Journal of Zhejiang University SCIENCE A
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Reliability analysis of diesel engine crankshaft based on 2D stress strength interference model

ZHOU Xun (周 迅), YU Xiao-li (俞小莉)

(*School of Mechanical and Energy Engineering, Zhejiang University, Hangzhou 310027, China*)

E-mail: zhouxun@zju.edu.cn; yuxl@zju.edu.cn

Received Mar. 24, 2005; revision accepted Nov. 19, 2005

Abstract: A 2D stress strength interference model (2D-SSIM) considering that the fatigue reliability of engineering structural components has close relationship to load asymmetric ratio and its variability to some extent is put forward. The principle, geometric schematic and limit state equation of this model are presented. Reliability evaluation for a kind of diesel engine crankshaft was made based on this theory, in which multi-axial loading fatigue criteria was employed. Because more important factors, i.e. stress asymmetric ratio and its variability, are considered, it theoretically can make more accurate evaluation for structural component reliability than the traditional interference model. Correspondingly, a Monte-Carlo Method simulation solution is also given. The computation suggests that this model can yield satisfactory reliability evaluation.

Key words: Fatigue reliability, 2-dimensional interference model (2D-SSIM), Monte-Carlo Method, Load asymmetric ratio, Multi-axial fatigue criteria

doi:10.1631/jzus.2006.A0391 **Document code:** A **CLC number:** TB114.3; TK422

INTRODUCTION

For critical structures and applications, requiring a given reliability, it is necessary to account for uncertainties and variability in material properties, loads and geometric tolerances.

Using various reliability analysis methods, we can determine the reliability of existing structures or systems; or design new structures or systems with certain desired reliability.

Manufacturing processes may involve many factors that have inherent uncertainties or random variations in them, such as geometrical tolerances, humidity and temperature changes, variations in chemical compositions and mechanical properties of raw materials. Such uncertainties may have an influence on the quality of the resulting product, but are difficult to quantify.

Using reliability methods, effects and relative importance of various manufacturing factors on the product quality can be quantified. With this knowledge, it is possible to: (a) increase control on those factors that have important effect on the product quality, and (b) relax over-stringent control on other factors that are not critical to the quality, thus to reduce cost without compromising quality.

For structural components working reliability estimation, stress strength interference model is one of the most commonly used methods. One-dimensional stress strength interference model was initially applied to fatigue reliability analysis by Freudenthal *et al*.(1966). The model was later extensively used in fatigue reliability analysis. In 1D stress strength interference model, applied stress and resistive strength are all reduced to probability distribution. When applied stress demand, S, and resistive strength capability, R, are defined by probability distribution, failure occurs when the tails of the two distributions overlap, as shown in Fig.1. The tail-overlap area suggests the probability that a weak resistive material will encounter an excessively applied stress to cause failure. The limit state equation is

$$Z = R - S = 0.$$

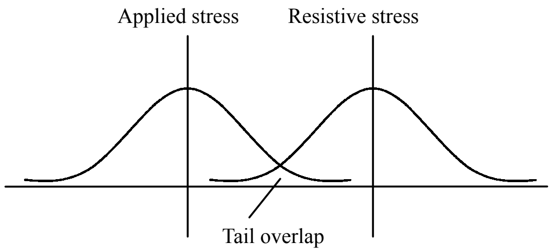

Fig.1 One-dimensional stress strength interference model

Suppose that applied and resistive stress probability density functions are all normal and independent and may be combined to form a third normal expression, known as the safety index

$$\beta=\frac{\mu_{\mathrm{R}}-\mu_{\mathrm{S}}}{\sqrt{\sigma_{\mathrm{R}}^{2}+\sigma_{\mathrm{S}}^{2}}},$$

where μ_{R} and σ_{R} is the mean and variance of resistive strength density function, respectively; μ_{S} and σ_{S} the mean and variance of applied stress density function, respectively. The relationship between the failure probability and the safety index is expressed as:

$$p_{\mathrm{f}}=\int_{-\infty}^{\frac{\mu_Z}{\sigma_Z}}\frac{1}{\sqrt{2\pi}}\exp(-t^2/2)\mathrm{d}t=\Phi(-\beta),$$

where μ_Z and σ_Z is the mean and variance of the security residual Z, respectively; Φ is the standard normal cumulative distribution function.

For fatigue reliability, it is well known that structural component's endurance limit has close relationship to the load feature, usually denoted by the stress asymmetric ratio:

$$r=\frac{s_{\min}}{s_{\max}}=\frac{s_{\mathrm{m}}-s_{\mathrm{a}}}{s_{\mathrm{m}}+s_{\mathrm{a}}},$$

where $s_{\min}$, $s_{\max}$, s_{m} and s_{a} is the minimum, maximum, mean and amplitude of applied stress, respectively. Empirically, the relationship between the stress asymmetric ratio and fatigue life is predicted by endurance limit curve. The Gerber relationship diagram is one of them, as shown in Fig.2.

Because the variability of the stress asymmetric ratio is ignored, the traditional stress strength interference model is a simplified and deficient method for reliability analysis. As the requirements of reliability are now becoming more and more critical, i.e., failure probability is now lower and lower, when estimating fatigue reliability of some special component used in complicated environment, such as engine crankshaft, satisfying estimation accuracy is often impossible with the use of this model. In this regard, the need to cope with many more uncertainties in a reliability engineering design has long been recognized. In the recent two decades, some complicated reliability assessment models or solution methodologies for them have been developed (Proschan, 1980; Ferdous *et al*., 1995; Liao *et al*., 1995). These models or methodologies made the reliability assessment more accurate and more effective in general or for some special applications. Yet, as an important factor of fatigue reliability, load feature, i.e. the asymmetric ratio of load and its variability, has not been thoroughly and clearly researched. So it is still necessary to find a way which can take this factor into account.

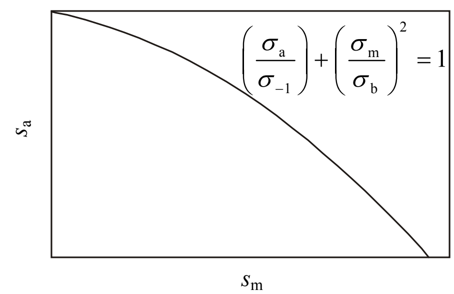

Fig.2 The Gerber relationship diagram

TWO-DIMENSIONAL STRESS STRENGTH INTERFERENCE MODEL

In order to get a more accurate analysis model, expression of loads on structural component has been developed as a joint density function with mean applied stress, s_{m}, and applied stress amplitude, s_{a}, as the variables. Assume that the border distribution of s_{m} is normal while the border distribution of s_{a} is semi-normal, the joint density equation is expressed as:

$$f(s_{\mathrm{a}},s_{\mathrm{m}})=\frac{\mathrm{e}^{\frac{-1}{2(1-\rho^2)}\left[\frac{(s_{\mathrm{a}}-\mu_{\mathrm{a}})^2}{\sigma_{\mathrm{a}}^2}-2\rho\frac{(s_{\mathrm{a}}-\mu_{\mathrm{a}})(s_{\mathrm{m}}-\mu_{\mathrm{m}})}{\sigma_1\sigma_2}+\frac{(s_{\mathrm{m}}-\mu_{\mathrm{m}})^2}{\sigma_{\mathrm{m}}^2}\right]}}{\pi\sigma_{\mathrm{a}}\sigma_{\mathrm{m}}\sqrt{1-\rho^2}},$$

where μ_a and σ_a is the mean and variance of applied stress amplitude, respectively; μ_m and σ_m is the mean and variance of mean applied stress, respectively; ρ ($|\rho|<1$) is the correlation coefficient of s_a and s_m. The distribution surface is as shown in Fig.3.

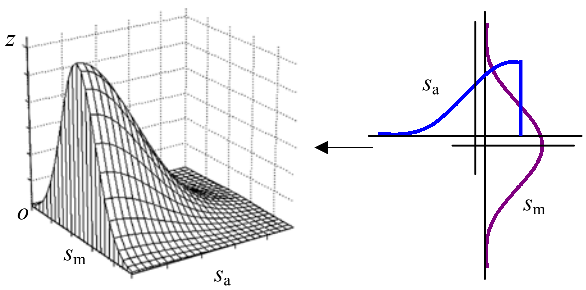

Fig.3 The joint density distribution surface of applied stress

Firstly ignore the variability of resistive strength. In the s_a-s_m coordinate system, the scanning face in z-orientation through the Gerber relationship curve dimidiates the joint density distribution surface. This is described graphically in Fig.4. According to the definition of endurance limit, the separated part inside the scanning face represents the survival region while the outside part is the failure region.

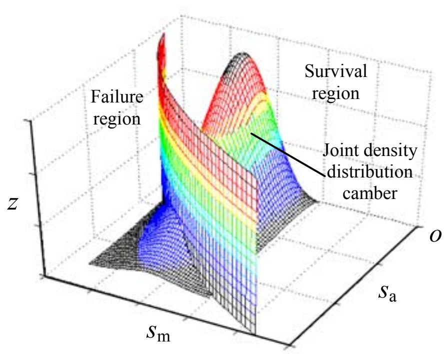

Fig.4 The joint density distribution surface dimidiated by the Gerber relationship scanning curved face

The volume covered by the joint density distribution surface of failure region is equal to failure probability. It is numerically calculated as

$$\gamma = \iint_{\Omega_f} f(s_a, s_m) \mathrm{d}s_a \mathrm{d}s_m,$$

where Ω_f represents the failure region.

When the variability of resistive strength is taken into account, the relationship between endurance limit and stress asymmetric ratio is not a single curve, but a surface made up by a series curve with different probability density. Fig.5 shows the example of Gerber relationship.

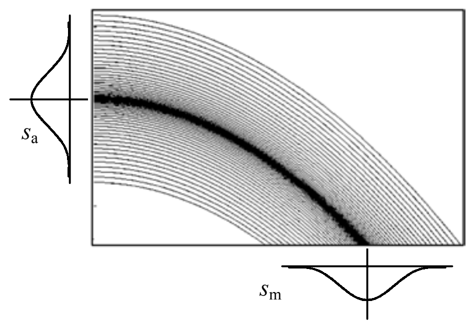

Fig.5 The density distribution surface of Gerber relationship

In this instance, it is supposed that both the mean resistive strength and the resistive strength amplitude follow normal distribution.

Drawing the density distribution surface of Gerber relationship and the density distribution surface of applied stress in the same coordinate system, a 2D stress strength interference contour is produced, as shown in Fig.6. The volume covered by the tail overlap curved face suggests the probability that a component with random resistive strength encounters random excessively applied stress, in both mean and amplitude, to cause failure because of insufficient strength. The probability may be analytically expressed as:

$$\gamma = \frac{V_{\text{overlap}}}{V_{\text{total}}} = \iint_{\Omega_o} f_r(s_a, s_m) f_a(s_a, s_m) \mathrm{d}s_a \mathrm{d}s_m,$$

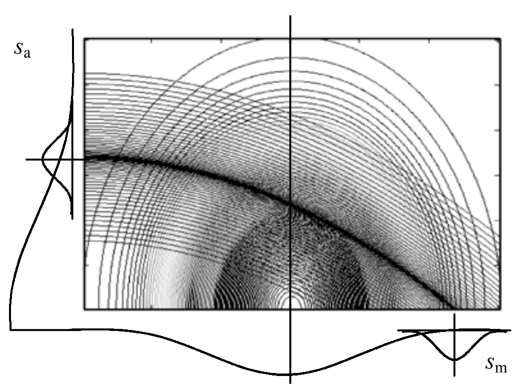

Fig.6 2D stress strength interference contour

where Ω_o represents the tail overlapped surface.

Because the variability of stress asymmetric ratio is considered, it is concluded that the 2D-SSIM describes the problem more practically, and calculate failure probability more accurately than the 1D model.

Similar to 1D stress strength interference model, the limit state equation of 2D-SSIM can be derived from Fig.7. The limit state expression becomes a system of equations in the 2D case:

$$\begin{cases} Z_m = \dfrac{-\left(\dfrac{\eta\sigma_b^2}{\sigma_{-1}}\right)+\sqrt{\left(\dfrac{\eta\sigma_b^2}{\sigma_{-1}}\right)^2+4\sigma_b^2}}{2} - s_m = 0, \\ Z_a = \dfrac{-\left(\dfrac{\eta^2\sigma_b^2}{\sigma_{-1}}\right)+\eta\sqrt{\left(\dfrac{\eta^2\sigma_b^2}{\sigma_{-1}}\right)^2+4\sigma_b^2}}{2} - s_a = 0, \end{cases}$$

where η is a factor determined by stress asymmetric ratio:

$$\eta = (r-1)/(r+1).$$

Then the failure probability is calculated by

$$P_f = P\ (Z_m<0 \cup Z_a<0).$$

It means that if either s_a or s_m is out of the survival area, the component fails.

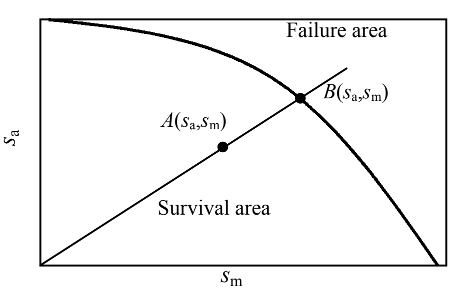

Fig.7 Graphical position of primary load magnitude in the Gerber relationship curve

ENGINE CRANKSHAFT RELIABILITY ANALYSIS WITH 2D-SSIM

Failure probability estimation for a diesel engine crankshaft was made based on 2D-SSIM. This component is made of ductile material, there is a correlative relationship between the tensile strength (σ_b) and the fatigue strength (σ_{-1}). As a general rule, the fatigue strength with increase in tensile strength is empirically expressed as:

$$\sigma_{-1}/\sigma_b = \rho_\sigma,$$

where ρ_σ is the correlation coefficient.

Tensile and fatigue strength experiments were conducted to get statistical data on σ_b, σ_{-1} and their correlation coefficient ρ_σ, the results are listed in Table 1. All stresses were measured at the round corner of the crankpin.

Table 1 Statistical data on measured crankshaft strength

Item	Distribution	Mean μ	Variability coefficient δ
σ_b	Normal	743 MPa	0.0152
σ_{-1}	Normal	215.25 MPa	0.0713
k_σ	Normal	0.288	0.0139

A crankshaft is usually considered to be under combined non-proportional out-of-phase bending and torsion (Fig.8). Like the stress measured point, stress at the round corner of the crankpin was calculated at the check point.

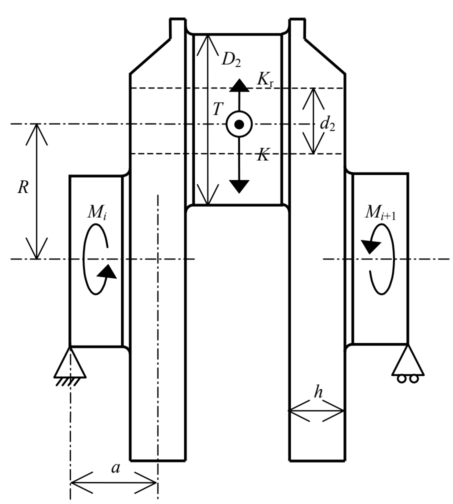

Fig.8 Mechanical calculation model of the testing crankshaft

Based on the dynamical analysis of the crankshaft, formulas for nominal normal and shear stress estimation at rated load are given in Table 2, where W_σ represents section modulus of bending, W_t represents section modulus of torque.

$$W_\sigma = \frac{bh^2}{6}, \quad W_t = \frac{\pi}{16} D_2^3 \left[1 - \left(\frac{d_2}{D_2}\right)^4\right].$$

It is suitable to use multi-axial fatigue criteria to evaluate crankshaft's fatigue reliability. For this application, Lee (1985)'s criterion was applied. To account for stress concentration factor and surface modification factor respectively, the mean and amplitude of stress on the round corner of the crankpin are evaluated by the following formulas:

$$\begin{cases} s_a = \dfrac{1}{\beta} k_\sigma (\sigma_n)_a \left\{1 + \left[1.71 \dfrac{k_\tau (\tau_n)_a}{k_\sigma (\sigma_n)_a}\right]^\xi\right\}^{1/\xi}, \\ s_m = \dfrac{1}{\beta} \sqrt{[k_\sigma (\sigma_n)_m]^2 + 3[k_\tau (\tau_n)_m]^2}. \end{cases}$$

The exponent ξ is calculated by $\xi=2(1+\gamma\sin\varphi)$. where φ is the phase difference between applied torsion and bending, and γ is a material constant.

In the calculation, most parameters, including geometrical tolerances, combustion pressure, weights and some relative factors, etc., are treated as stochastic quantities. Table 3 gives the details.

SOLUTION BY MCS

For 2D-SSIM, although we can easily and clearly build up the overlap surface graphically, it is still very difficult to write out its equations and analytically integrate the volume covered by it, which suggests the failure probability. So it is necessary to find a numerical method to solve the problem.

Among many methodologies introduced for structural components reliability analysis, each one

Table 2 Estimation on the stresses at rated load

Item	Position (°CA)	Estimation formula
Maximum of bending stress	365	$(\sigma_n)_{max} = [0.7809D^2 p_z - m_j R\omega^2 (0.9904 + 0.9791\lambda) - m_r R\omega^2]a / 2W_\sigma$
Minimum of bending stress	491	$(\sigma_n)_{min} = [-m_j R\omega^2 (0.5279 + 0.1119\lambda) - m_r R\omega^2]a / 2W_\sigma$
Mean/amplitude bending stress		$(\sigma_n)_a = [(\sigma_n)_{max} - (\sigma_n)_{min}]/2,\ (\sigma_n)_m = [(\sigma_n)_{max} + (\sigma_n)_{min}]/2$
Maximum of torque stress	385	$(\tau_n)_{max} = [0.4094D^2 p_z - m_j R\omega^2 (0.4724 + 0.3351\lambda)]R / W_t$
Minimum of torque stress	351	$(\tau_n)_{min} = [-0.1149D^2 p_z + m_j R\omega^2 (0.1936 + 0.1864\lambda)]R / W_t$
Mean/amplitude torque stress		$(\tau_n)_a = [(\tau_n)_{max} - (\tau_n)_{min}]/2,\ (\tau_n)_m = [(\tau_n)_{max} + (\tau_n)_{min}]/2$

Table 3 Description and distribution characters of those variables in working stress evaluation

Item	Parameter name	Distribution
a	Space between crank centre and crank journal centre	Truncated normal
b	Crank body width	Normal
D	Cylinder diameter	Truncated normal
D_2	Crankpin diameter	Truncated normal
d_2	Diameter of crankshaft unweight hole	Truncated normal
h	Crank body thickness	Normal
k_σ	Concentration factor of bending stress at round corner	Normal
k_τ	Concentration factor of torque stress at round corner	Normal
m_j	Back-forth mass of piston-rod mechanism	Truncated normal
m_r	Rotary mass of piston-rod mechanism	Truncated normal
L	Crank journal length	Truncated normal
ω	Crank angular velocity	Normal
p_z	Maximum combustion pressure	Normal
R	Crank radius	Truncated normal
β	Modification factor	Normal
λ	Crank radius-connection rod length ratio	Normal

has special applications and characters. In numerical methods, first order second moment (FOSM) reliability method is a commonly used, expedient and greatly developed one. Yet, this approach is only suitable for the cases of linear limit state equations and those with normal distributed random variables. When deal with nonlinear limit state equations or cases involving some random variables not obeying to normal distribution or variable correlations exist, necessary transformations and simplifies make FOSM to put out solutions with insufficient accuracy. Sometimes even may fail to make any sense especially to those components like crankshaft.

Monte-Carlo simulation (MCS) is another widely used method to estimate the failure probability of structural systems and components. MCS can be used for implicit and (or) non-linear state limit functions without any transformations and simplifies. Even for very complicated functions with multi random variables, it still can simulate directly. Estimation accuracy is only up to the simulation time. Additionally, the excellent performance of modern day's computer systems provides right conditions for accurate MCS processing.

An MCS solving program was written for this testing crankshaft. At each run of the MCS, random quantities, such as geometrical size, pressure and mass etc., are generated firstly for evaluating applied stresses at the round corner of crankpin, with this stress asymmetric ratio to be gain. Then generate σ_b, and ρ_σ from their distributions and calculate corresponding r_a and r_m. At last, state of current run time, failing or surviving, can be decide by comparing to see if the applied stress is inside the survival region according to the limit state function. The algorithm is explained in Fig.9.

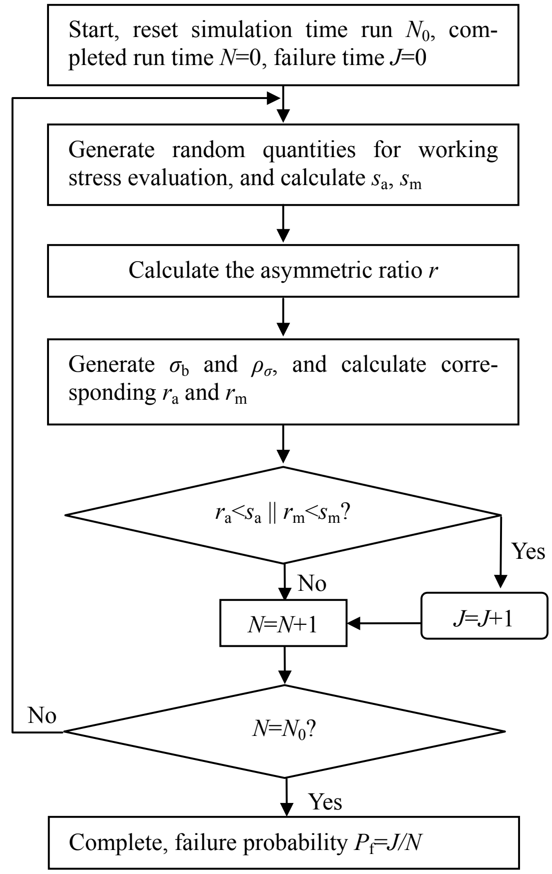

Fig.9 Flow chart of Monte-Carlo simulation

An MCS procedure without considering the variability of the stress asymmetric ratio was also conducted which yielded a failure probability of 2.15E−4. This example shows that the 2D-SSIM makes fatigue reliability assessment more realistic.

Figs.10 and 11 show the estimated relationship of failure probability to mean tensile stress and stress variability coefficient in a limited range. If the curves shown in Figs.10 and 11 are referred to, manufacturer's incorrect reliability assessment and faulty decision may be avoided. Maintenance scheduling and safety standards can also be rationalized accordingly.

ANALYSIS RESULTS

For this application, MCS was conducted with solving tolerance set as 1E−5. With the given working condition and tested resistive strength, the failure probability of the structural component is calculated to be 3.73E−4, while the factual failure probability is about 4.0E−4, according to the crankshaft manufacturer's practical investigation results.

SUMMARY

2D-SSIM is proposed as a complementary theory to cope with the problem of reliability analysis of complicated structural components where traditional

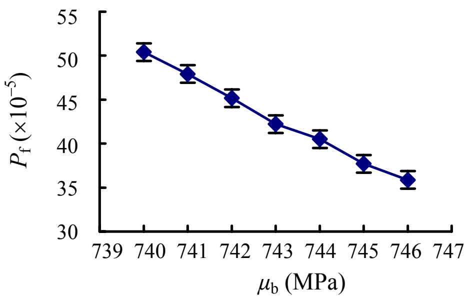

Fig.10 Relationship between the failure probability and mean tensile stress of the crankshaft (δ_b=0.015)

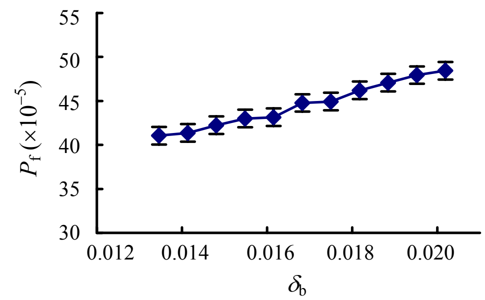

Fig.11 Relationship between the failure probability and the tensile stress variability coefficient of the crankshaft (μ_b=743 MPa)

1D interference model is insufficient. Because more important factors, i.e. stress asymmetric ratio and its variability, are considered, it theoretically can make more accurate evaluation for structural component reliability.

Fatigue failure probability of an engine crankshaft was evaluated based on this model. Calculation results of the application show that this methodology is feasible. A series of relationship curves were worked out as instructive references for manufacturer's planning.

References

Ferdous, J., Borhan, U., Pandey, M., 1995. Estimation of reliability of a component under multiple stresses. *Microelectronics Reliability*, **35**(2):279-283. [doi:10.1016/0026-2714(94)00051-O]

Freudenthal, A.M., Carrelts, M., Shinozuka, M., 1966. The analysis of structural safety. *Journal of Structure Division, ASCE*, **92**:267-325.

Lee, S.B., 1985. A Criterion for fully Reversed Out-of-phase Torsion and Bending. *In*: Miller, K.J., Brown, M.W.(Eds.), Multiaxial Fatigue. ASTM, Philadelphia, p.553-568.

Liao, M., Xu, X.F., Yang, Q.X., 1995. Cumulative fatigue damage dynamic interference statistical model. *Int. J. Fatigue*, **17**(8):559-566. [doi:10.1016/0142-1123(96)81216-3]

Proschan, F., 1980. A new approach to inference from accelerated life test. *IEEE Trans. Reliab.*, **29**:98-102.