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Unified expression for failure of reinforced concrete members in bridge*

HUANG Zhen (黄 真)†, LIU Xi-la (刘西拉), YANG Guang (阳 光)

(*Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai 200030, China*)

†E-mail: zhenhuang@sjtu.edu.cn

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Abstract: Reinforced concrete structural elements with box section are commonly used in the horizontal and vertical structure of bridges. The reinforced concrete structure in bridge often failed under the combined forces of bending, axial load, shear and torsion caused by wind and earthquake. It is very important to study the mechanism of RC box section structures subjected to a combination of forces. A theoretical study and deduction of the unified expression for failure of reinforced concrete members with box section under combined bending, shear, axial force and torsion were carried out with stress equilibrium assumption. Comparison of theoretical analysis results with experimental results showed that the unified expression for failure of reinforced concrete members with box section can be used for static calculation of such structure members.

Key words: Stress equilibrium, Box section, Unification, Axial load, Bending, Shear, Torsion

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INTRODUCTION

Reinforced concrete elements with box section are commonly used in horizontal subsystems and in vertical support of bridge structures. The horizontal structural members normally have box section and top flange. The vertical supports of the bridge usually have a box section also. To simplify the deduction of such structural member under combined forces, the box sections without top flange are discussed here. Fig.1 shows the typical section types of reinforced concrete members. The failures of such structures are mostly induced by different combination of axial forces, bending, shear and torsion caused by wind and earthquake. The behaviors of structures under combined actions of axial forces, bending, shear and torsion are complex and difficult to be expressed with a unified failure expression. Researches on the box section structure were normally limited to test investigations. The expressions of failure strength established with the statistic method were often not comprehensive enough, so that it is very important to find the failure mechanism and give a unified expression for the failure of RC box section structures under combined actions of axial forces, bending, shear and torsion.

In this work, theoretical study and deduction of a unified expression for the failure of reinforced concrete members with box section under combined bending, shear, axial force and torsion were carried out on the assumption of stress equilibrium. Comparisons between theoretical analysis results and experimental results on reinforced box girders under torsion, bending and shear forces conducted in TU Braunschweig/Germany showed good agreement, indicating that the unified expression for failure of reinforced concrete members with box section can be used in static calculation of such structure members.

SECTIONAL STRESSES YIELDING CRITERION

As a traditional combined material, reinforced

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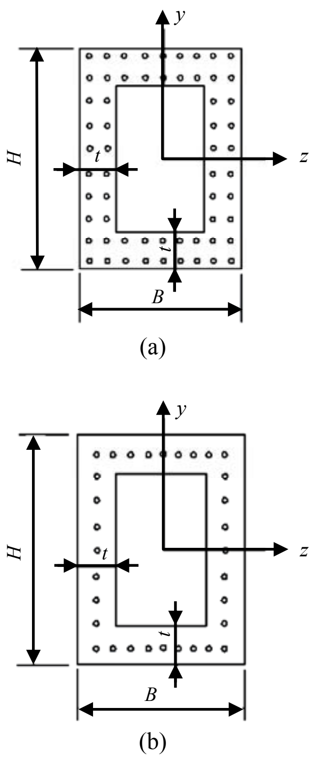

Fig.1 Typical box section types of reinforced concrete beams. (a) Box section with double reinforcements; (b) Box section with single reinforcements

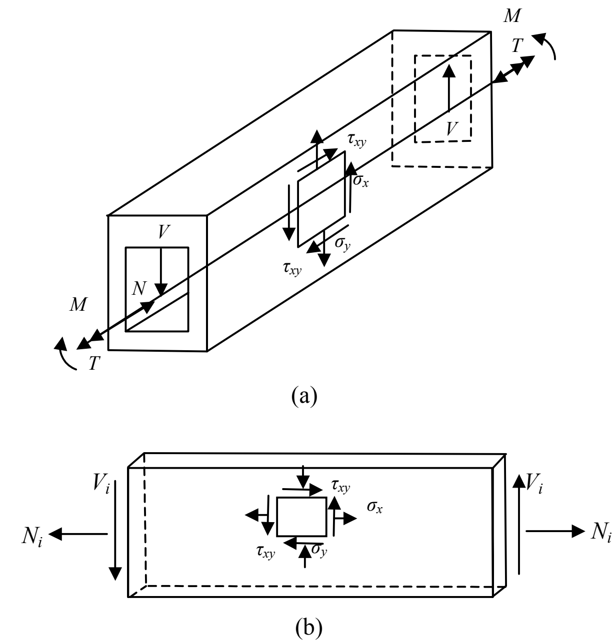

Fig.2 The simulated behaviour of RC box sectional beams and the equivalent flat slab under combined actions. (a) The RC box sectional beam; (b) The equivalent flat slab
σ_x, σ_y, τ_{xy} are the stresses of equivalent reinforced concrete along the axis of x, y and z

concrete has been used over 150 years. There were already many investigations on the behavior of reinforced concrete under the separate action of tension, compression, bending, shear and torsion (Liu, 2003; Thomas, 1993; 1996; 2001), but few investigations on the behavior of reinforced concrete under the combined action of tension, compression, bending, shear and torsion (Li and Liu, 1992). Most of the research results were statistical formulas based on the test data. It is necessary to find a unified theoretical expression for the failure of reinforced concrete elements under the combined actions of tension, compression, bending, shear and torsion.

In order to describe the bearing capacity of reinforced box sectional elements under combined actions, the yielding criterion of reinforced concrete should be established at first so that the relevant curves in stress spaces can be obtained. The simulated behavior of box sectional elements and equivalent flat slab elements under combined actions (Fig.2) can be used to deduce the yielding criterion of box sectional elements by introducing the yielding criterion of equivalent flat slabs (Nielson, 1999) and the concept of sectional reinforced degree (Huo and Liu, 2004). The detailed deductions are given below.

Assumptions

In order to establish the yielding criterion of box sectional elements, some assumptions on equivalent flat slabs should be used here:

(1) The deformation of reinforcements and concrete are coordinated;

(2) The dowel actions of reinforcement can be ignored;

(3) The tensile ability of concrete can be ignored;

(4) The reinforcements are uniformly distributed in the concrete, the reinforced concrete can be considered to be equivalent to uniform material.

Sectional reinforced degree

The sectional reinforced degree of box sectional elements can be expressed as ϕ with following expressions along the axis of x, y, z:

$$\phi_x = A_{sx} f_y / A_c f_c, \quad (1)$$

$$\phi_y = A_{sy} f_y / st f_c, \quad (2)$$

$$\phi_z = A_{sz} f_y / A_c f_c, \tag{3}$$

where, A_{sx}, A_{sy}, A_{sz} are the sectional areas of the reinforcements along the axis of x, y and z; f_y, f_c are the yielding strengths of reinforcements and concrete; A_c is the box sectional area of reinforced concrete elements; s, t are the distance of stirrup bars and the thickness of box section.

The stress of reinforcements is distributed uniformly in the concrete section and can be formulated as equivalent stress of reinforcements. In the same section, the stress of concrete is combined with the equivalent stress of reinforcement to give a total stress. This section is called as equivalent uniformed section of reinforced concrete.

If the yielding strength of longitudinal reinforcements is different from that of the horizontal stirrup bars, the different yielding strength of reinforcements should be used in the above expressions. The same method can be used in the situation when the thickness of flanges and webs of the box section are different. That means the relevant thickness of flanges or webs should be used in the above expressions.

Yielding criteria

Introducing the concept of sectional reinforced degree and assuming equivalent uniformed section, the yielding criterion of equivalent flat slab can be used in the reinforced box sectional elements under combined action of axial force, bending moment, shear and torsion to establish its yielding criterion. Fig.3 shows the yield criterion in two dimensions and the yield space in three dimensions of equivalent RC flat slab (Nielson, 1999).

For the box sectional elements, the following modified deduction of equivalent flat slab can be used to find the yielding criterion of box sectional elements.

If $-(1-\phi)f_c \le \sigma_x \le \phi f_c$, the curve can be expressed as:

$$\tau_{xy} = \pm\sqrt{\phi_y f_c (\phi_x f_c - \sigma_x)}. \tag{4}$$

If $-(1+\phi)f_c \le \sigma_x \le -(1-\phi)f_c$, the curve can be expressed as:

$$\tau_{xy} = \pm\sqrt{(1+\phi_y) f_c [(1+\phi_x) f_c + \sigma_x]}, \tag{5}$$

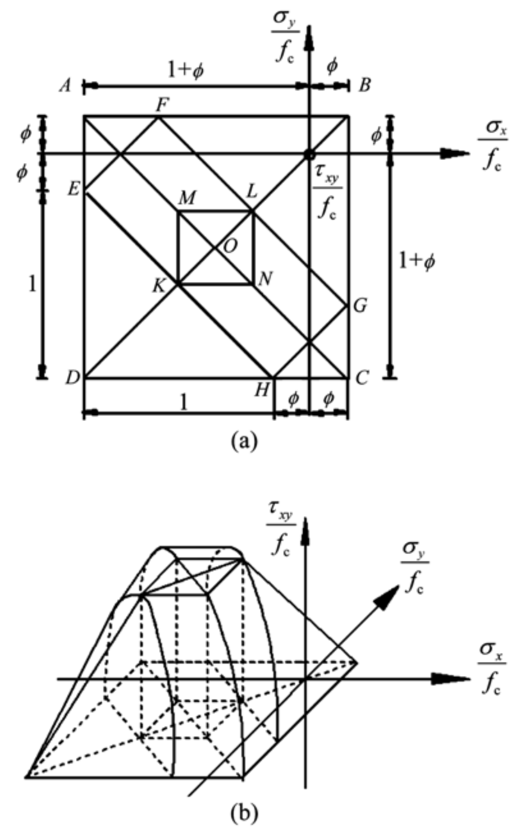

Fig.3 (a) The yield criterion of equivalent RC flat slab in 2D; (b) The yield space of equivalent RC flat slab in 3D

where ϕ is the sectional reinforced degree of box sectional elements expressed with Eqs.(1)~(3).

Furthermore, Eqs.(4) and (5) can be changed to

$$\tau_{xy}^2 + \phi_y f_c \sigma_x = (\phi_x f_c)(\phi_y f_c), \tag{6}$$

$$\tau_{xy}^2 - (1+\phi_y) f_c \sigma_x = [(1+\phi_x) f_c][(1+\phi_y) f_c]. \tag{7}$$

As the shear stress and axial stresses in the element are influenced by the combined actions of axial force, bending moment, shear and torsion, Eqs.(6)~(7) can be expressed as follows (Nielson, 1999):

$$\sum \tau_i^2 + \phi_y f_c \sum \sigma_i = (\phi_x f_c)(\phi_y f_c), \tag{8}$$

$$\sum \tau_i^2 - (1+\phi_y) f_c \sum \sigma_i = [(1+\phi_x) f_c][(1+\phi_y) f_c]. \tag{9}$$

RELATIONSHIPS OF LOADS AND STRESSES

In order to describe the bearing capacity of reinforced concrete under combined actions, the relevant curves or spaces are used and the relationships of the loads and stresses can be expressed as follows.

Axial forces

Assuming planar strain distribution, according to the force balance of loads and sectional stresses, the axial stress of the box section can be expressed as:

$$\sigma_N = \frac{N}{A} = \frac{N}{2t(H+B-2t)} \quad \text{(tension is positive)}, \tag{10}$$

where H is the height of the box section; B is the width of the box section; t is the thickness of the box section.

Bending moment

Under the action of bending moment, the stresses in the section are divided into a positive (tension) and a negative (compression).

$$\sigma_M^+ = \frac{M}{W} \quad \text{(tension)}, \tag{11}$$

$$\sigma_M^- = \frac{M}{W} \quad \text{(compression)}, \tag{12}$$

where W is the box section moment module considering the plastic deformation in the section. If the concrete cracks, the concrete in the tensile area will not work together to take loads that should be taken by the reinforcements in the tensile area. The reinforcement in the tensile area can be considered as an equivalent steel strip which should yield when the section fails to take loads. The strength of the steel strip can be expressed as the strength of concrete in order to get a uniform expression. From Fig.4, the moment module W can be calculated as follows:

(1) When $M/M_o \le 0.5$ (M_o is the maximal bending moment when the section is only forced by bending moment), it is supposed that the neutral axis is in the position of the central axis, and that the moment module consists of two parts of compressive concrete and tensile equivalent steel strip:

$$W = Bt(H-t)/2 + (H/2-t)^2 t + f_y f_c^{-1}[B_g t_g (H_g - t_g)/2 + (H_g/2 - t_g)^2 t_g]. \tag{13}$$

(2) When $M/M_o > 0.5$, it is supposed that the neutral axis is moved to the bottom of the box section's top flange and that the moment module is determined with compressive concrete in the top flange and tensile equivalent steel strip under the neutral axis:

$$W = Bt\frac{t}{2} + \frac{f_y}{f_c}\left(H - \frac{3t}{2}\right)\left(B_g t_g + t_g\left(H - \frac{3t}{2}\right)\right). \tag{14}$$

where H_g, B_g, t_g respectively denote the height, width and thickness of the equivalent steel strip.

If the box section is loaded by two axial bending moments in x and y directions, the stresses should be added to the two axis.

Shear force

The shear force is resisted mainly by the box section webs (Fig.5a). The average shear stress in the

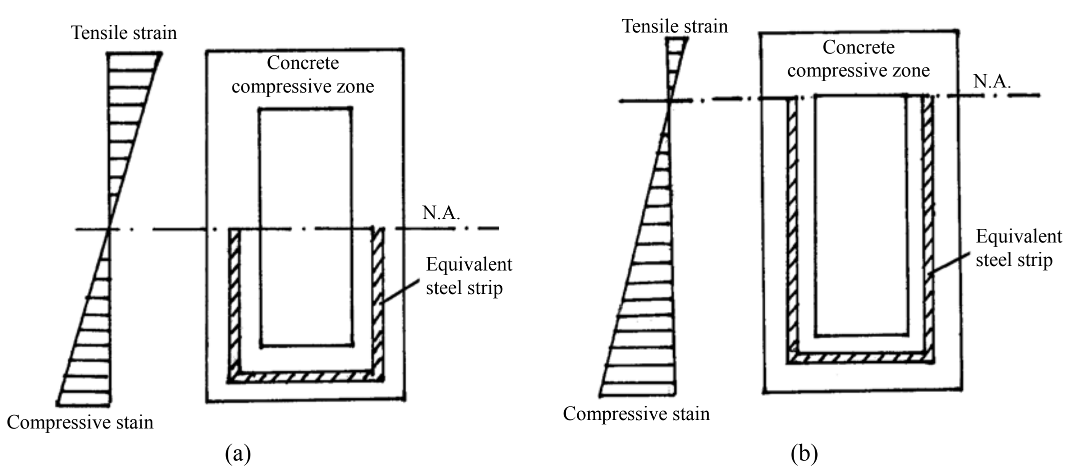

Fig.4 The calculation of sectional bending module. (a) The neutral axial in section center; (b) The neutral axial moved to flange's bottom

webs of box section can be simplified as

$$\tau_v = \frac{V}{2Ht}. \quad (15)$$

The maximal shear stress in the box section is

$$\tau_{\max} = 1.5\frac{0.9V}{2Ht} = 0.675\frac{V}{Ht}. \quad (16)$$

Torsion moment

The torsion action of box section induces the shear flow shown in Fig.5b. Assuming the average shear flow in the box section, the shear stress of torsion can be calculated as:

$$\tau_T = \frac{T}{2t(B-t)(H-t)}. \quad (17)$$

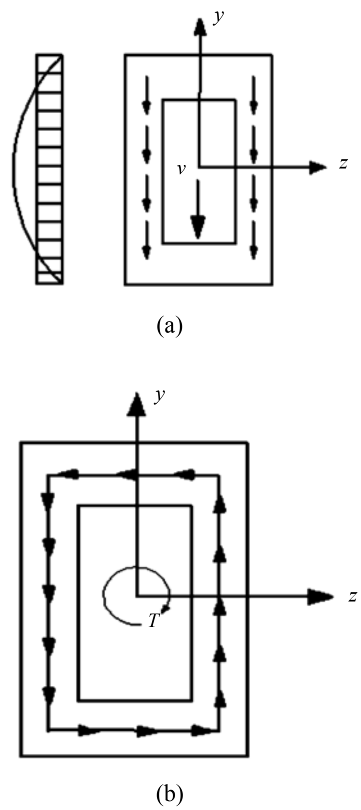

Fig.5 The shear flow under shear and torsion
(a) The shear flow under shear action; (b) The shear flow under torsion action

GENERAL YIELDING CRITERIA

According to the above analysis of the relationships of loads and stresses, and considering the yielding criterion of Eq.(8) and Eq.(9), the following two failure types can be expressed.

Failure induced by tension:

$$\tau_V^2 + \tau_T^2 + \phi_y f_c \sigma_N + \phi_y f_c \sigma_M^+ = (\phi_x f_c)(\phi_y f_c). \quad (18)$$

Failure induced by compression:

$$\tau_V^2 + \tau_T^2 - (1+\phi_y) f_c \sigma_N - (1+\phi_y) f_c \sigma_M^- = [(1+\phi_x) f_c][(1+\phi_y) f_c]. \quad (19)$$

The tension failure is caused by the tensile yielding of the steel strip. The failure induced by compression is caused by the compressive failure of concrete. The yielding criteria can be given as follows.

Failure induced by tension

If $\sigma^+ = \sigma_N + \sigma_M^+ = \frac{N}{A} + \frac{M}{W}$ is between $[-(1-\phi)f_c, \phi f_c]$:

$$\left(\frac{V}{2Ht}\right)^2 + \left(\frac{T}{2t(B-t)(H-t)}\right)^2 = \phi_y f_c \left(\phi_x f_c - \left(\frac{N}{A} + \frac{M}{W}\right)\right). \quad (20)$$

If $\sigma^+ = \sigma_N + \sigma_M^+ = \frac{N}{A} + \frac{M}{W}$ is between $[-(1+\phi)f_c, -(1-\phi)f_c]$:

$$\left(\frac{V}{2Ht}\right)^2 + \left(\frac{T}{2t(B-t)(H-t)}\right)^2 = \phi_y f_c \left(\phi_x f_c + \left(\frac{N}{A} + \frac{M}{W}\right)\right). \quad (21)$$

Failure induced by compression

If $\sigma^- = \sigma_N + \sigma_M^- = \frac{N}{A} + \frac{M}{W}$ is between $[-(1-\phi)f_c, \phi f_c]$:

$$\left(\frac{V}{2Ht}\right)^2 + \left(\frac{T}{2t(B-t)(H-t)}\right)^2$$

$$=(1+\phi_y)f_c\left((1+\phi_x)f_c-\left(\frac{N}{A}-\frac{M}{W}\right)\right). \qquad (22)$$

If $\sigma^- = \sigma_N + \sigma_M^- = \dfrac{N}{A}+\dfrac{M}{W}$ is between $[-(1+\phi)f_c, -(1-\phi)f_c]$:

$$\left(\frac{V}{2Ht}\right)^2+\left(\frac{T}{2t(B-t)(H-t)}\right)^2 =(1+\phi_y)f_c\left((1+\phi_x)f_c+\left(\frac{N}{A}-\frac{M}{W}\right)\right). \qquad (23)$$

General yielding criterion

From the above analysis of yielding criteria of box section under combined actions, the general yielding criterion can be expressed as

$$\left(\frac{V}{2Ht}\right)^2+\left(\frac{T}{2t(B-t)(H-t)}\right)^2=\phi_y f_c\left(\phi_x f_c-\left(\frac{N}{A}+\frac{M}{W}\right)\right), \qquad (24)$$

$$\left(\frac{V}{2Ht}\right)^2+\left(\frac{T}{2t(B-t)(H-t)}\right)^2=\phi_y f_c\left(\phi_x f_c+\left(\frac{N}{A}+\frac{M}{W}\right)\right), \qquad (25)$$

$$\left(\frac{V}{2Ht}\right)^2+\left(\frac{T}{2t(B-t)(H-t)}\right)^2 =(1+\phi_y)f_c\left((1+\phi_x)f_c-\left(\frac{N}{A}-\frac{M}{W}\right)\right), \qquad (26)$$

$$\left(\frac{V}{2Ht}\right)^2+\left(\frac{T}{2t(B-t)(H-t)}\right)^2 =(1+\phi_y)f_c\left((1+\phi_x)f_c+\left(\frac{N}{A}-\frac{M}{W}\right)\right). \qquad (27)$$

Expressed differently:

$$\frac{1}{\phi_x\phi_y}v^2+\frac{1}{\phi_x\phi_y}t^2+\frac{1}{\phi_x}n+\frac{1}{\phi_x}m=1, \qquad (28)$$

$$\frac{1}{\phi_x\phi_y}v^2+\frac{1}{\phi_x\phi_y}t^2-\frac{1}{\phi_x}n-\frac{1}{\phi_x}m=1, \qquad (29)$$

$$\frac{1}{(1+\phi_x)(1+\phi_y)}v^2+\frac{1}{(1+\phi_x)(1+\phi_y)}t^2 +\frac{1}{(1+\phi_x)}n-\frac{1}{(1+\phi_x)}m=1, \qquad (30)$$

$$\frac{1}{(1+\phi_x)(1+\phi_y)}v^2+\frac{1}{(1+\phi_x)(1+\phi_y)}t^2 -\frac{1}{(1+\phi_x)}n+\frac{1}{(1+\phi_x)}m=1, \qquad (31)$$

where

$$n=N/(Af_c), \qquad (32)$$

$$m=M/(Wf_c), \qquad (33)$$

$$v=V/(2Htf_c), \qquad (34)$$

$$t=\frac{T}{2t(B-t)(H-t)f_c}. \qquad (35)$$

From the general yielding criteria above, the relevant curves and spaces can be drawn. The common curves or the common spaces are the final yielding curves and spaces.

COMPARISONS WITH EXPERIMENTS

In order to check the theoretical model of the general yielding criterion of box section under combined actions, comparisons with experimental data are necessary. The experimental data of German TU Braunschweig's five box sectional concrete girders (SETMQ1, SETMQ2, TRAG1, TRAG2, TRAG3) under torsion, bending and shear (Falkner *et al*., 1997; Kordina *et al*., 1984) were used for comparison with the theoretical results.

Fig.6 shows the test section and the loading conditions. For test girders SETMQ1, SETMQ2, TRAG1, TRAG2, the height, width, thickness of the box section was H=600 mm, B=600 mm, t=120 mm respectively. The total sectional area of longitudinal steel bars of SETMQ1 was A_s=3711 mm^2, the web reinforcements was ∅10@100 mm, the concrete grade was C50, the yielding strength of longitudinal reinforcements was 800 N/mm^2, the yielding strength of web reinforcements was 210 N/mm^2. For test girder TRAG3, the height of the box section was H=400 mm, the width of box section was B=600 mm, the top and bottom thickness of the box section was t'=100 mm, and side thickness was t''=80 mm. The total sectional area of longitudinal steel bars was A_s=

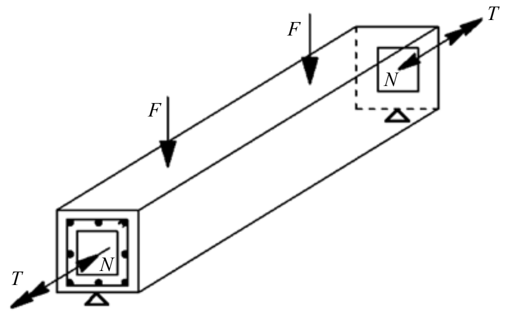

Fig.6 The section and loading of tested box beam

1416 mm^2, the web reinforcements was ∅10@100 mm, the concrete grade was C50, the yielding strength of longitudinal reinforcements was 1570 N/mm^2, the yielding strength of web reinforcements was 210 N/mm^2. The test girders SETMQ1 and SETMQ2 were monolithical girders with unbonded prestressing, the test girder TRAG1, TRAG2 and TRAG3 were segmental girders with unbonded prestressing. The girders SETMQ1, SETMQ2 and TRAG1 were loaded with bending moment, torsion moment and axial force. The girders TRAG2 and TRAG3 were loaded with bending moment, torsion moment, shear force and axial force. The test girder SETMQ1 was taken as sample girder for the comparison of test and theoritical results as follows:

$$\phi_x = A_s f_y/(Af_c) = 0.547,$$
$$\phi_y = A_w f_{yw}/(stf_c) = 4.01. \quad (36)$$

According to the general yielding criterion Eqs.(24)~(27), the test girders have the following yielding criteria:

$$0.456v^2 + 0.456t^2 + 1.828n + 1.828m = 1, \quad (37)$$
$$0.456v^2 + 0.456t^2 - 1.828n - 1.828m = 1, \quad (38)$$
$$0.129v^2 + 0.129t^2 + 0.646n - 0.646m = 1, \quad (39)$$
$$0.129v^2 + 0.129t^2 - 0.646n + 0.646m = 1, \quad (40)$$

where

$$n = \frac{N}{Af_c} = \frac{N}{5414400}, \quad (41)$$
$$m = \frac{M}{Wf_c} = \frac{M}{569200000}, \quad (42)$$

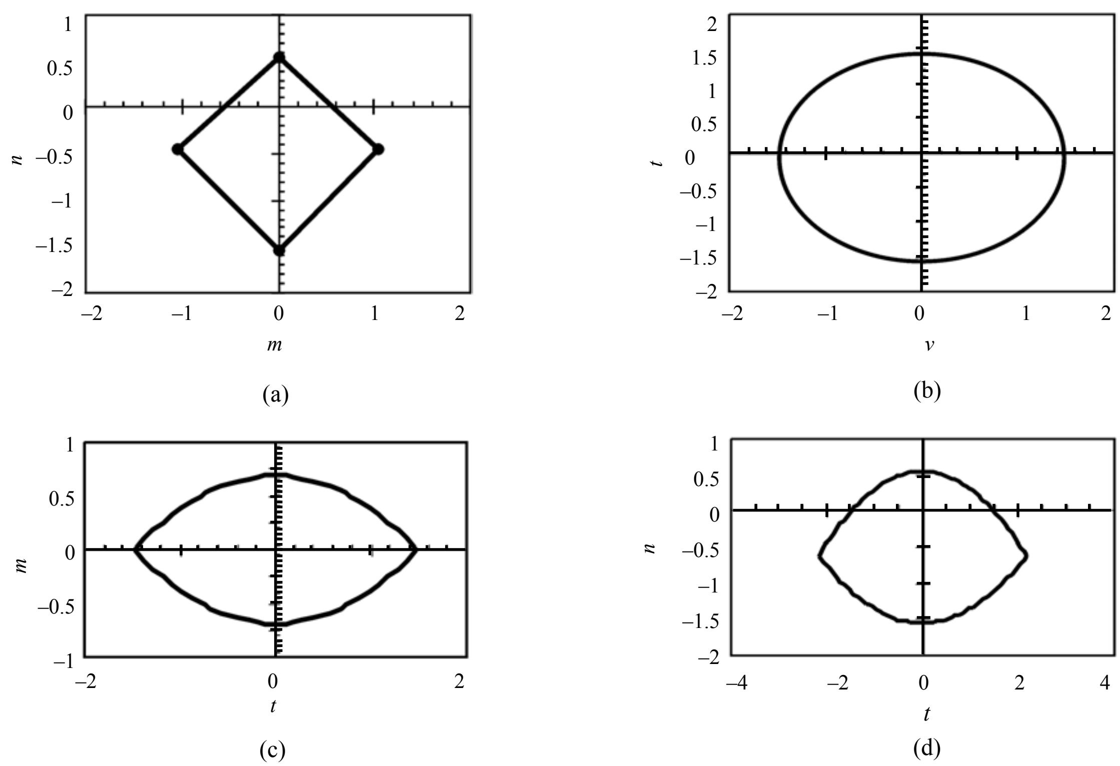

Fig.7 The relationships of box section girders under combined actions
(a) *n-m* curve; (b) *t-v* curve; (c) *m-t* curve; (d) *n-t* curve

Table 1 Comparison of theoretical results and test results

Name of girder	Section dimensions			Test results				Theoretical results				Comparison			
	H (mm)	B (mm)	t (mm)	N_e (KN)	M_e (KN·m)	V_e (KN)	T_e (KN·m)	N_t (KN)	M_t (KN·m)	V_t (KN)	T_t (KN·m)	$\frac{N_e}{N_t}$	$\frac{M_e}{M_t}$	$\frac{V_e}{V_t}$	$\frac{T_e}{T_t}$
SETMQ1	600	600	120	1152	700	0	240	1152	760	0	240	1.0	0.92		1.0
SETMQ2	600	120	600	1152	582	0	200	1152	710	0	200	1.0	0.82		1.0
TRAG1	600	120	600	1152	650	0	166	1152	758	0	166	1.0	0.86		1.0
TRAG2	600	120	400	1152	428	126	250	1152	424	126	250	1.0	1.01	1.0	1.0
TRAG3	600	80	100	775	227	63	125	775	279	63	125	1.0	0.81	1.0	1.0

$$v=\frac{V}{2Htf_c}=\frac{V}{3384000}, \tag{43}$$

$$t=\frac{T}{2t(B-t)(H-t)f_c}=\frac{T}{1299456000}. \tag{44}$$

The relationships above can be expressed as relevant curves or spaces shown in Fig.7 (see the proceding page). Table 1's comparison of test results (Falkner *et al*., 1997; Kordina *et al*., 1984) with theoretical results of box section under combined action of torsion, bending and shear show good agreement between the two results.

CONCLUSION

Reinforced concrete structural elements with box section are commonly used in the horizontal and vertical structure of bridges. The reinforced concrete structure in bridge often failed under the combined action of bending, axial load, shear and torsion caused by wind and earthquake.

This work investigated the mechanism of RC box section structures subjected to the action of combined forces. Theoretical study and derivation of a unified expression for the failure of reinforced concrete members with box section under combined bending, shear, axial force and torsion were on the assumption of stress equilibrium. Comparison of the results of our theoretical analysis with the experimental results of TU Braunschweig/Germany showed that the unified expression for the failure of reinforced concrete members with box section can be used in the static calculation of such structure members. For box section with top flange, further study should be conducted on the basis of the results obtained in this work.

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