Journal of Zhejiang University SCIENCE A
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Robust adaptive controller design for a class of nonlinear systems with unknown high frequency gains*

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Received May 20, 2005; revision accepted Nov. 25, 2005

Abstract: In this note, a robust adaptive control scheme is proposed for a class of nonlinear systems that have unknown multiplicative terms. Unlike previous results, except for the unknown control directions, we do not require a priori bounds on the unknown parameters. We also allow the unknown parameters to be time-varying provided that they are bounded. Our proposed robust adaptive controller is designed to identify on-line the unknown control directions and is a switching type controller, in which the controller parameters are tuned in a switching manner via a switching logic. Global stability of the closed-loop systems have been proved.

Key words: Robust control, Adaptive control, Logic-based switching, Nonlinear system
doi:10.1631/jzus.2006.A0315 **Document code:** A **CLC number:** TP273

INTRODUCTION

Many researchers have devoted their energy to the development of nonlinear system with unknown control directions during the past few years. The class of nonlinear system has been developed greatly since Mudgett and Morse (1985) relaxed the assumption that the unknown multiplicative terms are not only sign-invariant but also have known signs. A method called correction vector approach was proposed in (Lozano *et al*., 1990) and has been applied to design adaptive controller of first-order nonlinear systems with unknown control direction. Then, Kaloust and Qu (1995) developed a robust controller for a general second nonlinear system. The general nonlinear systems with unknown multiplicative terms were solved via the nussbaum-type gains technique in (Ye, 1999; Ge and Wang, 2003). Almost all of the works mentioned above were completed via backstepping approach (Kanellakopoulos *et al*., 1991).

In this paper, by incorporating variable structure control (Efe *et al*., 2004; Feng and Wu, 1996) and robust control (Ye, 2003; Zhang and Liu, 2005) into the backstepping approach, we can propose a new robust adaptive controller for a class of nonlinear uncertain system. The controller does not need both control directions and a priori bounds on the unknown parameters. It can identify on-line the control directions and tune its estimate parameters in a switching manner via logic-based switching at the same time.

The remainder of the paper is organized as follows. In Section I, formulation of our robust adaptive control problem of uncertain nonlinear system is presented. A robust adaptive controller design procedure is developed in Section II. The stability of the closed-loop systems is analyzed in Section III and a simulation example is given in Section IV. Finally, the note is concluded in Section V.

PROBLEM FORMULATION

Consider the global robust adaptive control of

* Project supported by the National Natural Science Foundation of China (No. 60474010), and the Scientific Research Foundation for the Returned Chinese Scholars, State Education Ministry, China

the following nonlinear parameter-strictly-feedback (PSF) systems:

$$\begin{cases} \dot{x}_i = \theta_{1,i} x_{i+1} + f_i(\bar{\boldsymbol{x}}_i, t) + \sum_{j=1}^{p} \phi_j g_{i,j}(\bar{\boldsymbol{x}}_i, t), 1 \le i \le n-1, \\ \dot{x}_n = \theta_{1,n} \beta(\boldsymbol{x}) u + f_n(\boldsymbol{x}, t) + \sum_{j=1}^{p} \phi_j g_{n,j}(\boldsymbol{x}, t), \end{cases} \tag{1}$$

where, $\boldsymbol{x} \in \mathbb{R}^n$ is the state, $\bar{\boldsymbol{x}}_i^{\mathrm{T}} = [x_1, \cdots, x_i]$, $u \in \mathbb{R}$ is the control input, $\boldsymbol{\phi}^{\mathrm{T}} \in \mathbb{R}^p$ is an unknown parameter vector, $\theta_{1,i}$ ($1 \le i \le n$) are unknown parameters which characterize the directions of control motion. The control directions, $\mathrm{sign}(\theta_{1,i})$, $1 \le i \le n$, and the bounds of the unknown parameters are unknown. The function $\beta(x) \ne 0$, $\forall \boldsymbol{x} \in \mathbb{R}^n$.

For system Eq.(1) the following assumptions are made:

(A.1) The unknown parameters $|\theta_{1,i}| > 0$, $1 \le i \le n$.

(A.2) The functions $f_i(\bar{\boldsymbol{x}}_i, t)$ and $g_i(\bar{\boldsymbol{x}}_i, t)$, $1 \le i \le n$, are known, smooth, bounded and vanish at $x=0$, i.e., $f_i(0,t)=0$, $g_i(0,t)=0$.

(A.3) System Eq.(1) has a classical solution under a continuous control u.

(A.4) The functions $f_i(\bar{\boldsymbol{x}}_i, t)$, $1 \le i \le n$, can be bounded by known functions that are uniformly bounded with respect to time and locally uniformly bounded with respect to the state, i.e., $|f_i(\bar{\boldsymbol{x}}_i, t)| \le \bar{f}_i(\bar{\boldsymbol{x}}_i)$, $1 \le i \le n$.

(A.5) The functions in system Eq.(1) are differentiable and their partial derivatives are bounded by known functions:

$$\left|\frac{\partial^l f_i}{\partial t^l}\right| + \left|\frac{\partial^l g_i}{\partial t^l}\right| \le \mu_i^l(\bar{\boldsymbol{x}}_i, t), \sum_{j=1}^{i}\left(\left|\frac{\partial^l f_i}{\partial x_j^l}\right| + \left|\frac{\partial^l g_i}{\partial x_j^l}\right|\right) \le \nu_i^l(\bar{\boldsymbol{x}}_i, t);$$
$$1 \le j \le i \le n-1 \text{ and } \max(i+l) = n.$$

With these assumptions, we can design in the next section a robust adaptive control u that makes the state $\boldsymbol{x}$ asymptotically converge to zero while keeping all closed-loop signals bounded.

ROBUST ADAPTIVE CONTROLLER DESIGN

In order to develop the robust adaptive controller, we first estimate the unknown high frequency gains $\theta_{1,i}$ ($1 \le i \le n$), and define the estimates as k_i ($1 \le i \le n$). Then we can design a robust adaptive controller that contains the tuning parameters k_i ($1 \le i \le n$), on the assumption that k_i ($1 \le i \le n$) are fixed. Finally we develop a switching mechanism to guarantee the controller given in the former step can make the closed-loop system asymptotically stable.

Robust adaptive controller design

The controller in this section is designed in a backstepping way and the estimate parameters k_i ($1 \le i \le n$) are assumed to be fixed. The procedure consists of n steps. At the ith step, $1 \le i \le n-1$, the state variable x_{i+1} is viewed as a fictitious control, for which a "reference" signal α_i is designed. At the nth step, the fictitious control equals to the actual control u which completes the design.

Step 1: Let $z_1 = x_1$, from Eq.(1) we have

$$\dot{z}_1 = \theta_{1,1} x_2 + f_1(x_1, t) + \sum_{j=1}^{p} \phi_j g_{1,j}(x_1, t). \tag{2}$$

We design the following "reference" signal α_1 for the fictitious control x_2

$$\begin{cases} \alpha_1 = k_1 \left[-z_1 - \left(z_1 + \frac{z_1}{\varepsilon + z_1^2} \right) \bar{f}_1(x_1) - \sum_{j=1}^{p} \hat{\phi}_{1,j} g_{1,j}(x_1, t) \right] \\ \dot{\hat{\phi}}_{1,j} = g_{1,j}(x_1, t) z_1, \quad 1 \le j \le p \end{cases} \tag{3}$$

and define the candidate Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \sum_{j=1}^{p} (\hat{\phi}_{1,j} - \phi_j)^2. \tag{4}$$

The time derivative of V_1 computed with Eqs.(2) and (3), is given by

$$\dot{V}_1 = \theta_{1,1} \alpha_1 z_1 + f_1 z_1 + \sum_{j=1}^{p} \phi_j g_{1,j} z_1 + \sum_{j=1}^{p} (\hat{\phi}_{1,j} - \phi_j)\ \dot{\hat{\phi}}_{1,j}$$
$$\le z_1 \left\{ [\theta_{1,1} k_1 - 1] \left[-\sum_{j=1}^{p} \hat{\phi}_{1,j} g_{1,j} - z_1 - \left(z_1 + \frac{z_1}{\varepsilon + z_1^2} \right) \bar{f}_1 \right] \right\}$$
$$+ \sum_{j=1}^{p} (\hat{\phi}_{1,j} - \phi_j)\ (\dot{\hat{\phi}}_{1,j} - g_{1,j} z_1) - z_1^2$$

$$=\Phi_1(z_1,k_1,t)-z_1^2\,, \tag{5}$$

where

$$\Phi_1=z_1[\theta_{1,1}k_1-1]\left[-\sum_{j=1}^{p}\hat{\phi}_{1,j}g_{1,j}-z_1-\left(z_1+\frac{z_1}{\varepsilon+z_1^2}\right)\bar{f}_1\right]. \tag{6}$$

The estimate parameter k_1 is designed in the next section to guarantee $\Phi_1(z_1,k_1,t)\le 0$.

However, x_2 is not the actual control, so there exists a difference z_2 between x_2 and its "reference" signal α_1. z_2 is defined as

$$z_2=x_2-\alpha_1. \tag{7}$$

Accordingly, expression Eq.(5) should be modified as

$$\begin{aligned}\dot{V}_1&=-z_1^2+\Phi_1(z_1,k_1,t)+\theta_{1,1}z_1z_2\\&\le-\frac{3}{4}z_1^2+\Phi_1(z_1,k_1,t)+\theta_{1,1}^2z_2^2\,,\end{aligned} \tag{8}$$

where the effect of $z_2(t)$ on $\dot{V}_1$ will be controlled at the next step. Thus, at Step 2, if $z_2(t)$ can be regulated such that it is square integrable and k_1 can be designed in the next section to guarantee $\Phi_1(z_1,k_1,t)\le 0$, then regulation of $z_1(t)$ can be achieved by using Barbalat's lemma. In this way, we can complete the design of the robust adaptive controller.

Step i ($2\le i\le n-1$): Using the definition for $z_1, \ldots, z_i, \alpha_1, \ldots, \alpha_{i-1}, \hat{\phi}_{1,1}, \ldots, \hat{\phi}_{i-1,p+i-2}$ and noting that k_i is time invariant, we have

$$\dot{z}_i=\theta_{1,i}x_{i+1}+\sum_{j=1}^{p}\phi_j\varphi_{i,j}+\sum_{j=1}^{i-1}\theta_{1,j}\varphi_{i,j+p}+\varphi_{i,i+p} \tag{9}$$

where $\varphi_{i,j}$ ($1\le j\le i+p$) are appropriately defined smooth functions of ($z_1, \ldots, z_i, k_1, \ldots, k_{i-1}, \hat{\phi}_{1,1}, \ldots, \hat{\phi}_{i-1,p+i-2}$).

We design the following "reference" signal α_i for the fictitious control x_{i+1}

$$\begin{cases}\alpha_i=k_i\left[-z_i-\left(z_i+\dfrac{z_i}{\varepsilon+z_i^2}\right)\bar{\varphi}_{i,i+p}-\displaystyle\sum_{j=1}^{p+i-1}\hat{\phi}_{i,j}\varphi_{i,j}\right]\\ \dot{\hat{\phi}}_{i,j}=\varphi_{i,j}z_i,\quad 1\le j\le p+i-1\end{cases} \tag{10}$$

and define

$$V_i=\frac{1}{2}z_i^2+\frac{1}{2}\sum_{j=1}^{p}(\hat{\phi}_{i,j}-\phi_j)^2+\frac{1}{2}\sum_{j=1}^{i-1}(\hat{\phi}_{i,j+p}-\theta_{1,j})^2\,, \tag{11}$$

$$z_{i+1}=x_{i+1}-\alpha_i\,. \tag{12}$$

The time derivative of V_i, computed with Eqs.(9), (10) and (12), is given by

$$\dot{V}_i\le-\frac{3}{4}z_i^2+\Phi_i(\bar{z}_i,k_i,t)+\theta_{1,i}^2z_{i+1}^2\,, \tag{13}$$

where

$$\begin{aligned}\Phi_i(\bar{z}_i,k_i,t)&=z_i[\theta_{1,i}k_i-1]\\&\times\left[-\sum_{j=1}^{p+i-1}\hat{\phi}_{i,j}\varphi_{i,j}-z_i-\left(z_i+\frac{z_i}{\varepsilon+z_i^2}\right)\bar{\varphi}_{i,i+p}\right],\end{aligned} \tag{14}$$

$\bar{\varphi}_{i,i+p}\ge|\varphi_{i,i+p}|$, the effect of $z_{i+1}(t)$ on $\dot{V}_i$ will be controlled at the next step. Thus, if $z_{i+1}(t)$ can be regulated such that it is square integrable and the estimate parameter k_i can guarantee $\Phi_i(\bar{z}_i,k_i,t)\le 0$, then $V_i(t)$ and $z_i(t)$ are all bounded. Furthermore, $z_i(t)$ is also square integrable on $[0,t_f]$.

Step n: Using the definition for $z_1, \ldots, z_n, \alpha_1, \ldots, \alpha_{n-1}, \hat{\phi}_{1,1}, \ldots, \hat{\phi}_{n-1,p+n-2}$ and noting that k_n is time invariant, we have

$$\dot{z}_n=\theta_{1,n}\beta(x)u+\sum_{j=1}^{p}\phi_j\varphi_{n,j}+\sum_{j=1}^{n-1}\theta_{1,j}\varphi_{n,j+p}+\varphi_{n,n+p}\,, \tag{15}$$

where $\varphi_{n,j}$ ($1\le j\le n+p$), are appropriately defined smooth functions of ($z_1, \ldots, z_n, k_1, \ldots, k_{n-1}, \hat{\phi}_{1,1}, \ldots, \hat{\phi}_{n-1,p+n-2}$).

We design the following actual robust adaptive control

$$\begin{cases}u=\dfrac{k_n}{\beta(x)}\left\{-\dfrac{3}{4}z_n-\displaystyle\sum_{j=1}^{p+n-1}\hat{\phi}_{n,j}\varphi_{n,j}-\left(z_n+\dfrac{z_n}{\varepsilon+z_n^2}\right)\bar{\varphi}_{n,n+p}\right\}\\ \dot{\hat{\phi}}_{n,j}=\varphi_{n,j}z_n,\quad 1\le j\le p+n-1\end{cases} \tag{16}$$

and define the candidate Lyapunov function as

$$V_n=\frac{1}{2}z_n^2+\frac{1}{2}\sum_{j=1}^{p}(\hat{\phi}_{n,j}-\phi_j)^2+\frac{1}{2}\sum_{j=1}^{n-1}(\hat{\phi}_{n,j+p}-\theta_{1,j})^2\,. \tag{17}$$

The time derivative of V_n, computed with Eqs.(15) and (16), is given by

$$\dot{V}_n \le -\frac{3}{4}z_n^2 + \Phi_n(\boldsymbol{z}, k_n, t) . \qquad (18)$$

If the estimate parameter k_n can guarantee $\Phi_n(\boldsymbol{z},k_n,t)\le 0$, then $V_n(t)$ and $z_n(t)$ are all bounded. Furthermore, $z_n(t)$ is also square integrable on $[0,t_f]$. Therefore we can conclude that all of $z_i(t)$ ($1\le i\le n$) are square integrable on $[0,t_f]$.

In the next section, we give the switching mechanism of the estimate parameters k_i ($1\le i\le n$) which can guarantee $\Phi_i(\bar{z}_i, k_i, t) \le 0,\ 1 \le i \le n$.

Design of the switching mechanism

In this section, our objective is to make $\Phi_i(\bar{z}_i, k_i, t) \le 0,\ 1 \le i \le n$.

To simplify the analysis of the closed-loop system and implementation of our adaptive controller, we tune the parameters k_i ($1\le i\le n$) in a piecewise constant way. Suppose we have chosen for k_i ($1\le i\le n$), a sequence $H=\{h(l):l=1,2,\ldots,\infty\}$. The only requirement for the function h is that it should be strictly increasing and satisfy $\lim_{l\to\infty} h(l) = \infty$. We start with $k=h(1)$.

1. Define switching condition

At each time $t>t_s$ if

$$V_i(t) > V_i(t_s) + c,\ 1 \le i \le n , \qquad (19)$$

or at each time $t>t_s+\tau$ if

$$V_i(t) - V_i(t_s^+) > -\frac{3}{4}\int_{t_s}^{t} z_i^2(s)\mathrm{d}s,\ 1 \le i \le n , \qquad (20)$$

where the start time $t_s=0$, $V_i(t_s^+)$ denote values of $V_i(t_s)$ ($1\le i\le n$), just after the switching; τ and c are two positive numbers to be designed.

2. Switching logic

When the condition Eq.(19) is satisfied, we can switch k_i ($1\le i\le n$) to its next element in the sequence H and change its sign(k_i), i.e., $k_i=-h(2)$ ($1\le i\le n$). Reset $t_s=t$, and the process is repeated.

When the condition Eq.(20) is satisfied only, we only switch k_i ($1\le i\le n$) to its next element in the sequence H, i.e., $k_i=h(2)$ ($1\le i\le n$). Reset $t_s=t$, and the process is repeated.

Remark 1 When the condition Eq.(19) is satisfied, the energy of the system is rapidly accumulated. The condition implies that the control may be positive feedback. But, when only the condition Eq.(20) is satisfied, the energy accumulation of the system is less than c during the time τ. This condition implies that the energy accumulation may be primarily due to insufficient extent of the control force.

STABILITY ANALYSIS

In this section, the proof of $\Phi_i(\bar{z}_i, k_i, t) \le 0$ is first given. Then, we prove k_i ($1\le i\le n$), can be switched only a finite number of times. Finally, the stability analysis is shown.

Proof First, we prove $\Phi_i(\bar{z}_i, k_i, t) \le 0$.

From Eq.(14), we can easily see that the sign of $\Phi_i(\bar{z}_i, k_i, t)$ ($1\le i\le n$), is determined by the sign of ($\theta_{1,i}k_i-1$). When $\Phi_i(\bar{z}_i, k_i, t)>0$, we can conclude that either k_i works in the wrong control direction or $|k_i|$ is less than $|1/\theta_i|$ from the definition of Eq.(14). Therefore, k_i must be rectified according to the switching logic. From Eq.(14), we have $\Phi_i(\bar{z}_i, k_i, t) \le 0$ when k_i was switched a finite number of times.

Next, we need to prove that k_i ($1\le i\le n$), can be switched only a finite number of times.

We begin with the condition that k_n can be switched only a finite number of times. Suppose on the contrary that k_n can be switched an infinite number of times. Let t_1 be a switching time such that $\Phi_n(\boldsymbol{z},k_n,t)\le 0$ and t_2 be the next switching time. Then, from our switching logic, we can derive that either

$$V_n(t_2) > V_n(t_1) + c , \qquad (21)$$

or

$$V_n(t_2) - V_n(t_1^+) > -\frac{3}{4}\int_{t_1}^{t_2} z_n^2(s)\mathrm{d}s . \qquad (22)$$

However, no switching occurs on the time interval (t_1, t_2). So we can conclude from Eq.(18) that

$$V_n(t_2) - V_n(t_1^+) \le -\frac{3}{4}\int_{t_1}^{t_2} z_n^2(s)\mathrm{d}s . \qquad (23)$$

Thus, Eq.(23) implies that neither Eq.(21) nor Eq.(22) holds, a contradiction of both. Therefore, k_n must be switched only a finite number of times. In this way, we can conclude that k_{n-1}, k_{n-2}, …, k_1, can also only be switched a finite number of times.

Finally, we give the stability analysis.

According to the switching logic, between one and its next switching time for any k_i ($1\le i\le n$), a closed-loop solution exists when $V_i(t)$ ($1\le i\le n$) has increased c or when τ time units have passed and Eq.(20) holds. So long as k_i ($1\le i\le n$) can be switched a finite number of times, the closed-loop solution clearly exists. If the solution's maximum interval of existence is on $[0,t_1)$, no finite-time escape phenomenon can occur on $[0,t_1)$.

When t_1 is the time when the final switching occurs, according to our switching logic, we must have

$$V_i(t)\le V_i(t_1)+c,\quad 1\le i\le n,\quad \text{for all } t>t_1$$

and

$$V_i(t)\le V_i(t_1^+)-\frac{3}{4}\int_{t_1}^{t} z_i^2(s)\mathrm{d}s,\ \text{for all } t>t_1+\tau.$$

Otherwise further switching would occur. Thus, $V_i(t)$ and $z_i(t)$ ($1\le i\le n$) are all bounded for all $t>t_1$ and we can easily conclude that $z_i(t)$ ($1\le i\le n$) are square integrable on $[t_1,\infty)$ and that $u(t)$, $\dot{z}_i(t)$ ($1\le i\le n$), and $\alpha_i(t)$ ($1\le i\le n-1$), are all bounded on $[t_1,\infty)$. Therefore, by Barbalat's lemma, we have $\lim_{t\to\infty} z(t)=0$. From the transformation $z_1=x_1$, $z_i=x_i-\alpha_{i-1}$, $2\le i\le n$, we have that $x(t)$ is bounded on $[t_1,\infty)$ and that $\lim_{t\to\infty} x(t)=0$.

Now we can summarize the previous results in the following theorem.

Theorem 1 Suppose the robust adaptive controller presented in this paper is applied to system Eq.(1). Then, for all initial conditions, all closed-loop states are bounded on $[0,\infty)$ and asymptotic regulation is achieved, i.e., $\lim_{t\to\infty} x(t)=0$.

ILLUSTRATIVE EXAMPLE

Consider the following second-order nonlinear system:

$$\begin{cases}\dot{x}_1=\theta_{1,1}x_2+\phi_1 x_1^3\\ \dot{x}_2=\theta_{1,2}u+x_2^2\cos(t)\end{cases}\tag{24}$$

where $[\theta_{1,1},\theta_{1,2}]$ is an unknown parameter vector and we have no a priori knowledge bound on it; ϕ_1 is also an unknown parameter.

According to the method proposed in this paper, we can construct the following robust adaptive controller

$$\begin{cases}u=k_2\left[-\frac{3}{4}z_2-\left(z_2+\frac{z_2}{\varepsilon+z_2^2}\right)\bar{\varphi}_{2,3}-\sum_{j=1}^{2}\hat{\phi}_{2,j}\varphi_{2,j}\right]\\ \dot{\hat{\phi}}_{2,j}=\varphi_{2,j}z_2,\quad j=1,2\end{cases}\tag{25}$$

where

$$\varphi_{2,1}=-k_1[1+3\hat{\phi}_{1,1}z_1^2]z_1^3,\qquad \varphi_{2,2}=-k_1[1+3\hat{\phi}_{1,1}z_1^2]x_2,$$
$$\varphi_{2,3}=-k_1z_1^7+x_2^2\cos(t),\qquad |\varphi_{2,3}|\le\bar{\varphi}_{2,3}=|k_1z_1^7|+x_2^2,$$

$z_2=x_2-\alpha_1$ and α_1 is given as

$$\begin{cases}\alpha_1=k_1[-z_1-\hat{\phi}_{1,1}z_1^3]\\ \dot{\hat{\phi}}_{1,1}=z_1^4\end{cases}\tag{26}$$

with k_i ($1\le i\le 2$) adaptively tuned by the switching mechanism developed in this paper.

Simulation was carried out with the following choices: the positive numbers are $\tau=0.1$, $c=0.1$, $\varepsilon=0.0001$; the unknown parameter is $\phi_1=1$ and $[\theta_{1,1}(t),\theta_{1,2}(t)]=[1,-1]$; the initial condition is $\hat{\phi}_{1,1}(0)=\hat{\phi}_{2,1}(0)=\hat{\phi}_{2,2}(0)=0$ and $[x_1(0), x_2(0)]=[-1, 2]$; the strictly increasing sequence is $H=\{0.3i{:}i=1,2,3,\ldots\}$. Figs.1~5 depict the simulation results.

These simulation results clearly showed that the robust adaptive controller presented in this paper guarantees the boundedness and convergence of all the states in the closed-loop system.

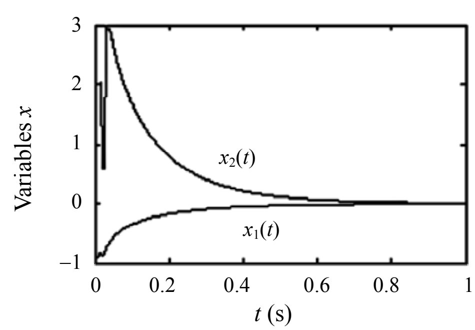

Fig.1 State variables $x_1(t)$ and $x_2(t)$

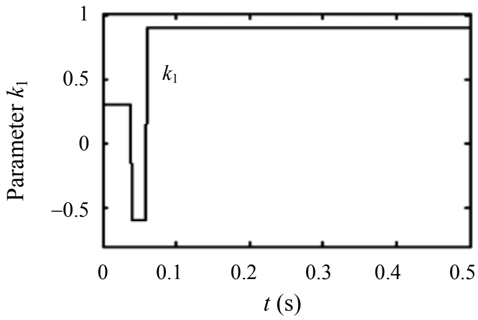

Fig.2 Adaptive parameter k_1

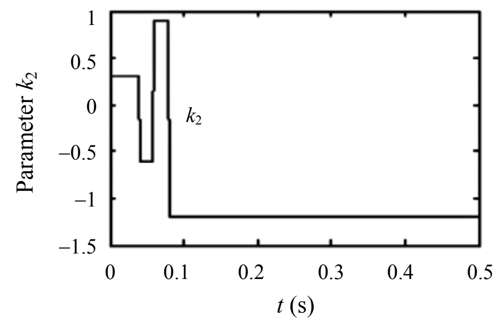

Fig.3 Adaptive parameter k_2

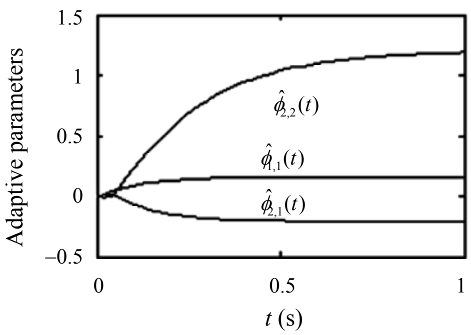

Fig.4 Adaptive parameter $\hat{\phi}_{1,1}(t)$, $\hat{\phi}_{2,1}(t)$, $\hat{\phi}_{2,2}(t)$

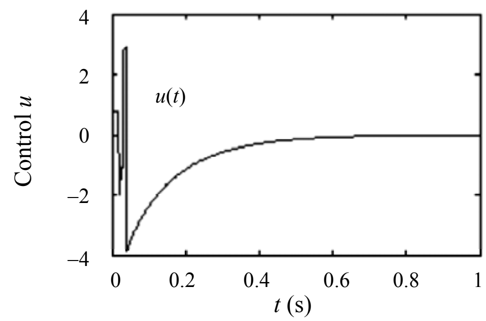

Fig.5 Control $u(t)$

CONCLUSION

In this note, we develop a robust adaptive controller for a class of nonlinear system that has unknown multiplicative terms which include not only its control directions but also its bounds. Our proposed robust adaptive controller is a switching-type controller whose parameters are tuned via a switching logic manner.

On the one hand, the smaller c is chosen, the faster k_i ($1 \le i \le n$) changes its sign and more effective is the adaptive controller for stabilizing the system. However, the frequency of chattering will be higher. On the other hand, the bigger c is chosen, the larger the state changes its magnitude and the larger the overshoot of the state. Therefore, the key to achieving good control results is to choose c properly and the most common way is to choose possibly smaller c.

The simulation results indicated the feasibility of the design procedure.

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