	24.532530	1.0×10^{-3}	2.6×10^{-4}	1.8×10^{-8}	9.6×10^{-11}
0.7	0.7	29.964100	9.9×10^{-4}	2.4×10^{-4}	1.6×10^{-8}	2.6×10^{-10}
0.8	0.8	36.598234	7.5×10^{-4}	1.9×10^{-4}	1.4×10^{-8}	3.9×10^{-10}
0.9	0.9	40.447304	3.4×10^{-4}	8.6×10^{-5}	1.1×10^{-8}	5.1×10^{-10}

have $(L_t(u)-L_t(v), w)\le C(M)\|u-v\|\|w\|$ for every $w\in H$.

Theorem (Sufficient condition of convergence for the nonlinear 2D time-dependent diffusion equation)

The ADM applied to the equation below:

$$L_t u=\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+f(u)$$

without initial and boundary conditions, converges towards a particular solution.

Proof To prove the theorem, we will verify the conditions (H_1) and (H_2) of convergence. Firstly, we will verify the convergence hypotheses (H_1) for the operator $L_t(u)$:

$$(L_t(u)-L_t(v),u-v)=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+f(u)-\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2}$$
$$-f(v)=\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 v}{\partial y^2}+f(u)-f(v) \quad (36)$$

Therefore,

$$(L_t(u)-L_t(v),u-v)=\left(\frac{\partial^2}{\partial x^2}(u-v),(u-v)\right)$$
$$+\left(\frac{\partial^2}{\partial y^2}(u-v),(u-v)\right)+\big(f(u)-f(v),u-v\big) \quad (37)$$

But there exists two real values δ_1, δ_2>0, such that

$$\left(\frac{\partial^2}{\partial x^2}(u-v),(u-v)\right)\ge\delta_1\|u-v\|^2, \quad (38)$$

$$\left(\frac{\partial^2}{\partial y^2}(u-v),(u-v)\right)\ge\delta_2\|u-v\|^2, \quad (39)$$

and according the Schwartz inequality, we get

$$(f(u)-f(v),u-v)\le\alpha\|u-v\|^2, \quad (40)$$

where α>0 is the Lipschitzian constant and therefore:

$$\begin{aligned}&(f(u)-f(v),u-v)\le\alpha\|u-v\|^2\\&\Leftrightarrow -(f(u)-f(v),u-v)\ge\alpha\|u-v\|^2\end{aligned} \quad (41)$$

Substituting Eqs.(38)~(41) into Eq.(37) we yields

$$\begin{aligned}(L_t(u)-L_t(v),u-v)&\ge(\delta_1+\delta_2+\alpha)\|u-v\|^2\\&=k\|u-v\|^2,\end{aligned} \quad (42)$$

where $k=\delta_1+\delta_2+\alpha$. Then the hypothesis ($H_1$) holds.

Then we verify the convergence hypotheses (H_2) for the operator $L_t(u)$. For that we have

$$(L_t(u)-L_t(v),w)=\left(\frac{\partial^2}{\partial x^2}(u-v),w\right)+\left(\frac{\partial^2}{\partial y^2}(u-v),w\right)$$
$$+(f(u)-f(v),w)\le4\|u-v\|\|w\|+\alpha\|u-v\|\|w\|$$
$$=(4+\alpha)\|u-v\|\|w\|=C(M)\|u-v\|\|w\| \quad (43)$$

where $C(M)=4+\alpha$ and the hypotheses (H_2) holds.

CONCLUSION

In this paper, we calculated the approximate solutions of 2D and 3D parabolic differential equations by using ADM. We demonstrated that the decomposition procedure is quite efficient for determining solution in closed form by using initial condition or boundary conditions. Our present methods avoid the tedious work needed by traditional techniques. In the studies by Ho and Yang (2001), and Dehghan (2002; 2003a; 2003b) much time was spent and boring operations were done by WGM, FTCS, BTCS, (1,13) fully explicit method, 9-point finite difference method and C-N to get approximate solutions. In our study, however, we can get better approximately accurate solutions than the other methods by using easy calculations by using our method. The method avoids the difficulties and extensive computational work that usually arise from the WG, Finite difference methods, Parallel techniques method and the C-N method.

It is possible to solve this problem by using both the initial and the various boundary conditions. Relevant studies (Lesnic, 2002b) showed that the use of the Dirichlet, Robin, Neumann and mixed boundary conditions will be sufficient for finding approximately accurate solutions.

References

Abbaoui, K., Cherruault, Y., 1999. The decomposition method applied to the Cauchy problem. *Kybernetes*, **28**(1):68-74.

Abbaoui, K., Pujol, M.J., Cherruault, Y., Himoun, N., Grimalt, P., 2001. A new formulation of Adomian method, convergence results. *Kybernetes*, **30**(9/10):1183-1191.

Adjedj, B., 1999. Application of the decomposition method to the understanding of HIV immune dynamics. *Kybernetes*, **28**(3):271-283.

Adomian, G., 1989. Nonlinear Stochastic Systems and Applications to Physics. Kluwer Academic Press, Boston.

Adomian, G., 1994. Solving Frontier Problems of Physics: The Decomposition Method. Kluwer Academic Press, Boston.

Adomian, G., Cherruault, Y., Abbaoui, K., 1996. A nonperturbative analytical solution of immune response with time-delays and possible generalization. *Math. Comput. Modelling*, **20**(10):89-96.

Cannon, J.R., Hoek, J., 1986. Diffusion subject to specification of mass. *J. Math. Anal. Applic.*, **115**:517-529.

Cannon, J.R., Matheson, A.L., 1993. A numerical procedure of diffusion subject to the specification of mass. *Int. J. Eng. Sci.*, **31**:347-354.

Cannon, J.R., Lin, Y., Wang, S., 1990. An implicit finite difference scheme for diffusion equation subject to mass specification. *Int. J. Eng. Sci.*, **28**(7):573-578.

Capso, V., Kunisch, K., 1988. A reaction-diffusion system arising in modeling man-environment diseases. *Quart. Appl. Math.*, **46**:431-449.

Cherruault, Y., 1989. Convergence of Adomian's method. *Kybernetes*, **18**(1):31-38.

Cherruault, Y., Adomian, G., 1993. Decomposition method: A new proof of convergence. *Math. Comput. Modelling*, **18**:103-106.

Dehghan, M., 2002. Fully explicit finite-difference methods for two-dimensional diffusion with an integral condition. *Nonlinear Analysis*, **48**:637-650.

Dehghan, M., 2003a. Parallel techniques for a boundary value problem with non-classic boundary conditions. *Appl. Math. Comput.*, **137**:399-412.

Dehghan, M., 2003b. Numerical solution of a parabolic equation with non-local boundary specification. *Appl. Math. Comput.*, **145**:185-194.

El-Sayed, S.M., Abdel-Aziz, M.R., 2003. A comparison of Adomian's decomposition method and Wavelet-Galerkin method for solving integro-differential equations. *Appl. Math. Comput.*, **136**:151-159.

Guellal, S., Cherruault, Y., 1995. Application of the decomposition method to identify the disturbed parameters of an elliptical equation. *Math. Comput. Modelling*, **21**(4):51-55.

Guellal, S., Grimalt, P., Cherruault, Y., 1997. Numerical study of Lorentz's equation by the Adomian method. *Comput. Math. Applic.*, **33**(3):25-29.

Ho, S.L., Yang, S.Y., 2001. Wavelet-Galerkin method for solving parabolic equations in finite domains. *Finite Elements in Analysis and Design*, **37**:1023-1037.

Laffez, P., Abbaoui, K., 1996. Modelling of the thermic exchanges during a drilling. Resolution with Adomian's method. *Math. Comput. Modelling*, **21**(4):51-55.

Lesnic, D., 2002a. Convergence of Adomian's method: Periodic temperatures. *Comput. Math. Applic.*, **44**:13-24.

Lesnic, D., 2002b. The decomposition method for forward and backward time-dependent problems. *J. Comput. Appl. Math.*, **147**:27-39.

Ndour, M., Abbaoui, K., Ammar, H., Cherruault, Y., 1996. An example of an interaction model between two species. *Kybernetes*, **25**(4):106-118.

Ngarhasta, N., Some, B., Abbaoui, K., Cherruault, Y., 2002. New numerical study of Adomian method applied to a diffusion model. *Kybernetes*, **31**(1):61-75.

Sanchez, F., Abbaoui, K., Cherruault, Y., 2000. Beyond the thin-sheet approximation: Adomian's decomposition. *Optics Commun.*, **173**:397-401.

Wang, S., 1990. The numerical method for the heat conduction subject to moving boundary energy specification. *Numer. Heat. Transfer*, **130**:35-38.