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A stable image reconstruction algorithm for ECT

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Abstract: Most image reconstruction algorithms developed for electrical capacitance tomography (ECT) can only reconstruct qualitative images. Stabled quantitative image reconstruction is necessary for many applications. To get stable ECT image, the authors constructed a compressive operator and developed a new iterative algorithm, which can overcome the semi-convergence occurring in the Landweber iteration reconstruction technique. Experimental results showed that the stability and quality of reconstructed images are improved significantly.

Key words: Capacitance tomography, Inverse problem, Convergence, Stability
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INTRODUCTION

Electrical capacitance tomography (ECT) is based on measuring the variation in capacitance between pairs of electrodes surrounding a subject, and reconstructing the distribution of dielectric properties in the sensing area using the measured data. For image reconstruction with ECT, the linear back-projection (LBP) algorithm is the most popular (Xie *et al*., 1992). However, it can only produce qualitative images. In many cases, it is necessary to provide quantitative measurements, e.g. for metering the volume flow rate of oil in a pipeline or the volume flow rate of solids in a pneumatic conveyor (Liu *et al.*, 1997). To enhance image quality and to provide quantitative images, various iterative image reconstruction algorithms have been developed for ECT, including Chen *et al*.(1993)'s method, which requires the updating of sensitivity maps, Isaksen and Nordtvedt (1993)'s model-based method, Reinecke and Mewes (1996)'s algebraic reconstruction technique (ART), Landweber iteration, which was first used in ECT by Yang *et al*.(1999), and Liu *et al.* (1999)'s an optimization method for iterative image reconstruction. Yang and Peng (2003) reviewed the existing algorithms for ECT. The fast image reconstruction algorithm proposed recently is called online iterative image reconstruction (Wang *et al.*, 2004; Liu *et al.*, 2004).

One problem using iterative algorithms in ECT is semi-convergence wherein the reconstructed image may approach to an acceptable level, and then diverge afterwards. For example, with the Landweber iteration, although the capacitance error may continuously decrease, the image error would increase after a certain number of iterations (Liu *et al*., 1999; Fu *et al*., 2000). It is difficult to determine when the iterative process should stop. Therefore, it is necessary to investigate how to improve the convergence of iterative image reconstruction algorithms.

This paper presents a new method to address the above problem. Because one reason for semi-convergence is that the operator relative to ECT is not compressive, we will focus on constructing a compressive operator and then forming a stable iterative image reconstruction algorithm based on the

Landweber iteration.

FUNDAMENTALS OF IMAGE RECONSTRUCTION

A typical ECT sensor has 8 or 12 measurement electrodes. The capacitance measured from two electrodes may be described by an integral equation.

$$C = -\frac{1}{V}\iint_{\Gamma} \varepsilon(x,y)\nabla\phi(x,y)\mathrm{d}\Gamma \tag{1}$$

Because it is too complicated to deal with the integral equation, it is common practice to simplify the system to be linear by approximation.

$$\boldsymbol{SG} = \boldsymbol{C} \tag{2}$$

where, $\boldsymbol{C}^{\mathrm{T}}=(C_1,C_2,\ldots,C_M)$ is the measured capacitance vector, $\boldsymbol{G}^{\mathrm{T}}=(G_1,G_2,\ldots,G_N)$ is the image, which represents distribution of dielectric properties in the sensing area, and $\boldsymbol{S}_{M\times N}$ is the sensitivity matrix, which reflects the effect of dielectric properties distribution at each pixel on the inter-electrode capacitance.

The task of image reconstruction is to solve the inverse problem, i.e. to obtain an image $\boldsymbol{G}$ from Eq.(2). Obviously, it is an ill-posed problem (Shi *et al.*, 2001).

Two orthogonal unity matrixes $\boldsymbol{U}$ and $\boldsymbol{V}$ may be found by singular value deposition (SVD), to express $\boldsymbol{S}$.

$$\boldsymbol{S} = \boldsymbol{U\Sigma V}^{\mathrm{T}} \tag{3}$$

where $\boldsymbol{\Sigma}$ is a diagonal matrix and $\sigma_1\geq\sigma_2\geq\ldots\geq\sigma_p>0$, i.e.

$$\boldsymbol{\Sigma}=\mathrm{diag}[\sigma_1,\sigma_2,\ldots,\sigma_p] \tag{4}$$

Combining Eqs.(2) and (3) yields

$$\boldsymbol{\Sigma V}^{\mathrm{T}}\boldsymbol{G} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{C} \tag{5}$$

Note that

$$\boldsymbol{V}^{\mathrm{T}}\boldsymbol{G} \equiv \boldsymbol{G}_V \text{ and } \boldsymbol{U}^{\mathrm{T}}\boldsymbol{C} \equiv \boldsymbol{C}_U \tag{6}$$

thus

$$\boldsymbol{\Sigma G}_V = \boldsymbol{C}_U \tag{7}$$

where $\boldsymbol{G}_V^{\mathrm{T}}=(G_{V1},G_{V2},\ldots,G_{VN})$, $\boldsymbol{C}_U^{\mathrm{T}}=(C_{U1},C_{U2},\ldots,C_{UN})$.

Divide both sides of Eq.(7) by σ_p, and denote $S_c\equiv\sigma_1/\sigma_p$, which is called the condition number of a sensitivity matrix $\boldsymbol{S}$. It can be seen that a relatively small change σ_1 of G_{V1} can result in a significant change of $S_c\delta_1$ times in C_{U1}, In general, the condition number for ECT S_c is much larger than 10 (Yang and Peng, 2003). This indicates that the measured data $\boldsymbol{C}$ is sensitive to errors in the image $\boldsymbol{G}$. In practice, there is always noise ε_p in the measured data C_{Up}, which results in ε_p/σ_p times change in the image element G_{Vp}. Even a very small ε_p can lead to a significant effect on the image because σ_p usually approaches to zero. This indicates that the SVD algorithm is sensitive to noise, i.e. small noise in the measured data can result in significant errors in the reconstructed image. So it is necessary to develop an algorithm to improve the stability of image reconstruction.

LANDWEBER ITERATION AND ITS SEMI-CONVERGENCE

To reduce the effect of noise in solving Eq.(2), the least-squares algorithm for minimising $\|\boldsymbol{SG}-\boldsymbol{C}\|^2$ is commonly used. It may be expressed as follows:

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{SG} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} \tag{8}$$

In the Landweber iteration, an image is obtained by the following iterative process (Yang *et al.*, 1999):

$$\boldsymbol{G}_{k+1} = \boldsymbol{G}_k + \alpha_k\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{C}-\boldsymbol{SG}_k) \tag{9}$$

where $\boldsymbol{G}_k$ is the image in the kth step, α_k is the relaxation factor in the kth step, k=0, 1, 2, …, n, and $\boldsymbol{G}_0=\alpha_0\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}$ is the first estimate of the image.

Eq.(9) can be rewritten as

$$\boldsymbol{G}_{k+1} = \alpha_0\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} + (\boldsymbol{I}-\alpha_k\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S})\boldsymbol{G}_k \tag{10}$$

where, $\boldsymbol{I}$ is a unity matrix. If a fixed relaxation factor α_k is used, i.e. $\alpha_k\equiv\alpha$, a residual may be defined as

$$\boldsymbol{R} \equiv \boldsymbol{I} - \alpha \boldsymbol{S}^{\mathrm{T}} \boldsymbol{S} \tag{11}$$

Considering

$$\boldsymbol{G}_0 \equiv \alpha \boldsymbol{S}^{\mathrm{T}} \boldsymbol{C} \tag{12}$$

Eq.(10) can be expressed as a series:

$$\boldsymbol{G}_{k+1} = (\boldsymbol{I} + \boldsymbol{R} + \boldsymbol{R}^2 + \boldsymbol{R}^3 + \ldots + \boldsymbol{R}^{k+1})\boldsymbol{G}_0 \tag{13}$$

In theory, the norm of $\boldsymbol{R}$ should be less than 1, in order to guarantee the convergence, i.e.

$$\|\boldsymbol{R}\| \equiv \|\boldsymbol{I} - \alpha \boldsymbol{S}^{\mathrm{T}} \boldsymbol{S}\| < 1 \tag{14}$$

where the norm of $\boldsymbol{R}$ is defined by

$$\|\boldsymbol{R}\| \equiv \max \frac{\|\boldsymbol{RG}\|}{\|\boldsymbol{G}\|} \quad \text{for arbitrary} \quad \boldsymbol{G} \neq 0 \tag{15}$$

Commonly, α which satisfies $0<\alpha<2/\|\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\|$ is chosen to obtain an acceptable solution of Eq.(10). In practice, however, the solution suffers from semi-convergence problem. One of the possible reasons is that the inverse problem is ill-conditioned, i.e. the number of measured data is less than the number of pixels in the image, the rank of the adjoint matrix $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}$ is not full. This means that some eigenvalues of $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}$ are equal to zero. Therefore, in most cases, the condition of Eq.(14) cannot be met, i.e. $\|\boldsymbol{R}\| \equiv \|\boldsymbol{I}-\alpha\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\| \geq 1$. This is why the convergence of the iterative process cannot be guaranteed. The norm of operator $\boldsymbol{R}$ is equal to the largest amplification to image $\boldsymbol{G}$. $\|\boldsymbol{R}\| \geq 1$ indicates that the residual $\boldsymbol{R}$ is not a compressive operator, which may increase the data noise and computation error. This is why the Landweber iterative process is considered semi-convergent.

A STABLE IMAGE RECONSTRUCTION ALGORITHM

To overcome the semi-convergence of the Landweber iterative algorithm, a new algorithm with a new operator has been developed as follows:

$$\overline{\boldsymbol{R}} \equiv \lambda \boldsymbol{I} - \beta \boldsymbol{S}^{\mathrm{T}} \boldsymbol{S} \tag{16}$$

where β is a parameter related to the sensitivity matrix and the noise level of an ECT system, and the parameters λ and β are so selected such that the norm of operator satisfies

$$\|\overline{\boldsymbol{R}}\| \leq \lambda < 1 \tag{17}$$

in which $\lambda=1-n_\alpha$ is a positive parameter; and n_α is a factor related to the noise level of the ECT system, which satisfies $0<n_\alpha<1$. An operator whose norm is less than one is called a compressive operator.

From the new operator $\overline{\boldsymbol{R}}$, the reconstructed image sequence $\{\overline{\boldsymbol{G}}_k\}$ can be obtained by

$$\overline{\boldsymbol{G}}_{k+1} = \overline{\boldsymbol{G}}_0 + \overline{\boldsymbol{R}}\overline{\boldsymbol{G}}_k \tag{18}$$

It can be seen that the new operator $\overline{\boldsymbol{R}}$ from Eq.(17) is a compressive operator, and therefore the reconstructed image sequence $\{\overline{\boldsymbol{G}}_k\}$ is convergent. Similar to Eq.(10), Eq.(18) can be expressed as a series:

$$\overline{\boldsymbol{G}}_{k+1} = (\boldsymbol{I} + \overline{\boldsymbol{R}} + \overline{\boldsymbol{R}}^2 + \overline{\boldsymbol{R}}^3 + \ldots + \overline{\boldsymbol{R}}^{k+1})\overline{\boldsymbol{G}}_0 \tag{19}$$

where $\overline{\boldsymbol{G}}_0 \equiv \alpha \boldsymbol{S}^{\mathrm{T}} \boldsymbol{C}$.

EXPERIMENTAL RESULT AND ITS ANALYSIS

The new image reconstruction algorithm has been tested with measured data acquired from an ECT system with a 12-electrode sensor. The tested media were plastic beads (ε_r=2.6, similar to these of oil). The plastic beads filled half the plastic tube as shown in Fig.1. Fig.2 shows two serials of reconstructed images using the Landweber iteration described by Eq.(10) and the stable image reconstruction algorithm described by Eq.(18) respectively, where k=0,1,...,41, λ=0.98, β=0.0000695.

As shown in Fig.2a, with the traditional Landweber iteration, the first few images show the beads (the highlighted part) adhering to the plastic tube wall in the beginning and then approaching gradually to the shape of the test model. However, the reconstructed image becomes worse and appears some artifacts after a certain number of iterations, which indicates that the Landweber iteration is unstable and semi-convergent. In contrast, the iterative process described by Eq.(18) caused the stability and convergence of the reconstructed image (see Fig.2b).

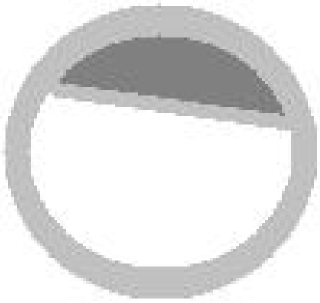
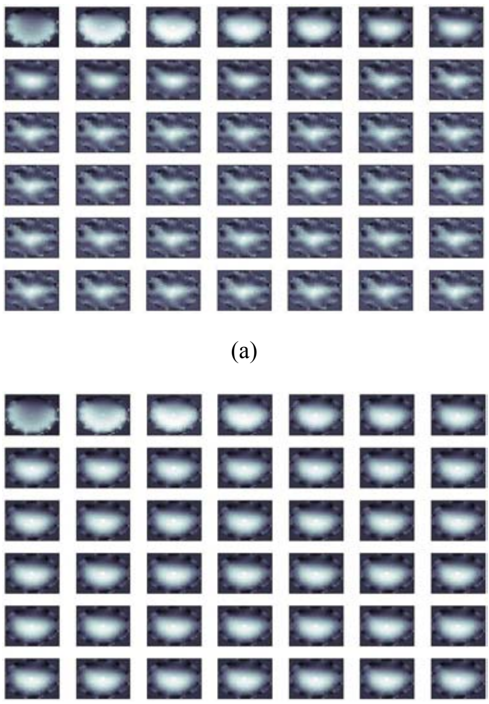


Fig.1 Test model



(a)

(b)

Fig.2 Comparison of reconstructed images (a) using Landweber iteration; (b) using stable image reconstruction algorithm

CONCLUSION

A stable iterative algorithm for ECT is presented in this paper. The operator used in the algorithm is compressive. The experiment results showed that the algorithm could converge, and that the quality of reconstructed image can be improved significantly. More work is needed to investigate how to choose parameters λ and β of the constructed operator $\bar{\boldsymbol{R}}$.

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