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LDPC based time-frequency double differential space-time coding for multi-antenna OFDM systems*

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Abstract: Differential space-time coding was proposed recently in the literature for multi-antenna systems, where neither the transmitter nor the receiver knows the fading coefficients. Among existing schemes, double differential space-time (DDST) coding is of special interest because it is applicable to continuous fast time-varying channels. However, it is less effective in frequency-selective fading channels. This paper's authors derived a novel time-frequency double differential space-time (TF-DDST) coding scheme for multi-antenna orthogonal frequency division multiplexing (OFDM) systems in a time-varying frequency-selective fading environment, where double differential space-time coding is introduced into both time domain and frequency domain. Our proposed TF-DDST-OFDM system has a low-complexity non-coherent decoding scheme and is robust for time- and frequency-selective Rayleigh fading. In this paper, we also propose the use of state-of-the-art low-density parity-check (LDPC) code in serial concatenation with our TF-DDST scheme as a channel code. Simulations revealed that the LDPC based TF-DDST OFDM system has low decoding complexity and relatively better performance.

Key words: Differential space-time coding, OFDM, Time-varying channels, Frequency-selective fading, LDPC
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INTRODUCTION

High data rate and high quality multimedia services are required in beyond third generation (B3G) and fourth generation (4G) mobile communications. Space-time coding scheme has been proposed (Tarokh *et al*., 1998; 1999) to achieve higher capacity and data rate. So far, most research on space-time coding assumed that accurate channel estimations are available at the receivers. However, accurate channel estimation is difficult and too many training symbols are required in a rapidly changing mobile environment (Shan *et al*., 2004).

Single differential space-time (SDST) coding schemes were proposed to achieve diversity gains without channel state information (CSI) (Hughes, 2000). SDST coding schemes allow for slowly changing channels that have to remain invariant within two consecutive symbols. So it is less effective in rapidly fading environments. In order to allow for fast time-varying fading channels, double differential space-time coding (DDST) was proposed (Liu *et al.*, 2001). However, in DDST, channel delay resulting from multi-path fading is assumed to be smaller than the symbol duration, which cannot be guaranteed in frequency-selective fading wireless channels. To combat frequency-selective fading, Yao and Howlader (2002) proposed a DDST-OFDM system, in which DDST is introduced only into time domain and the differential process is performed between adjacent OFDM frames in the same sub-carrier.

In this paper, a novel time-frequency double differential space-time (TF-DDST) coding scheme is proposed for multi-antenna OFDM systems, in which double differential space-time coding is introduced into both time domain and frequency domain. A cor-

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responding low-complexity non-coherent decoding scheme is also proposed. Our proposed TF-DDST-OFDM system is robust for time- and frequency-selective Rayleigh fading channels, and requires less decoding delay compared with DDST-OFDM system.

From the information theoretic perspective, it is necessary to apply channel coding to approach further the channel capacity limit. So, channel coding is indispensable in a practical communication system. Recently, state-of-the-art low-density parity-check (LDPC) codes (Gallager, 1962; 1963) with good performance close to the Shannon limit, have attracted much attention. LDPC codes have been applied to space-time coded OFDM system (Futaki and Ohtsuki, 2003; Lu *et al*., 2002) and unitary space-time coded OFDM system (Yoshimochi *et al*., 2003). In this paper, LDPC code is adopted as the channel code of our proposed TF-DDST-OFDM system. The decoding algorithm of LDPC-TF-DDST-OFDM system is derived in the paper via analysis of TF-DDST decoding scheme. System performance is evaluated and compared with TF-DDST-OFDM system via simulations.

SYSTEM MODEL

Fig.1 depicts the block diagram of our proposed TF-DDST-OFDM system with N_c subcarriers, N_t transmit antennas, and N_r receive antennas. At the transmitter, each information symbol is mapped into a space-time code word, which spans N_x adjacent OFDM symbols and one subcarrier. We define the N_x OFDM symbol intervals as one OFDM time block and denote TF-DDST code matrix as $\boldsymbol{C}(k,i)$, in which k is the subcarriers index and i is the index of OFDM time blocks. $\boldsymbol{C}(k,i)$ is a $N_x \times N_t$ matrix, as shown below:

$$\boldsymbol{C}(k,i)=\begin{pmatrix} c_1(iN_x) & \cdots & c_{N_t}(iN_x) \\ \vdots & \ddots & \vdots \\ c_1(iN_x+N_x-1) & \cdots & c_{N_t}(iN_x+N_x-1) \end{pmatrix}. \quad (1)$$

Consider the time-varying channel response between the mth transmit antenna and the nth receive antenna. Following (Proakis, 1995), the time-domain channel impulse response can be modeled as a tapped delay line.

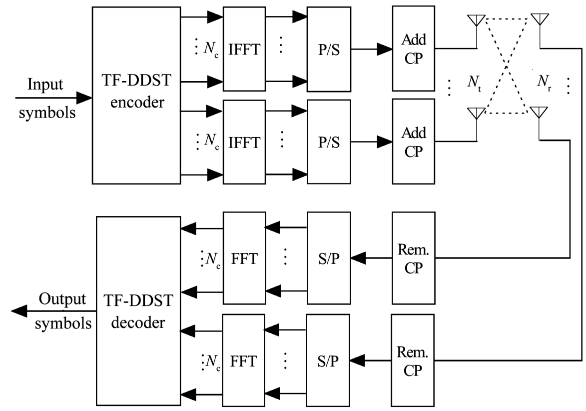

Fig.1 Block diagram of TF-DDST-OFDM system

$$h_{mn}(\tau;t)=\sum_{p=1}^{L}\alpha_{mn}(p;t)\delta\left(\tau-\frac{n_p}{N_c\Delta f}\right), \quad (2)$$

where $\delta(\cdot)$ is the Dirac delta function, L denotes the number of nonzero taps and $\alpha_{mn}(p;t)$ is the complex amplitude of the pth nonzero tap, whose delay is $n_p/(N_c\Delta f)$, where n_p is an integer and Δf is the tone spacing of the OFDM system. Similar to the description in (Yao and Howlader, 2002), we assume that the time-varying effect is small and that the Doppler frequency shift is invariable during the period of maximum channel delay. Then $\alpha_{mn}(p;t)$ can be expressed as:

$$\alpha_{mn}(p;t)=\tilde{h}_{mn}(p)\exp(\mathrm{j}2\pi f_n t), \quad (3)$$

where f_n is the Doppler frequency shift caused by relative motion between the transmit antennas and receive antennas. After discrete Fourier transform (DFT), the frequency-domain channel response can be derived from the time-domain response as:

$$\begin{aligned} H_{mn}(k;t) &= \sum_{p=1}^{L}\alpha_{mn}(p;t)\exp(-\mathrm{j}2\pi k n_p/N_c) \\ &= \left[\sum_{p=1}^{L}\tilde{h}_{mn}(p)\exp(-\mathrm{j}2\pi k n_p/N_c)\right]\cdot\exp(\mathrm{j}2\pi f_n t) \\ &= \tilde{H}_{mn}(k)\cdot\exp(\mathrm{j}2\pi f_n t). \end{aligned} \quad (4)$$

It is assumed that $\tilde{H}_{mn}(k)$ remains invariant during at least two consecutive OFDM time blocks for the kth sub-carrier and that the Doppler frequency f_n is common to all transmit antennas, which is valid if the multi-path components originate far away from the receiver so that they all share a common angle of arrival. For simplicity of description, $\tilde{H}_{mn}(k)$ is called fading component and $\exp(\mathrm{j}2\pi f_n t)$ is called Doppler component in the rest of the paper. At the nth receive antenna, DFT is applied to the signals received from N_t transmit antennas. The received signal $\boldsymbol{X}_n(k,i)$ at the kth sub-carrier and during the ith OFDM time block, is obtained as:

$$\boldsymbol{X}_n(k,i) = \mathrm{e}^{\mathrm{j}2\pi f_n i N_x} \boldsymbol{\varLambda}_n \boldsymbol{C}(k,i)\boldsymbol{H}_n(k) + \boldsymbol{Z}_n(k,i), \tag{5}$$

where $\boldsymbol{\varLambda}_n := \mathrm{diag}(1, \mathrm{e}^{\mathrm{j}2\pi f_n}, \ldots, \mathrm{e}^{\mathrm{j}2\pi f_n (N_x-1)}), \boldsymbol{H}_n(k) := (\tilde{H}_{1n}(k), \tilde{H}_{2n}(k), \ldots, \tilde{H}_{N_\mathrm{t}n}(k))^\mathrm{T}$, $\boldsymbol{Z}_n(k,i)$ is the noise defined as $\boldsymbol{Z}_n(k,i) := (z_n(k,iN_x),\ldots,z_n(k,iN_x+N_x-1))^\mathrm{T}$, which is circularly symmetric complex Gaussian distributed with variance N_0.

TF-DDST CODING AND DECODING

The method proposed in (Yao and Howlader, 2002), in which OFDM is concatenated with double differential space-time coding (DDST), is an effective way to suppress time-selective and frequency-selective fading. In their scheme, DDST is introduced in consecutive OFDM symbols in the same sub-carrier and the fading component of frequency-domain channel response $\tilde{H}_{mn}(k)$ is assumed to be constant during three consecutive OFDM time blocks. In this paper, a novel time-frequency double differential space-time coding scheme is proposed for multi-antenna OFDM system, adopting DDST in both time and frequency domains. The new coding scheme has relatively better performance and needs only two OFDM time blocks to detect the information symbols.

TF-DDST encoding scheme

TF-DDST code matrix, denoted as $\boldsymbol{C}(k,i)$, is chosen as a code matrix with orthogonal columns, i.e.

$$\boldsymbol{C}^\mathrm{H}(k,i)\boldsymbol{C}(k,i) = N_x \boldsymbol{I}_{N_\mathrm{t}} \; (i \geq 1, k = 1,2,\ldots,N_\mathrm{c}). \tag{6}$$

In order to maximize the transmission rate, $N_x=N_\mathrm{t}=N$ is assumed.

DDST is introduced into both time-domain and frequency-domain and our TF-DDST code matrices are designed to satisfy:

$$\boldsymbol{C}(k,i) = \boldsymbol{G}(k,i)\boldsymbol{C}(k,i-1) \quad (i \geq 2, k = 1,2,\ldots,N_\mathrm{c}), \tag{7}$$

where the generating matrix $\boldsymbol{G}(k,i)$ obeys

$$\begin{aligned} &\boldsymbol{G}(k,i) = \boldsymbol{F}(k,i)\boldsymbol{G}(k-1,i), \quad (k = 2,3,\ldots,N_\mathrm{c}, i \geq 1), \\ &\boldsymbol{G}(k,1) = \boldsymbol{I}_N, \quad (k = 1,2,\ldots,N_\mathrm{c}), \end{aligned} \tag{8}$$

where k is the index of sub-carriers and i is the index of OFDM time blocks. The matrix $\boldsymbol{F}(k,i)$ is mapped from the information symbol one-to-one. In the design of the transmit matrices, we consider diagonal unitary space-time coding, i.e. $\boldsymbol{F}^\mathrm{H}(k,i)\boldsymbol{F}(k,i)=\boldsymbol{I}_N$ $(\forall \boldsymbol{F}(k,i) \in \boldsymbol{\varOmega})$, where $\boldsymbol{\varOmega}$ is the group of $N\times N$ unitary and diagonal matrices.

Sub-optimal TF-DDST decoding

From the description of generation of TF-DDST code $\boldsymbol{C}(k,i)$ (Eq.(7) and Eq.(8)) and the expression of the received signal in the nth receive antenna (Eq.(5)), four corresponding receive matrices $\boldsymbol{X}_n(k-1,i-1)$, $\boldsymbol{X}_n(k-1,i)$, $\boldsymbol{X}_n(k,i-1)$, $\boldsymbol{X}_n(k,i)$ that are adjacent in time and frequency domains can be written as:

$$\begin{aligned} \boldsymbol{X}_n(k-1,i-1) &= \mathrm{e}^{\mathrm{j}2\pi f_n (i-1) N_x} \boldsymbol{\varLambda}_n \boldsymbol{C}(k-1,i-1)\boldsymbol{H}_n(k-1) \\ &\quad + \boldsymbol{Z}_n(k-1,i-1), \\ \boldsymbol{X}_n(k-1,i) &= \mathrm{e}^{\mathrm{j}2\pi f_n i N_x} \boldsymbol{G}(k-1,i)\boldsymbol{\varLambda}_n \boldsymbol{C}(k-1,i-1)\boldsymbol{H}_n(k-1) \\ &\quad + \boldsymbol{Z}_n(k-1,i), \\ \boldsymbol{X}_n(k,i-1) &= \mathrm{e}^{\mathrm{j}2\pi f_n (i-1) N_x} \boldsymbol{\varLambda}_n \boldsymbol{C}(k,i-1)\boldsymbol{H}_n(k) + \boldsymbol{Z}_n(k,i-1), \\ \boldsymbol{X}_n(k,i) &= \mathrm{e}^{\mathrm{j}2\pi f_n i N_x} \boldsymbol{G}(k,i)\boldsymbol{\varLambda}_n \boldsymbol{C}(k,i-1)\boldsymbol{H}_n(k) + \boldsymbol{Z}_n(k,i). \end{aligned} \tag{9}$$

Considering the complexity of ML detection, a sub-optimal yet low-complexity non-coherent decoding approach is proposed in this section. First, the unknown fading components $\boldsymbol{H}_n(k-1)$ and $\boldsymbol{H}_n(k)$ are removed by use of the characteristic of differential process between two adjacent OFDM time blocks in the same sub-carrier, which is described as

$$\begin{aligned} &\boldsymbol{X}_n(k-1,i) - \boldsymbol{Z}_n(k-1,i) \\ &\quad = \mathrm{e}^{\mathrm{j}2\pi f_n N_x} \boldsymbol{G}(k-1,i)[\boldsymbol{X}_n(k-1,i-1) \end{aligned}$$

$$\begin{aligned}&-\boldsymbol{Z}_n(k-1,i-1)]\boldsymbol{X}_n(k,i)-\boldsymbol{Z}_n(k,i)\\&=\mathrm{e}^{\mathrm{j}2\pi f_n N_x}\boldsymbol{G}(k,i)\left[\boldsymbol{X}_n(k,i-1)-\boldsymbol{Z}_n(k,i-1)\right].\end{aligned}\tag{10}$$

And then, the Doppler component is eliminated by performing the outer product to get:

$$\begin{aligned}&\mathrm{diag}\left\{[\boldsymbol{X}_n(k-1,i)-\boldsymbol{Z}_n(k-1,i)][\boldsymbol{X}_n^{\mathrm{H}}(k,i)-\boldsymbol{Z}_n^{\mathrm{H}}(k,i)]\right\}\\&=\boldsymbol{G}(k-1,i)\mathrm{diag}\left\{[\boldsymbol{X}_n(k-1,i-1)-\boldsymbol{Z}_n(k-1,i-1)]\right.\\&\quad\left.\times[\boldsymbol{X}_n^{\mathrm{H}}(k,i-1)-\boldsymbol{Z}_n^{\mathrm{H}}(k,i-1)]\right\}\boldsymbol{G}^{\mathrm{H}}(k-1,i)\boldsymbol{F}^{\mathrm{H}}(k,i)\\&=\mathrm{diag}\left\{[\boldsymbol{X}_n(k-1,i-1)-\boldsymbol{Z}_n(k-1,i-1)]\times[\boldsymbol{X}_n^{\mathrm{H}}(k,i-1)\right.\\&\quad\left.-\boldsymbol{Z}_n^{\mathrm{H}}(k,i-1)]\right\}\boldsymbol{F}^{\mathrm{H}}(k,i).\end{aligned}\tag{11}$$

For simplicity, we define

$$\begin{aligned}&\boldsymbol{X}_n(k-1,i-1)=\boldsymbol{X}_{n1},\quad \boldsymbol{X}_n(k-1,i)=\boldsymbol{X}_{n2},\\&\boldsymbol{X}_n(k,i-1)=\boldsymbol{X}_{n3},\qquad \boldsymbol{X}_n(k,i)=\boldsymbol{X}_{n4}.\end{aligned}\tag{12}$$

$\boldsymbol{Z}_{n1}$, $\boldsymbol{Z}_{n2}$, $\boldsymbol{Z}_{n3}$ and $\boldsymbol{Z}_{n4}$ are also defined similar to Eq.(12), and $\boldsymbol{F}(k,i)$ is denoted as $\boldsymbol{F}$. Then, Eq.(11) can be rewritten as

$$\begin{aligned}&\mathrm{diag}[(\boldsymbol{X}_{n2}-\boldsymbol{Z}_{n2})(\boldsymbol{X}_{n4}^{\mathrm{H}}-\boldsymbol{Z}_{n4}^{\mathrm{H}})]\\&\quad=\mathrm{diag}[(\boldsymbol{X}_{n1}-\boldsymbol{Z}_{n1})(\boldsymbol{X}_{n3}^{\mathrm{H}}-\boldsymbol{Z}_{n3}^{\mathrm{H}})]\boldsymbol{F}^{\mathrm{H}}.\end{aligned}\tag{13}$$

We collect all noise terms in Eq.(13) to the right-hand side and get:

$$\mathrm{diag}(\boldsymbol{X}_{n2}\boldsymbol{X}_{n4}^{\mathrm{H}})=\mathrm{diag}(\boldsymbol{X}_{n1}\boldsymbol{X}_{n3}^{\mathrm{H}})\boldsymbol{F}^{\mathrm{H}}+\boldsymbol{N}_n,\tag{14}$$

where

$$\begin{aligned}\boldsymbol{N}_n=&\,\mathrm{diag}[\boldsymbol{X}_{n2}\boldsymbol{Z}_{n4}^{\mathrm{H}}+\boldsymbol{Z}_{n2}\boldsymbol{X}_{n4}^{\mathrm{H}}+\boldsymbol{Z}_{n1}\boldsymbol{Z}_{n3}^{\mathrm{H}}-\boldsymbol{Z}_{n2}\boldsymbol{Z}_{n4}^{\mathrm{H}}\\&-\boldsymbol{X}_{n1}\boldsymbol{Z}_{n3}^{\mathrm{H}}-\boldsymbol{Z}_{n1}\boldsymbol{X}_{n3}^{\mathrm{H}}].\end{aligned}\tag{15}$$

For notational simplicity, Eq.(14) can be re-written as

$$\boldsymbol{r}_n(i)=\boldsymbol{r}_n(i-1)\boldsymbol{F}^{\mathrm{H}}+\boldsymbol{N}_n,\tag{16}$$

where

$$\begin{aligned}&\boldsymbol{r}_n(i)=\mathrm{diag}(\boldsymbol{X}_{n2}\boldsymbol{X}_{n4}^{\mathrm{H}})=\mathrm{diag}(\boldsymbol{X}_n(k-1,i)\boldsymbol{X}_n^{\mathrm{H}}(k,i)),\\&\boldsymbol{r}_n(i-1)=\mathrm{diag}(\boldsymbol{X}_{n1}\boldsymbol{X}_{n3}^{\mathrm{H}})\\&\qquad=\mathrm{diag}(\boldsymbol{X}_n(k-1,i-1)\boldsymbol{X}_n^{\mathrm{H}}(k,i-1)).\end{aligned}$$

Then, the signal of all the receive antennas can be written as

$$\boldsymbol{R}(i)=\boldsymbol{R}(i-1)[\boldsymbol{I}_{N_\mathrm{r}}\otimes\boldsymbol{F}^{\mathrm{H}}]+\boldsymbol{N},\tag{17}$$

where $\boldsymbol{R}(i)=\mathrm{diag}[\boldsymbol{r}_1(i),\boldsymbol{r}_2(i),\ldots,\boldsymbol{r}_{N_\mathrm{r}}(i)]$, $\boldsymbol{N}=\mathrm{diag}(\boldsymbol{N}_1,\boldsymbol{N}_2,\ldots,\boldsymbol{N}_{N_\mathrm{r}})$, $\otimes$ stands for Kronecker product.

In order to derive the sub-optimal decoding rule, the second-order noise term in $\boldsymbol{N}$ is ignored and the noise $\boldsymbol{N}$ is approximated as a zero-mean complex Gaussian vector whose covariance matrix is denoted as

$$\boldsymbol{\Sigma}=\begin{pmatrix}\boldsymbol{\Sigma}_1&\cdots&\boldsymbol{0}\\\vdots&\ddots&\vdots\\\boldsymbol{0}&\cdots&\boldsymbol{\Sigma}_{N_\mathrm{r}}\end{pmatrix},\tag{18}$$

where

$$\begin{aligned}\boldsymbol{\Sigma}_n=&\,E(\boldsymbol{N}_n\boldsymbol{N}_n^{\mathrm{H}})=N_0\mathrm{diag}(\boldsymbol{X}_{n1}\boldsymbol{X}_{n1}^{\mathrm{H}}\\&+\boldsymbol{X}_{n2}\boldsymbol{X}_{n2}^{\mathrm{H}}+\boldsymbol{X}_{n3}^{\mathrm{H}}\boldsymbol{X}_{n3}+\boldsymbol{X}_{n4}^{\mathrm{H}}\boldsymbol{X}_{n4}).\end{aligned}\tag{19}$$

Because $\boldsymbol{N}$ is complex Gaussian distributed, so the probability function of $\boldsymbol{R}(i)$ conditioned on $\boldsymbol{R}(i-1)$ and $\boldsymbol{F}$ can be written as

$$\begin{aligned}&\mathrm{Pr}(\boldsymbol{R}(i)\,|\,\boldsymbol{R}(i-1),\boldsymbol{F})=\frac{1}{\pi^{N_\mathrm{t}N_\mathrm{r}}\det(\boldsymbol{\Sigma})}\exp\left\{-\mathrm{tr}\left\{[\boldsymbol{R}(i)\right.\right.\\&\left.\left.-\boldsymbol{R}(i-1)(\boldsymbol{I}_{N_\mathrm{r}}\otimes\boldsymbol{F}^{\mathrm{H}})]^{\mathrm{H}}\times\boldsymbol{\Sigma}^{-1}[\boldsymbol{R}(i)-\boldsymbol{R}(i-1)(\boldsymbol{I}_{N_\mathrm{r}}\otimes\boldsymbol{F}^{\mathrm{H}})]\right\}\right\},\end{aligned}\tag{20}$$

where "tr" and "H" denote the trace and conjugate-transpose respectively. TF-DDST decoder chooses $\boldsymbol{F}$ to maximize the probability $\mathrm{Pr}(\boldsymbol{R}(i)|\boldsymbol{R}(i-1),\boldsymbol{F})$. Then the estimate of $\boldsymbol{F}$ is

$$\begin{aligned}\hat{\boldsymbol{F}}&=\arg\max_{\boldsymbol{F}\in\boldsymbol{\Omega}}\mathrm{Re}\,\mathrm{tr}[(\boldsymbol{I}_{N_\mathrm{r}}\otimes\boldsymbol{F})\boldsymbol{R}^{\mathrm{H}}(i-1)\boldsymbol{\Sigma}^{-1}\boldsymbol{R}(i)]\\&=\arg\max_{\boldsymbol{F}\in\boldsymbol{\Omega}}\sum_{n=1}^{N_\mathrm{r}}\mathrm{Re}\,\mathrm{tr}\left\{\boldsymbol{F}\cdot[\mathrm{diag}(\boldsymbol{X}_{n1}\boldsymbol{X}_{n3}^{\mathrm{H}})]^*\times[\mathrm{diag}(\boldsymbol{X}_{n1}\boldsymbol{X}_{n1}^{\mathrm{H}}\right.\\&\quad\left.+\boldsymbol{X}_{n2}\boldsymbol{X}_{n2}^{\mathrm{H}}+\boldsymbol{X}_{n3}^{\mathrm{H}}\boldsymbol{X}_{n3}+\boldsymbol{X}_{n4}^{\mathrm{H}}\boldsymbol{X}_{n4})]^{-1}\times\mathrm{diag}(\boldsymbol{X}_{n2}\boldsymbol{X}_{n4}^{\mathrm{H}})\right\}.\end{aligned}\tag{21}$$

It is easy to recover the information symbols from $\hat{\boldsymbol{F}}$, since the mapping between the information symbol and matrix $\boldsymbol{F}$ is one-to-one. From the description above, it is evident that our TF-DDST-OFDM system needs only two OFDM time blocks to detect the information symbols, while DDST-OFDM system needs at least three OFDM time blocks.

PERFORMANCE ANALYSIS

Performance analysis of our TF-DDST-OFDM system is conducted in this section, based on the calculation of the pairwise error probability (PEP), defined as the probability of transmitting $\boldsymbol{F}$ and deciding in favor of $\boldsymbol{F}'$ at the decoder.

In this section, we only focus on the nth receive antenna, and our derivations are based on the following assumptions: (1) $\tilde{h}_{mn}(p)$'s in Eq.(3) are i.i.d. Gaussian distributed with variance $1/2L$ per dimension and mean 0; (2) High SNR. Based on Eq.(16), the conditional PEP for the nth receive antenna can be approximately by

$$P\left(\boldsymbol{F}\to\boldsymbol{F}'\,|\,\boldsymbol{r}_n(i-1)\right)\le\exp\left(-d^2(\boldsymbol{F},\boldsymbol{F}')N^2/16N_0\right),\quad(22)$$

in which,

$$d^2(\boldsymbol{F},\boldsymbol{F}')=\left\|\boldsymbol{r}_n(i-1)(\boldsymbol{F}-\boldsymbol{F}')^{\mathrm{H}}\right\|^2 =\mathrm{tr}\left\{\boldsymbol{e}\boldsymbol{r}_n^{\mathrm{H}}(i-1)\boldsymbol{r}_n(i-1)\boldsymbol{e}^{\mathrm{H}}\right\},$$

where $\|\cdot\|$ denotes Frobenius norm and $\boldsymbol{e}=\boldsymbol{F}-\boldsymbol{F}'$.

At high SNR, we can ignore the noise term in Eq.(9), and $d^2(\boldsymbol{F},\boldsymbol{F}')$ can be expressed as:

$$d^2(\boldsymbol{F},\boldsymbol{F}')\approx\mathrm{tr}\left\{\boldsymbol{H}^{\mathrm{H}}\boldsymbol{e}\boldsymbol{C}_2^{\mathrm{H}}\boldsymbol{C}_1^{\mathrm{H}}\boldsymbol{C}_1\boldsymbol{C}_2\boldsymbol{e}^{\mathrm{H}}\boldsymbol{H}\right\},\quad(23)$$

where $\boldsymbol{H}=\mathrm{diag}\{\boldsymbol{H}_n(k-1)\boldsymbol{H}_n^{\mathrm{H}}(k)\}$, $\boldsymbol{C}_1=\boldsymbol{C}(k-1,i-1)$, $\boldsymbol{C}_2=\boldsymbol{C}(k,i-1)$.

Let vec$\{\boldsymbol{A}\}$ denote a vector obtained from the diagonal elements of matrix $\boldsymbol{A}$. Then, Eq.(23) can be rewritten as:

$$d^2(\boldsymbol{F},\boldsymbol{F}')\approx\boldsymbol{h}^{\mathrm{H}}\left(\boldsymbol{e}\boldsymbol{C}_2^{\mathrm{H}}\boldsymbol{C}_1^{\mathrm{H}}\boldsymbol{C}_1\boldsymbol{C}_2\boldsymbol{e}^{\mathrm{H}}\right)\boldsymbol{h},\quad(24)$$

where $\boldsymbol{h}=\mathrm{vec}(\boldsymbol{H})$. From Eq.(6), we have $\boldsymbol{C}_1^{\mathrm{H}}\boldsymbol{C}_1=\boldsymbol{C}_2^{\mathrm{H}}\boldsymbol{C}_2=N\boldsymbol{I}_N$. So, $d^2(\boldsymbol{F},\boldsymbol{F}')$ can be expressed as:

$$d^2(\boldsymbol{F},\boldsymbol{F}')\approx(\boldsymbol{h}')^{\mathrm{H}}\boldsymbol{\Phi}(\boldsymbol{h}'),\quad(25)$$

where $\boldsymbol{h}'=\boldsymbol{h}/\sqrt{2}$, $\boldsymbol{\Phi}=2N^2\boldsymbol{e}\boldsymbol{e}^{\mathrm{H}}$. With Assumption (1), it is easy to derive that $\boldsymbol{h}'$ is complex Gaussian with zero-mean and that covariance matrix $E\{\boldsymbol{h}'(\boldsymbol{h}')^{\mathrm{H}}\}=E\{\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}\}/2=\boldsymbol{I}_N$. Substituting Eq.(25) into Eq.(22) and taking the statistical expectation with respect of $\boldsymbol{h}'$, we have:

$$P\left(\boldsymbol{F}\to\boldsymbol{F}'\right)\le\left(\frac{N^2}{16N_0}G_{\mathrm{c}}\right)^{-G_{\mathrm{d}}},\quad(26)$$

where $G_{\mathrm{d}}=\min\limits_{\forall\boldsymbol{F}\neq\boldsymbol{F}'}\mathrm{rank}(\boldsymbol{\Phi})$, $G_{\mathrm{c}}=\min\limits_{\forall\boldsymbol{F}\neq\boldsymbol{F}'}\left[\det(\boldsymbol{\Phi})\right]^{1/\mathrm{rank}(\boldsymbol{\Phi})}$. G_{d} and G_{c} are defined as diversity gain and coding gain respectively. Then, the code design criteria for our TF-DDST-OFDM system are:

(1) (Diversity gain criterion) Design optimal diagonal unitary group $\boldsymbol{\Omega}$ such that $\boldsymbol{e}=\boldsymbol{F}-\boldsymbol{F}'$ has full rank, for $\forall\boldsymbol{F}$, $\boldsymbol{F}'\in\boldsymbol{\Omega}$ and $\boldsymbol{F}\neq\boldsymbol{F}'$;

(2) (Coding gain criterion) Design $\boldsymbol{\Omega}$ such that $|\det(\boldsymbol{F}-\boldsymbol{F}')|$ is maximized for $\forall\boldsymbol{F}$, $\boldsymbol{F}'\in\boldsymbol{\Omega}$ and $\boldsymbol{F}\neq\boldsymbol{F}'$.

This implies that if a diagonal unitary group is optimal for DST-OFDM system, it is also optimal for our TF-DDST-OFDM system. So, we can adopt the group codes shown in (Tarokh *et al*., 1999; Hughes, 2002; Gallager, 1963) for the code construction in our TF-DDST-OFDM system.

LDPC-TF-DDST SCHEME

LDPC codes

Low-density parity-check (LDPC) codes were first proposed by Gallager in 1962 and recently reexamined in (MacKay and Neal, 1996; MacKay, 1999). It had shown that these codes achieve remarkable performance with iterative decoding that is very close to the Shannon limit (MacKay and Neal, 1996).

An LDPC code is a linear block code characterized by a very sparse parity-check matrix. The parity-check matrix $\boldsymbol{H}$ for an (n, k) LDPC code of rate $R=k/n$ is an $(n-k)\times n$ matrix. Both the number of 1's per column (column weight) and the number of 1's per row (row weight) are very small compared to the block length n. Apart from these constraints, the ones are placed randomly in $\boldsymbol{H}$. When the number of ones in every column is the same, the code is known as a regular LDPC code; otherwise, it is called an irregular LDPC code.

The algorithm used for LDPC decoding is an iterative message algorithm known as the sum-product algorithm (SPA) (MacKay, 1999). It determines a posteriori probabilities for bit values based on a priori information, improving the accuracy of these calculations at each iteration. The initialization of SPA is important for LDPC decoding, whose task is to compute the first likelihood rate of the received signal.

Algorithm for LDPC-TF-DDST scheme

Fig.2 shows the block diagram of LDPC-TF-DDST scheme. For simplicity, we employ finite-geometry LDPC codes (Kou *et al.*, 2001) using the parameters shown in Table 1. If other better LDPC codes are adopted, the system performance will be improved. In the receiver, the received signal can be expressed by $\boldsymbol{R}(i)$ and $\boldsymbol{R}(i-1)$ as shown in Eq.(17). In order to perform the decoding, the first likelihood rate of code bits "1" and "0" for all the code bits corresponding to received signals should be computed. And then, the sum-product algorithm is used to decode iteratively. The algorithm for computation of first likelihood rate is as follows.

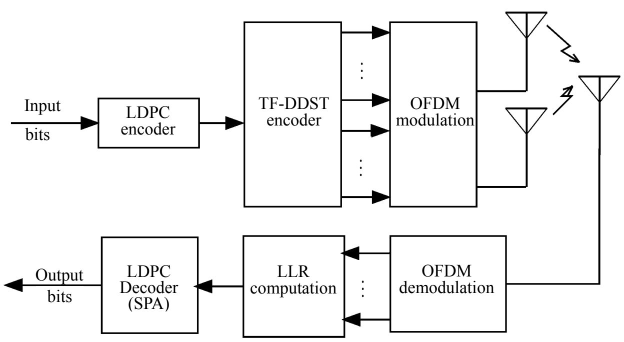

Fig.2 Block diagram of LDPC-TF-DDST scheme

Table 1 Parameters of finite-geometry LDPC code

Code	Type-I EG-LDPC
n	255
k	175
Iteration number	5
Column weight	16
Row weight	16
Decoding algorithm	SPA

The received signals correspond to $\log_2|\boldsymbol{\Omega}|$ bits, where $|\boldsymbol{\Omega}|$ denotes the cardinality of group $\boldsymbol{\Omega}$. We denote the transmitted bits as $\boldsymbol{b}=(b_1,\ldots,b_l,\ldots,b_{\log_2|\boldsymbol{\Omega}|})$. The log-likelihood rate for the lth bit in $\boldsymbol{b}$ is given as:

$$L(b_l)=\log\frac{\Pr[b_l=1\,|\,\boldsymbol{R}(i-1),\boldsymbol{R}(i)]}{\Pr[b_l=0\,|\,\boldsymbol{R}(i-1),\boldsymbol{R}(i)]}, \tag{27}$$

which can be written as:

$$L(b_l)=\frac{\sum_{\boldsymbol{b}:b_l=1}\Pr[\boldsymbol{R}(i-1),\boldsymbol{R}(i),\boldsymbol{b}\text{ is transmitted}]}{\sum_{\boldsymbol{b}:b_l=0}\Pr[\boldsymbol{R}(i-1),\boldsymbol{R}(i),\boldsymbol{b}\text{ is transmitted}]}. \tag{28}$$

Because different $\boldsymbol{b}$ is mapped into different $\boldsymbol{F}$, and $\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b})$, we get

$$L(b_l)=\frac{\sum_{\boldsymbol{F}:\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b}),b_l=1}\Pr[\boldsymbol{R}(i-1),\boldsymbol{R}(i),\boldsymbol{F}\text{ is transmitted}]}{\sum_{\boldsymbol{F}:\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b}),b_l=0}\Pr[\boldsymbol{R}(i-1),\boldsymbol{R}(i),\boldsymbol{F}\text{ is transmitted}]}. \tag{29}$$

Assuming all the constellation points are equiprobable, Eq.(29) can be rewritten as

$$L(b_l)=\frac{\sum_{\boldsymbol{F}:\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b}),b_l=1}\Pr[\boldsymbol{R}(i)\,|\,\boldsymbol{R}(i-1),\boldsymbol{F}]}{\sum_{\boldsymbol{F}:\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b}),b_l=0}\Pr[\boldsymbol{R}(i)\,|\,\boldsymbol{R}(i-1),\boldsymbol{F}]}. \tag{30}$$

Since $\Pr(\boldsymbol{R}(i)\,|\,\boldsymbol{R}(i-1),\boldsymbol{F})$ has been evaluated in Eq.(20), $L(b_l)$ can be derived in Eq.(31).

$$L(b_l)=\log\frac{\sum_{\boldsymbol{F}:\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b}),b_l=1}\frac{1}{\pi^{N_\mathrm{t}N_\mathrm{r}}\det(\boldsymbol{\Sigma})}E'}{\sum_{\boldsymbol{F}:\boldsymbol{F}=\boldsymbol{\Phi}(\boldsymbol{b}),b_l=0}\frac{1}{\pi^{N_\mathrm{t}N_\mathrm{r}}\det(\boldsymbol{\Sigma})}E'}, \tag{31}$$

where

$$E'=\exp\{-\mathrm{tr}\{[\boldsymbol{R}(i)-\boldsymbol{R}(i-1)(\boldsymbol{I}_{N_\mathrm{r}}\otimes\boldsymbol{F}^\mathrm{H})]^\mathrm{H}\cdot\boldsymbol{\Sigma}^{-1}[\boldsymbol{R}(i)-\boldsymbol{R}(i-1)(\boldsymbol{I}_{N_\mathrm{r}}\otimes\boldsymbol{F}^\mathrm{H})]\}\}.$$

Because $\boldsymbol{R}(i)$, $\boldsymbol{R}(i-1)$, $\boldsymbol{F}$ and $\boldsymbol{\Sigma}$ are all diagonal matrices, the computation of $L(b_l)$ has low complexity. After the derivation of $L(b_l)$, iterative decoding is executed as sum-product algorithm. Moreover, this decoding scheme is not only suitable for finite-geometry LDPC codes, but is also applicable for other kinds of LDPC codes.

SIMULATION RESULTS

In this section, first, we provide computer simulation results to illustrate the performance of our proposed TF-DDST-OFDM system over a time- and frequency-selective fading channel, namely two-tap equal-power Rayleigh fading channel with maximum Doppler frequency f_d, compared with SDST-OFDM and DDST-OFDM systems. The channel coefficients are generated based on Jakes model (Jakes, 1974). The optimal (4;1;1) group codes (Liu *et al.*, 2001) with code rate R=1 are employed for all the systems.

Fig.3 illustrates the performance of the three systems in a fast fading channel, whose maximum Doppler frequency f_d is chosen to satisfy f_dT=0.05, where T denotes the period of one OFDM time block. We can see from Fig.3 that TF-DDST-OFDM system has better performance than SDST-OFDM system with gain of about 8 dB at BER of 10^{-2} and outperforms DDST-OFDM by about 1 dB at BER of 10^{-3}.

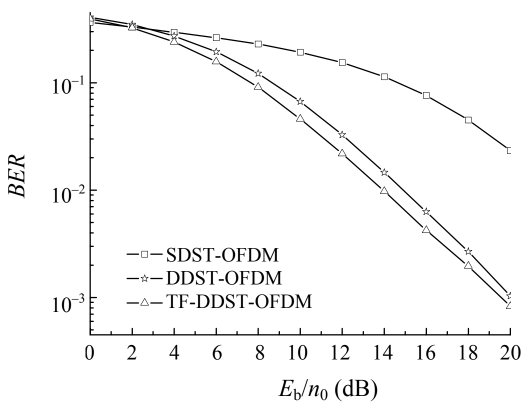

Fig.3 Performance comparison for fast fading channel (f_dT=0.05)

Fig.4 shows the performance of the three systems in a very fast fading channel, in which maximum Doppler frequency is chosen to satisfy f_dT=0.1. At this time, SDST-OFDM system has a very high BER even at high SNR, which means SDST-OFDM system is not suitable for very fast fading channel. The performance of TF-DDST-OFDM system nearly does not degrade compared with the performance in fast fading channel shown in Fig.3 and our TF-DDST-OFDM system keeps better performance compared with the DDST-OFDM system.

A conclusion can be drawn from Fig.3 and Fig.4 that TF-DDST-OFDM system is robust to time-selective and frequency-selective fading and has better performance than DDST-OFDM system.

Fig.5 shows the BER performance of LDPC-TF-DDST scheme using the parameters shown in Table 1. We assume that f_dT is 0.1, and that the channel model is also two-tap equal-power Rayleigh fading channel. The channel coefficients are also generated based on Jakes model (Jakes, 1974). Fig.5 shows that the LDPC-TF-DDST scheme can obtain high coding gains and improve the BER performance considerably compared to TF-DDST scheme. LDPC-TF-DDST scheme provides gain of about 6 dB at BER of 10^{-3}.

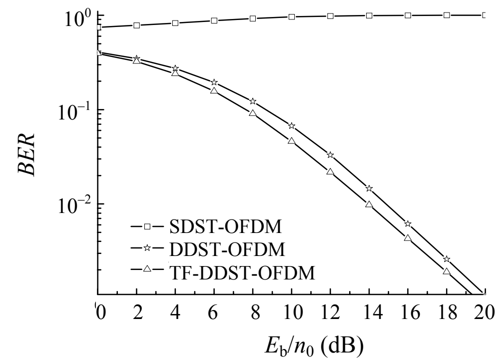

Fig.4 Performance comparison for very fast fading channel (f_dT=0.1)

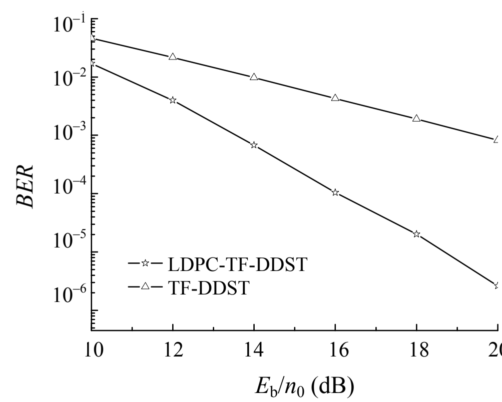

Fig.5 BER Performance of LDPC-TF-DDST Scheme

CONCLUSION

In this paper, first, a novel time-frequency double differential space-time (TF-DDST) coded OFDM system is developed, which is suitable for time-selective frequency-selective channel. In transmitter, TF-DDST encoding scheme is proposed, in which double differential space-time coding is introduced into both time domain and frequency domain. At receiver, a sub-optimal yet low-complexity non-coherent decoding scheme is proposed. TF-DDST-OFDM system has proved to be robust for the time-selective frequency-selective fading by performance analysis and computer simulations. Moreover, our TF-DDST scheme has better performance and less decoding delay than DDST-OFDM system. In this work, we also study the concatenation scheme of channel codes with our TF-DDST-OFDM system, and adopt state-of-the-art low-density parity-check (LDPC) code as our channel code. The decoding scheme for LDPC-TF-DDST-OFDM system is given in the paper. Simulation results proved that LDPC-TF-DDST-OFDM system achieves better BER performance than TF-DDST-OFDM, especially at high SNR. Moreover, increase of code length and iteration number will improve the system performance considerably.

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