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An algorithm for frequency estimation of signals composed of multiple single-tones

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Abstract: The high-accuracy, wide-range frequency estimation algorithm for multi-component signals presented in this paper, is based on a numerical differentiation and central Lagrange interpolation. With the sample sequences, which need at most 7 points and are sampled at a sample frequency of 25600 Hz, and computation sequences, using employed a formulation proposed in this paper, the frequencies of each component of the signal are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz with the amplitudes of each component of the signal varying from 1 V to 200 V and the phase angle of each component of the signal varying from 0° to 360°. The proposed algorithm needs at most a half cycle for the frequencies of each component of the signal under noisy or non-noisy conditions. A testing example is given to illustrate the proposed algorithm in Matlab environment.

Key words: Multi-component signal, Frequency estimation, Numerical differentiation, Lagrange interpolation
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INTRODUCTION

Frequency estimation is the basis of other parameter measurements of signals in intelligent instruments and meters. Frequency estimation of sinusoidal or nonsinusoidal signals is easier to realize under non-noisy conditions than under noisy conditions. Under noisy conditions, the frequency estimation becomes more difficult work due to distortion in sample data. Some pioneering algorithms, such as zero crossing technique (Moore *et al.*, 1996; Begovic *et al.*, 1993), level crossing technique (Nguyen and Srinivasan, 1984), least squares error technique (Kamwa and Grondin, 1992; Sachdev and Giray, 1985; Giray and Sachdev, 1989), Newton method (Terzija *et al.*, 1994), Kalman filter (Sachdev *et al.*, 1985; Girgis and Hwang, 1984; Girgis and Peterson, 1990; Lobos and Rezmer, 1997; Phadke *et al.*, 1983), Fourier transform (Yang and Liu, 2000; 2001; Lu *et al.*, 1998; Moore and Johns, 1994; Szafran and Rebizant, 1998; Kuang and Morris, 2002; Girgis and Ham, 1982), wavelet transform (Moore and Johns, 1994), are used for frequency estimation of signals with harmonics and noises. Such algorithms need much more time and computation for estimation if they are applied in some real-time measurement cases.

A novel algorithm proposed in this paper was developed for frequency estimation of multi-component signals using numerical differentiation and central Lagrange interpolation with multi-points. The proposed algorithm can achieve at an accuracy of 0.001% of estimation over a larger range and only spends at most 1 cycle. Compared with other algorithms, this high accuracy algorithm spends less time and computation over a wide range at and can be adaptively applied in real-time, intelligent measurement and control cases.

FORMULATION

Numerical differentiation

Given a function of voltage signal:

$$v(t)=0. \tag{1}$$

At discrete points such as (t_i, v_i) and (t_j, v_j), $i=0,1,2,\ldots,M-1$, $j=0,1,2,\ldots,M-1$ (M is derivative number), the Taylor series expansion is expressed as:

$$v(t_i)=v(t_j)+\Delta t\left.\frac{\mathrm{d}v}{\mathrm{d}t}\right|_{t=t_j}+\frac{(\Delta t)^2}{2!}\left.\frac{\mathrm{d}^2v}{\mathrm{d}t^2}\right|_{t=t_j}+\frac{(\Delta t)^3}{3!}\left.\frac{\mathrm{d}^3v}{\mathrm{d}t^3}\right|_{t=t_j}+\frac{(\Delta t)^M}{M!}\left.\frac{\mathrm{d}^Mv}{\mathrm{d}t^M}\right|_{t=t_j}+\ldots, \tag{2}$$

where $\Delta t=t_i-t_j$.

Without consideration of M order and higher order derivatives, we have central difference formulas such that:

$$v(t_{i+j})=v(t_i)+(t_{i+j}-t_i)v'(t_i)+\frac{(t_{i+j}-t_i)^2}{2!}v''(t_i)+\frac{(t_{i+j}-t_i)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(t_{i+j}-t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i),$$
$$j=k,k-1,\ldots,2,1,-1,-2,\ldots,-(k-1),-k, \tag{3}$$

where $k=floor(M/2)$. $floor(A)$ rounds the elements of A to the nearest integers less than or equal to A.

Given (t_0,v_0), (t_1,v_1), …, (t_M,v_M) with regularly spaced h, we have the following relationship:

t_0, $t_1=t_0+h$, $t_2=t_0+2h$, $t_3=t_0+3h$, …, $t_M=t_0+Mh$.

So that Eq.(3) becomes:

$$v(t_{i+j})=v(t_i)+jhv'(t_i)+\frac{(jh)^2}{2!}v''(t_i)+\frac{(jh)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(jh)^{M-1}}{(M-1)!}v^{(M-1)}(t_i),$$
$$j=k,k-1,\ldots,2,1,-1,-2,\ldots,-(k-1),-k. \tag{4}$$

With $v(t_{i-k})$–$v(t_{i+k})$ known, the solution of 1st and M–1 order derivatives can be obtained with the help of Eq.(4).

For any M, a general matrix form may be obtained as:

$$\boldsymbol{D}=\boldsymbol{C}\boldsymbol{r}, \tag{5}$$

where

$\boldsymbol{D}=[d_j]_{j=1:g}$, $\boldsymbol{r}=[r_k]_{k=1:g}$, $\boldsymbol{C}=[c_{ij}]_{i,j=1:g}$, $r_k=v^{(k)}(p)$,

$d_j=v(p+j)-v(p-j)$, $c_{ij}=2((ih)^{2j+1}/(2j+1)!)$,

$$g=\frac{M-1}{2}-2\left(\frac{M-1}{2}-floor\left(\frac{M-1}{2}\right)\right).$$

Therefore, the odd derivatives can be formulated as Eq.(6):

$$\boldsymbol{r}=\boldsymbol{C}^{-1}\boldsymbol{D}. \tag{6}$$

Similar form could be derived for even derivatives.

For example, with M=7, the 1st and 2nd order derivative of $v(t)$ at point p is expressed as follows:

$$v'(p)=\{[v(p+3)-v(p-3)]-9[v(p+2)-v(p-3)]+45[v(p+1)-v(p-3)]\}/60, \tag{7}$$

$$v''(p)=\{2[v(p+3)+v(p-3)]-27[v(p+2)+v(p-3)]+270[v(p+1)+v(p-3)]-490v(p)\}/180h^2, \tag{8}$$

where p is central point for derivative, h is regular space.

Frequency estimation

Under without noise condition, the multiple-component signal is expressed by Eq.(9):

$$v(t)=\sum_{k=1}^{K}V_k\sin(2\pi f_k t+\phi_k), \tag{9}$$

where f_k, V_k, ϕ_k is the frequency, magnitude and phase of the kth component sinusoid, K is the number of power system signal sinusoid.

The discrete-time sequence of the signal can be rewritten as Eq.(10):

$$v(n)=\sum_{k=1}^{K}V_k\sin\left(2\pi f_k\frac{nT}{N}+\phi_k\right)=\sum_{k=1}^{K}V_k\sin(2\pi f_k t_s+\phi_k), \tag{10}$$

where $t_s=nT/N$, N is the sample size, T is sample period.

Without loss of generality, a multiple-component signal with 3-components is taken into consideration, and the discrete-time sequence of the signal can be written as Eq.(11):

$$v(n)=V_1\sin(2\pi f_1 t_s+\phi_1)+V_2\sin(2\pi f_2 t_s+\phi_2)+V_3\sin(2\pi f_3 t_s+\phi_3). \quad (11)$$

The 1st-order differentiation of $v(n)$ is formulated as Eq.(12):

$$v'(n)=2\pi f_1 V_1\cos(2\pi f_1 t_s+\phi_1)+2\pi f_2 V_2\cos(2\pi f_2 t_s+\phi_2)+2\pi f_3 V_3\cos(2\pi f_3 t_s+\phi_3). \quad (12)$$

The 2nd-order differentiation of $v(n)$ is formulated as:

$$v''(n)=-V_1(2\pi f_1)^2\sin(2\pi f_1 t_s+\phi_1)-V_2(2\pi f_2)^2\times\sin(2\pi f_2 t_s+\phi_2)-V_3(2\pi f_3)^2\sin(2\pi f_3 t_s+\phi_3). \quad (13)$$

From Eqs.(12) and (13), we can obtain the following general Eq.(14):

$$v^{(i)}(n)=\sum_{r=1}^{K}V_r(2\pi f_r t_s)^i\sin(2\pi f_r t_s+\phi_r+i\pi/2),\quad i=0,1,2,... \quad (14)$$

So, we get the 4th-order differentiation of $v(n)$ as:

$$v^{(4)}(n)=V_1(2\pi f_1)^4\sin(2\pi f_1 t_s+\phi_1)+V_2(2\pi f_2)^4\times\sin(2\pi f_2 t_s+\phi_2)+V_3(2\pi f_3)^4\sin(2\pi f_3 t_s+\phi_3), \quad (15)$$

where

$$v_1(n)=V_1\sin(2\pi f_1 t_s+\phi_1). \quad (16)$$

From Eqs.(11), (13) and (15), we obtain the formula of component 1, namely $v_1(n)$ of the signal as:

$$v_1(n)=\{36(2\pi f_1)^4 v(n)+13(2\pi f_1)^2 v''(n)+v^{(4)}(n)\}/(2\pi f_1)^4. \quad (17)$$

In the same way, we obtain the formula of component 2, $v_2(n)$ and 3, $v_3(n)$ respectively as:

$$v_2(n)=\{36(2\pi f_2)^4 v(n)+13(2\pi f_2)^2 v''(n)+v^{(4)}(n)\}/(2\pi f_2)^4, \quad (18)$$

$$v_3(n)=\{36(2\pi f_3)^4 v(n)+13(2\pi f_3)^2 v''(n)+v^{(4)}(n)\}/(2\pi f_3)^4. \quad (19)$$

From Eq.(17), we can obtain the 1st-order differentiation of $v_1(n)$ as:

$$v_1'(n)=\{36(2\pi f_1)^4 v'(n)+13(2\pi f_1)^2 v^{(3)}(n)+v^{(5)}(n)\}/(2\pi f_1)^4. \quad (20)$$

In the same way, we obtain the 1st-order differentiation of $v_2(n)$ and $v_3(n)$ respectively as:

$$v_2'(n)=\{36(2\pi f_2)^4 v'(n)+13(2\pi f_2)^2 v^{(3)}(n)+v^{(5)}(n)\}/(2\pi f_2)^4, \quad (21)$$

$$v_3'(n)=\{36(2\pi f_3)^4 v'(n)+13(2\pi f_3)^2 v^{(3)}(n)+v^{(5)}(n)\}/(2\pi f_3)^4. \quad (22)$$

From Eq.(16), the 1st and 2nd-order differentiation of $v_1(n)$ can be obtained respectively as:

$$v_2'(n)=V_1 2\pi f_1\cos(2\pi f_1 t_s+\phi_1), \quad (23)$$

$$v_2''(n)=-V_1(2\pi f_1)^2\cos(2\pi f_1 t_s+\phi_1)=-(2\pi f_1)^2 v_1(n). \quad (24)$$

Using Eq.(24), the frequency of component 1 of the signal is estimated by:

$$f_1=\frac{1}{2\pi}\sqrt{\frac{-v_1''(n)}{v_1(n)}}. \quad (25)$$

In the same way, the frequency of components 2 and 3 of the signal is estimated by Eq.(26) and Eq.(27) respectively:

$$f_2=\frac{1}{2\pi}\sqrt{\frac{-v_2''(n)}{v_2(n)}}, \quad (26)$$

$$f_3=\frac{1}{2\pi}\sqrt{\frac{-v_3''(n)}{v_3(n)}}. \quad (27)$$

ALGORITHM IMPLEMENTATION

The steps for implementation of frequency estimation of multi-component signals are:

Step 1: Sample the multi-component signal at sample frequency of N×50 Hz.

Step 2: Use central numerical differentiation of 7 points to compute the 1st to 6th-order differentiation of the signal using Eqs.(5)~(10).

Step 3: Calculate the frequency f_{e11} of the 1st component sinusoid using Eq.(28):

$$f_{e1j} = \eta_1 \frac{1}{2\pi}\sqrt{\frac{-v_1''(n)}{v_1(n)}} \quad (j=1,2,3), \tag{28}$$

where η_k (k=1,2,…,K) is compensation coefficient of the kth component sinusoid. f_{ekn} is the nth estimated value of the frequency of the kth component sinusoids.

Step 4: Compute the signal sequences using Eq.(29):

$$v(n) = \sum_{k=1}^{K} V_{kc} \sin(2\pi k f_1 nT/N + \phi_{kc}), \tag{29}$$

where T=1/f_{e11}; V_{kc}, ϕ_{kc} is the computation coefficient of the magnitude and phase angle of the kth component sinusoid respectively.

Calculate the estimated frequency f_{e12} using Eq.(28).

Step 5: Compute the signal sequences using Eq.(29) with T=1/f_{e12}. Calculate the estimated frequency f_{e13} using Eq.(28).

Step 6: Compute the signal sequences using on Eq.(29) with T=1/f_{e13}. Calculate the estimated frequency f_{e14} using Eq.(28). f_{e14} is the measured value f_1 of the 1st component sinusoid.

Step 7: Set k=2.

Step 8: Calculate the estimated frequency f_{ek1} of the kth component sinusoid using the following equation:

$$f = \eta_k \frac{1}{2\pi}\sqrt{\frac{-v_{k1}''(n)}{v_k(n)}}. \tag{30}$$

Step 9: Compute the signal sequences using on the following formulation:

$$v(n) = \sum_{k=1}^{K} V_{kc} \sin(2\pi k f_k nT/N + \phi_{kc}), \tag{31}$$

where T=1/f_{ek1}.

Calculate the estimated frequency f_{ek2} using Eq.(30).

Step 10: Compute the signal sequences basing on Eq.(31) with T=1/f_{ek2}. Calculate the estimated frequency f_{ek3} using Eq.(30).

Step 11: Compute the signal sequences using on Eq.(31) with T=1/f_{ek3}. Calculate the estimated frequency f_{ek4} using Eq.(30). f_{ek4} is the measured value f_k of the kth component sinusoid.

Step 12: k=k+1. If k is not equal to K, then go to Step 13. If k=K, then it is the end of the computation.

NUMERICAL TEST

To illustrate the proposed algorithm, a signal with 3 components is given:

$$\begin{aligned} v(t) &= V_1\sqrt{2}\sin(2\pi f_1 t + \phi_1) + V_2\sqrt{2}\sin(2\pi f_2 t + \phi_2) \\ &\quad + V_3\sqrt{2}\sin(2\pi f_3 t + \phi_3), \end{aligned} \tag{32}$$

where the value of f_1, f_2, f_3, the amplitudes and phase angles of 3 harmonics are all unknown. In this paper, the simulation for the proposed algorithm is carried out with Matlab software.

In any case, the signal is sampled with an initial sample frequency of 25600 Hz, and a sample sequence obtained from this sample is used for frequency estimation.

The frequencies of the components 1, 2 and 3 of the signal are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz, as shown in Table 1. The simulation is accomplished in 5 cases:

Case 1: the amplitude of the components 1, 2 and 3 of the signal are set to vary from 1 V to 200 V at random respectively and the phase angle of the components 1, 2 and 3 of the signal are set to a fixed value, say ϕ_1=30°, ϕ_2=160°, ϕ_3=320°. The results of the simulation show that the accuracy for frequency estimation of the components 1, 2 and 3 are all 0.001% with the amplitude of the 1st, 2nd and 3rd harmonic varying from 1 V to 200 V at random. It is noteworthy that the accuracy is still not less than 0.001% in cases the amplitude of the 1st, 2nd and 3rd harmonic is all 1 V or 200 V, or is 1 V, 200 V, 200 V, or 1 V, 1 V, 200 V respectively.

Case 2: the phase angle of the components 1, 2

Table 1 Frequency estimation

f_1 (Hz)		f_2 (Hz)		f_3 (Hz)	
Real	Measure	Real	Measure	Real	Measure
1	0.99386	1	0.99386	1	0.99386
6	5.99715	18	17.97246	28	27.94138
73	73.00768	5	5.00106	10	10.00210
100	100.0020	200	200.0065	150	150.0041
400	400.0190	300	300.0365	500	499.9727
1000	999.9997	800	800.0098	2000	1999.999
5000	5000.080	2000	1999.999	3000	3000.027
3000	3000.048	6000	5999.997	8000	7999.996
10000	9999.997	8000	7999.989	20000	19999.99
50000	49999.84	70000	69999.78	60000	59999.81
100000	99999.69	200000	199999.3	150000	149999.5
200000	200000.5	200000	200000.5	200000	200000.5
300000	300000.8	200000	199999.4	400000	399999.3
500000	499998.6	400000	400000.9	500000	499998.6
500000	499998.6	500000	499998.6	500000	499998.6
800000	800006.7	600000	599998.6	700000	700008.0

and 3 of the signal is set to vary randomly from 0° to 360° respectively and the amplitude of the components 1, 2 and 3 of the signal are set to a fixed value, say V_1=200 V, V_2=10 V, V_3=120 V. In this case, the frequencies of the components 1, 2 and 3 of the signal are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz.

Case 3: the amplitude of the components 1, 2 and 3 of the signal is to vary randomly from 1 V to 200 V respectively, and the phase angle of the components 1, 2 and 3 of the signal is also set to vary randomly from 0° to 360° respectively. In this case, the frequencies of the components 1, 2 and 3 of the signal are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz.

Case 4: the amplitude fluctuation of the white noise is set to vary from 0.5 V to 400 V and the amplitude of the components 1, 2 and 3 of the signal are set to a fixed value, say V_1=200 V, V_2=10 V, V_3=120 V, the phase angle of the components 1, 2 and 3 of the signal are set to a fixed value, say ϕ_1=30°, ϕ_2=160°, ϕ_3=320°. In this case, the frequencies of the components 1, 2 and 3 of the signal are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz.

Case 5: the amplitude fluctuation of the white noise is set to vary randomly from 0.5 V to 400 V, the amplitude of the components 1, 2 and 3 of the signal are set to vary from 1 V to 200 V respectively, the phase angle of the components 1, 2 and 3 of the signal is set to vary randomly from 0° to 360° respectively. In this case, the frequencies of the components 1, 2 and 3 of the signal are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz.

Table 2 shows mean squared errors of frequency estimation of the first sinusoid component under different Signal-to-Noise Ratio (*SNR*) using the proposed algorithm. The mean squared errors are very small for *SNR* of 107.87255 dB, 29.63209 dB, 15.76915 dB and 4.01341 dB.

Table 2 Mean squared error of frequency estimation in noise

f_1 (Hz)	*SNR* (dB)			
	107.87255	29.63209	15.76915	4.01341
1	0.0001680	0.0001694	0.0001685	0.0001600
6	0.0032827	0.0060386	0.0055044	0.0062800
73	0.0004757	0.0001396	0.0001673	0.0001900
100	0.0015241	0.0002592	0.0003056	0.0003513
400	0.0000023	0.0043304	0.0049993	0.0058061
1000	0.0000428	0.0000066	0.0000064	0.0000072
3000	0.0002715	0.0035005	0.0000911	0.0001656
5000	0.0003303	0.0002270	0.0002791	0.0097457
10000	0.0025787	0.0006361	0.0006336	0.0008715
50000	0.0000151	0.0000060	0.0001985	0.0000001
100000	0.0956671	0.0009483	0.0020417	0.0007260
200000	0.0004324	0.0001408	0.0113145	0.0004350
300000	0.0070367	0.0001940	0.0071031	0.0002456

In Table 3, the time for frequency estimation of the first sinusoid component using NDBA (Numerical Differentiation Based Algorithm, proposed in this paper) is compared to other techniques. The computation time of DFT, SDFT and CWT algorithm are all obtained from (Yang *et al.*, 2000). In a DSP system with running frequency being greater than 20 MHz, the time spent for frequency estimation of the first sinusoid component by the algorithm proposed in this paper always needs at most 20 signal periods (less than 0.4 s). It is seen that the algorithm proposed in this paper is the fastest method for frequency estimation.

Table 3 Estimation time compared to that of other techniques

	NDBA	SDFT	DFT	Prony
Time (s)	≤ 0.40	0.54	0.71	2.03

In any cases, the proposed algorithm for estimating the frequencies of the components 1, 2 and 3 of the signal with noise or without noise needs at most

1 cycle. Because of use of numerical differentiation and central Lagrange interpolation, less computation work needs to be done so that much more time for computation is saved. The saved time depends on the speed of the microprocessor used in PC and the points used in numerical differentiation and central Lagrange interpolation.

CONCLUSION

A based on numerical differentiation and central Lagrange interpolation with multi-points algorithm for frequency estimation of multiple signals is presented in this paper.

The frequencies of the components 1, 2 and 3 of the signal under noisy or not noisy condition are all estimated at an accuracy of 0.001% over 1 Hz to 800 kHz with the amplitudes of the components 1, 2 and 3 of the signal varying from 1 V to 200 V and the phase angle of the components 1, 2 and 3 of the signal varying from 0° to 360°. As a whole, the proposed algorithm needs at most half a cycle for estimation of the frequencies of the components 1, 2 and 3 of the signal under noisy or non-noisy conditions.

The proposed algorithm is adaptive to any cases, in which the parameters, such as frequency, amplitude and phase of the signal are all unknown with the frequency of signal varying from 1 Hz to 800 kHz, the amplitude varying from 1 V to 200 V, and the phase angle varying from 0° to 360°. The proposed algorithm is also adaptive to any cases with or without noise. In cases where the amplitude fluctuation of the noises is up to 400 V, the accuracy for frequency estimation of the signals is still maintained at 0.001%.

Compared with other algorithms, this algorithm has higher accuracy and spends less time and computation over a wide range at a high accuracy, so that the proposed algorithm is adaptive to any intelligent measurement and control.

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