Journal of Zhejiang University SCIENCE A
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Novel resonator based on composite right/left-handed transmission lines*

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Received Oct. 30, 2005; revision accepted Nov. 22, 2005

Abstract: A new approach was introduced to analyze composite right/left-handed transmission lines (CRLH TLs). The Bloch impedance and the dispersion relations are directly obtained from the *S* parameters of the unit cells. The LH and RH frequency bands are then identified by the real parts of the Bloch impedance and the phase delay of the unit cells. The new approach has some advantages over the LC parameters extraction method introduced by Caloz *et al*.(2004). Based on the new approach, a novel resonator is designed using CRLH TLs. The simulation and experimental results accorded well with the theoretical analysis. The novel resonator may have potential applications in filters with high harmonic suppression and compact structures.

Key words: Composite right/left-handed transmission lines (CRLH TLs), Resonator
doi:10.1631/jzus.2006.A0085 **Document code:** A **CLC number:** TN75

INTRODUCTION

Left-handed materials (LHMs) are characterized by simultaneously negative permittivity and permeability. It was first investigated theoretically by Veselago (1968). The experimental realization of LHM with resonance structures was demonstrated by Smith *et al*.(2000). Since then, LHM has gained significant interest and started to be integrated into novel microwave and optical applications. The transmission-line (TL) approach was recently proposed to realize LHMs (Eleftheriades *et al*., 2002). LH-TL is the dual of the conventional RH-TL in which the inductance and capacitance have been interchanged. All practical LH structures include RH parasitic effects, and are therefore composite right/left-handed (CRLH) in reality (Sanada *et al*., 2004). The parasitic RH property can be expressed by inserting series inductance and shunt capacitance into the unit cell. CRLH can be realized by microstrip circuits which can be conveniently implemented in microwave circuits and antennas. A useful approach to analyze microstrip-based CRLH is the LC parameters extraction approach first introduced by Caloz *et al*.(2004). This paper presents a new approach developed to analyze the characteristics of CRLH. The Bloch impedance and the dispersion relations are directly obtained from the *S* parameters of the unit cells. The new approach has some advantages over the LC parameters extraction method. The new approach was applied to design a novel resonator. The simulation and experimental results accorded well with the theoretical analysis.

CRLH TL

The CRLH TL is configured as a periodic network of the unit cell (shown in Fig.1). L_L and C_L represent the shunt inductance and series capacitance. Similarly, L_R and C_R represent the series inductance and shunt capacitance. Expressing the unit cell in terms of a four-port *ABCD* matrix and solving the eigen-

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* Project supported by the National Basic Research Program (973) of China (No. 2004CB719800) and the National Natural Science Foundation of China (Nos. 60271027 and 60501018)

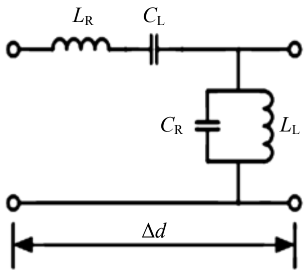


Fig.1 Unit cell of CRLH TL

value problem with the Bloch-Floquet periodic boundary conditions, we can obtain the dispersion relation:

$$\cos(\beta\Delta d)=1-\frac{1}{2}\left[\frac{1}{\omega^2 L_L C_L}+\omega^2 L_R C_R-\left(\frac{L_R}{L_L}+\frac{C_R}{C_L}\right)\right], \tag{1}$$

where β is the wave number. This dispersion relation is the keystone of CRLH structures. The group velocity v_g=dω/dβ can be calculated from Eq.(1). In the frequency band where the group velocity is negative for positive β, the wave has anti-parallel group and phase velocities and therefore CRLH support backward waves (LH waves) but support forward waves in the frequency band where the group velocity is positive for positive β. One good use of CRLH TL is microstrip implementation, which potentially provides better performances at high frequency. The unit cell of the microstrip CRLH TL proposed by Caloz *et al.*(2004) consists of a series interdigital capacitor and a shorted stub inductor as shown in Fig.2.

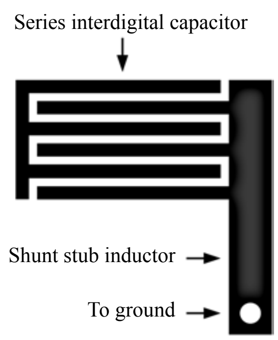


Fig.2 Microstrip implementation of CRLH TL

With the LC parameter extraction approach (Caloz *et al.*, 2004), the unit cell can be mapped to the model of Fig.1. Then the characteristics of the CRLH in different frequency bands can be analyzed based on Eq.(1).

A NEW APPROACH TO ANALYZE CRLH TL

In this section, a new approach will be introduced for analyzing the CRLH TL characteristics. First, the *S* parameters of the unit cell are obtained by full-wave simulation or experimental measurement. Since the CRLH TL is in the form of a periodic network of unit cells, Bloch-Floquet theorem can be used to obtain the eigen modes directly from the *S* parameters of the unit cell (Fig.3).

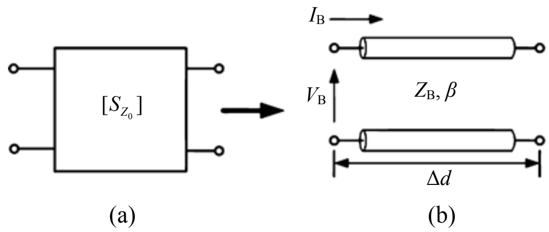


Fig.3 (a) The *S* parameters of the unit cell (normalized to the port impedance Z_0); (b) The eigenmode of the CRLH TL

Two important factors related to the eigen modes are the Bloch impedance Z_B and the wave number β. The Bloch impedance Z_B represents the ratio of the voltage and the current in the eigen modes,

$$Z_B= V_B/ I_B. \tag{2}$$

Hence the power of the eigen modes can be represented by

$$P_B=\frac{1}{2}\mathrm{Re}(V_B I_B^*)=\frac{1}{2}\mathrm{Re}(Z_B)\left|I_B\right|^2. \tag{3}$$

This means the sign of Re(Z_B) is related to the direction of the energy flow in eigen modes. On the other hand, the real part of β represents the phase propagation. Therefore the LH and RH frequency bands can be identified by the sign of Re(β)Re(Z_B). In the frequency band where Re(β)Re(Z_B)<0, the directions of energy flow and wave propagation are anti-parallel. This means the eigen modes are backward waves and the frequency bands are LH bands. In contrast, the RH band is identified by Re(β)Re(Z_B)>0. This new approach has some advantages over the LC parameters extraction approach.

(1) The new approach is suitable for unit cells with arbitrary structures. The LC parameters extraction approach is only suitable for unit cells with LC components.

(2) In the new approach, Z_B and β are directly obtained from the S parameters of the unit cell. It naturally takes the dispersion of the microstrip components into account and the results may be accurate over a wide frequency band.

(3) Under lossy case, Eq.(1) no longer works. In the new approach, LH and RH bands can still be identified by the sign of Re(β)Re(Z_B).

If the structure of the unit cell is symmetrical, the Bloch impedance and the wave number can be analytically expressed by the S parameters below:

$$\beta(\omega) = a\cos\left(\frac{1-S_{11}^2+S_{22}^2}{2S_{21}}\right), \tag{4}$$

$$Z_B(\omega) = \frac{jS_{21}\sin\beta}{1-S_{11}^2-S_{22}^2}. \tag{5}$$

For symmetrical unit cells, if we renormalized the S parameters in terms of the Bloch impedance, we can obtain

$$[S]_{Z_B} = \begin{bmatrix} 0 & \exp(-j\beta d) \\ \exp(-j\beta d) & 0 \end{bmatrix}. \tag{6}$$

It is seen that the unit cell is equivalent to a segment of a TL with characteristic impedance Z_B and phase delay βd. If the CRLH TL is open-ended, its input impedance is

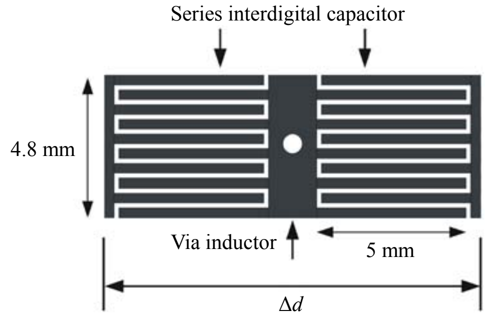
$$Z_{in} = -jZ_B\cot(\beta d), \tag{7}$$

while the input impedance of the shorted CRLH TL is

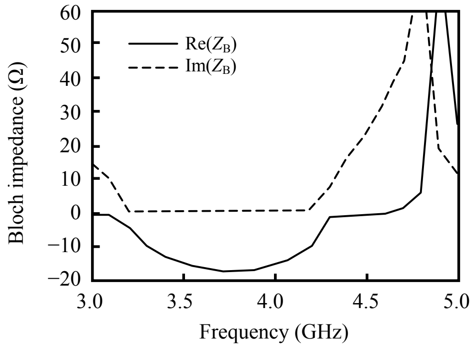
$$Z_{in} = jZ_B\tan(\beta d). \tag{8}$$

DESIGN OF A NOVEL RESONATOR BASED ON CRLH TLS

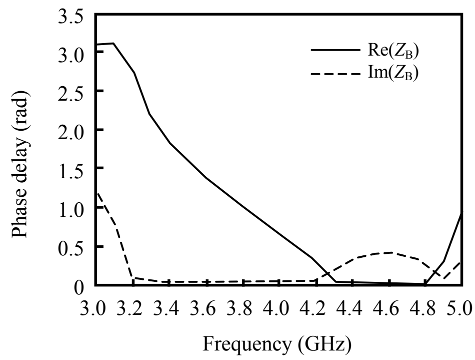
With a little modification, the microstrip-based unit cell becomes symmetrical. Fig.4a shows the new structure and its dimensions. It consists of two series interdigital capacitors embedded with an inductor and was fabricated in a substrate with thickness d=1.5 mm and relative permittivity ε_r=2.65. Using the new approach described in the above section, the Bloch impedance Z_B and phase delay βd were obtained and plotted in Figs.4b and 4c, showing that the LH band exists between 3.2~4.2 GHz.



(a)



(b)



(c)

Fig.4 The symmetrical unit cell structure. (a) Structure; (b) Bloch impedance; (c) Phase delay

In LH band, Z_B is negative and β is positive. If βd<π/2, the open-ended CRLH TL is equivalent to an inductance. This is different from the conventional TLs. These unusual properties can be applied to design a novel resonator whose topology is shown in Fig.5.

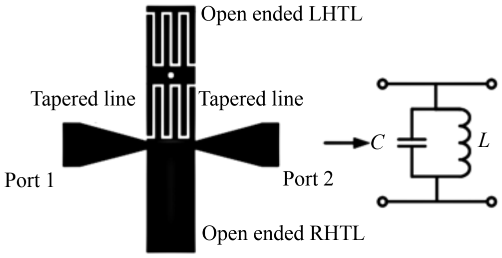


Fig.5 The topology of the novel resonator with open ended TL

The upper part is open-ended LH TL equivalent to an inductance and the lower part is open-ended RH TL equivalent to a capacitance. Two tapered microstrips are used to match the feed lines at the two ports. The total structure serves as a parallel connected LC resonator. The novel resonator was fabricated in substrate with thickness h=1.5 mm and relative permittivity ε_r=2.65. Its photograph is shown in Fig.6. It is measured with Agilent 8722E network analyzer and the result is plotted in Fig.7.

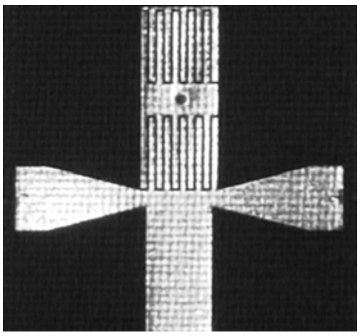


Fig.6 Photograph of the novel resonator with open ended TL

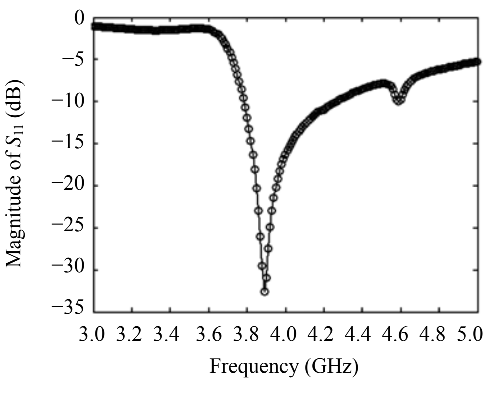


Fig.7 Measured S_{11} of the novel resonator with open ended TL

The resonant frequency is 3.89 GHz and lies in the LH frequency band of the CRLH TL. Theoretically speaking, the resonance frequency is not dependent on the total length of the resonators, but on the ratio of the length of the LH and RH parts. This property is similar to the resonator introduced by Engheta (2002). Since the CRLH TL is dispersive, the novel resonator is generally not resonant in the harmonic frequencies. It may have potential applications in compact filters with high harmonic suppression.

CONCLUSION

In this work, a new approach was developed for analyzing the characteristics of CRLH. The Bloch impedance Z_B and the wave number β are directly obtained from the S parameters of the unit cells. The LH and RH frequency bands are identified by the sign of Re(β)Re(Z_B). The new approach has some advantages over the LC parameters extraction method and was used to design a novel resonator. The simulation and experimental results accorded well with the theoretical analysis. The novel resonator may have potential applications in compact filters with high harmonic suppression.

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