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Rate-equation-based VCSEL thermal model and simulation*

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Abstract: In this paper, we present a simple thermal model of Vertical-Cavity Surface-Emitting Laser (VCSEL) light-current (LI) characteristics based on the rate-equation. The model can be implemented in conventional SPICE-like circuit simulators, including HSPICE, and be used to simulate the key features of VCSEL. The results compare favorably with experimental data from a device reported in the literature. The simple empirical model is especially suitable for Computer Aided Design (CAD), and greatly simplifies the design of optical communication systems.

Key words: Carrier leakage, Vertical-Cavity Surface-Emitting Lasers (VCSELs), Rate-equation, Thermal model, SPICE simulation

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INTRODUCTION

Vertical-Cavity Surface-Emitting Lasers (VCSELs) are a new generation of semiconductor lasers that differ considerably from the conventional edge-emitting lasers. In recent years, the characteristics of VCSELs have improved enormously (Badilita *et al*., 2004). Today's VCSELs have low-threshold current densities and high output power. Moreover, their circular output beam profiles and the suitability to be integrated into 2D arrays make them most promising candidates for short-range optical communications and optical interconnects.

Due to the large resistance introduced by the distributed Bragg reflectors (DBRs) and their poor heat dissipation, VCSELs have strong dependence on their thermal effect. Temperature effect on semiconductor lasers is an important factor to be considered in laser design and application. Obviously, in order to design VCSELs with even better performance, it is important to be able to model the devices with optical, electrical and thermal effects.

In this paper, we present a thermal VCSEL model, based on the well-known rate-equations. Although it is really simple, it can model basic laser behavior under both dc and non-dc conditions, so it can inherently describe light-current (LI) characteristics over a wide temperature range without referring to detailed thermal physics. The simple empirical model is especially suitable for Computer Aided Design (CAD), and greatly simplifies the design of optical communication systems.

RATE-EQUATION THERMAL MODEL

Our VCSEL model provides accurate description of the VCSEL's terminal characteristics. First, we discuss the foundation of the model based on the rate-equations which account for the spatial dependence of the VCSEL's behavior. Second, we present simple empirical expression for the thermally dependent active layer and get the spatially independent rate-equation. We then present a thermal rate-equation that models the device temperatures as a function of dissipated heat, and expressions that model the VCSEL's output power and electrical characteristics.

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characteristics.

The strong thermal dependence of VCSEL can account for a few of the mechanisms. Although Auger recombination and optical losses of inter-valence band absorption play a role in the thermal behavior (Mao *et al*., 2002), the dominating effects are due to the temperature dependent optical gain and the carrier leakage of the active region.

Spatially dependent rate-equation

It is a comprehensive technique that adopts the rate-equation to describe the laser's characteristic (Mena *et al*., 1999). The first equation, which describes the carrier number N in the active region, is

$$\frac{\partial N(\boldsymbol{r},t)}{\partial t}=\frac{\eta_{\mathrm{i}}I(\boldsymbol{r},t)}{q}-\frac{N(\boldsymbol{r},t)}{\tau_{\mathrm{n}}}-G(\boldsymbol{r},t,T)S-\frac{I_{\mathrm{l}}(N,T)}{q}, \tag{1}$$

where I is the injection current; S is the total photon number; η_{i} is the current injection efficiency; T is VCSEL's temperature; G is the optical gain; q is the electron charge; τ_{n} is the carrier lifetime; and I_{l} is the thermal leakage current.

The second equation, which describes the total photon number S, is

$$\frac{\partial S(t)}{\partial t}=-\frac{S(t)}{\tau_{\mathrm{p}}}+\frac{\beta N(\boldsymbol{r},t)}{\tau_{\mathrm{n}}}+G(\boldsymbol{r},t,T)S(t), \tag{2}$$

where β and τ_{p} are spontaneous-emission coefficient and the photon lifetime, respectively.

In addition to supplying a means for describing a VCSEL's spatially dependent behavior, Eqs.(1) and (2) also provide a way with which the carrier leakage and thermally dependent gain can be modelled. In the above equations, the carrier leakage is modelled with the current I_{l} and the gain is modelled via G. By using simple empirical expressions for these two terms, we can avoid complex computational process due to detailed physical descriptions.

Temperature dependent gain

The temperature dependence of gain can be explained as follows. As VCSEL's temperature rises, its gain peak shifts to longer wavelength and gains spectrum broadens. Therefore, the optical gain will change with temperature as the gain peak and wavelength become mismatched.

Just commonly done in other rate-equation-based VCSEL models, we model the thermally dependent optical gain G as a linear function of the carrier number N and incorporate the gain saturation with gain compression factor (Mena *et al*., 1999):

$$G(T)=\frac{G_{\mathrm{T}}(T)(N-N_{\mathrm{tr}}(T))}{(1+\varepsilon S)^{b}}, \tag{3}$$

where G_{T} is the optical temperature-dependent gain; N_{tr} is the temperature-dependent transparency carrier number; ε is the gain compression factor; and b is a constant, in our paper b=1 for Two-Level model.

By making G_{T} and N_{tr} functions of temperature, the gain's thermal-dependence can be described in a simple equation without resorting to complex calculations. As for edge-emitting lasers, G_{T} and N_{tr} are modelled as linear functions of temperature or exponential functions of temperature $\exp(\pm\alpha T)$. In VCSELs, we use similar thermal expressions. Based on the model of Badilita *et al*.(2004) and Scott *et al*.(1993), we develop a more analytical model. They are described with the following more generally accepted empirical equations:

$$G_{\mathrm{T}}(T)=G_{0}\left(\frac{a_{0}+a_{1}T+a_{2}T^{2}}{b_{0}+b_{1}T+b_{2}T^{2}}\right), \tag{4}$$

$$N_{\mathrm{tr}}(T)=N_{\mathrm{tr}0}(c_{0}+c_{1}T+c_{2}T^{2}), \tag{5}$$

where G_0 and $N_{\mathrm{tr}0}$ are temperature-independent gain constant and transparency carrier number, respectively; a_0~a_2, b_0~b_2 and c_0~c_2 are empirical matching constants. The values of different devices are obtained from the thermal gain figures with repeating contrast.

Thermal carrier leakage

Besides the temperature dependent gain, thermal leakage of carriers out of the active region is the most severe thermally dependent mechanism in VCSELs (Badilita *et al*., 2004). As the leakage reduces the VCSEL's total efficiency, we include it in our model via subtracting thermal leakage current I_{l} in Eq.(1). Scott *et al*.(1993) assumed the carrier leakage could be modelled with a function of the quasi-Fermi level separation ΔE_{fer}. They used a curve to fit the carrier's thermal dependence of (ΔE_{fer}–E_{gB}) with the equation ($-d_0+d_1N+d_2NT-d_3/N$), where E_{gB} is the bandgap of the confinement layer around the active region; N is the carrier number and d_0~d_3 are constants. As we can model the carrier leakage using an approximate homojunction diode relationship $\exp[(\Delta E_{fer}-E_{gB})/T]$, the following equation is obtained:

$$I_1 = I_{10}\exp\left[\left(-d_0+d_1N+d_2NT-d_3/N\right)/T\right]. \quad (6)$$

Fig.1 shows plots of Eq.(6) at temperature from 200 K to 450 K using values of d_0~d_3 based on data in (Scott *et al*., 1993). As we see, at 200 K and 250 K the leakage is negligible for small carrier number, and increases dramatically after some threshold value. In the practical region of VCSELs, Eq.(7) is suitable for the experimental device data so that we used Eq.(6) in our thermal model.

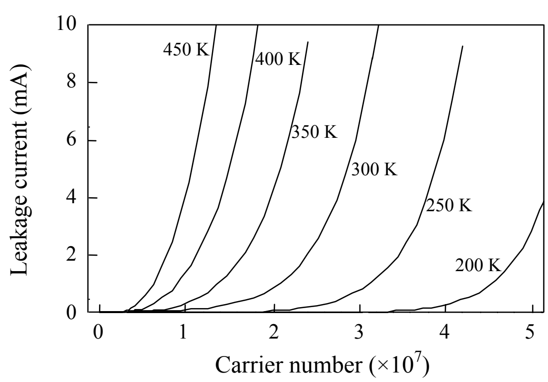

Fig.1 Leakage current as calculated by Eq.(6) at temperatures from 200 to 450 K based on data in (Scott *et al.*, 1993)

Thermally dependent model

As they require the whole spatial calculations, the spatially dependent rate Eqs.(1) and (2) are not suitable for the circuit-level model. As mentioned previously, the thermally dependent gain is the dominating factor which affects VCSEL's characteristics. For simplifying the model to be more computable, we neglect the spatial effect, then we get the equations below:

$$\frac{\partial N}{\partial t} = \frac{\eta_i I}{q} - \frac{N}{\tau_n} - \frac{G_T(T)\left(N-N_{tr}(T)\right)S}{1+\varepsilon S} - \frac{I_1(N,T)}{q}, \quad (7)$$

$$\frac{\partial S}{\partial t} = -\frac{S}{\tau_p} + \frac{\beta N}{\tau_n} + \frac{G_T(T)\left(N-N_{tr}(T)\right)S}{1+\varepsilon S}, \quad (8)$$

where G_T and N_{tr} are defined with Eqs.(4) and (5), respectively. The optical output power can be denoted with $P_o=\rho S$, where ρ is a factor which represents the output-power coupling coefficient of VCSELs.

Next, we must construct a suitable expression for the VCSEL temperature T. A much simpler method is to describe the temperature via a thermal rate-equation that results in the transient temperature increase as the result of heat dissipation. Based on this approach, we get

$$T = T_0 + (I_{tot}V - P_o)R_{th} - \tau_{th}\frac{dT}{dt}, \quad (9)$$

where R_{th} is the VCSEL's thermal impedance; τ_{th} is the thermal time constant (generally in the order of 1 μs); T_0 is the environment temperature; I_{tot} is the total current flowing through the device and V is the device voltage. We assumed all power except optical output is dissipated as heat.

Finally, we can model the current-voltage (IV) relationship based on the VCSEL's diode-similar character. But for simplicity, we modelled them using the following empirical function of current and temperature:

$$V = f(I,T), \quad (10)$$

where we assumed the current corresponding to the voltage V is the cavity injection current I. By introducing a capacitor or other parasitic components in parallel with the voltage, we can model the complete electrical characteristics of the VCSEL. The advantage of this technique is that Eq.(10) can be determined on the corresponding specific device. This simplified approach avoides the complex analysis of physical mechanism. With the empirical equation above and the experimental data, we can model the VCSELs more accurately.

Because our main goal is establishing a VCSEL model that can be used in CAD of optoelectronic systems, we try to implement this model in SPICE-like simulators. Then we can simulate the VCSELs combined with other electrical devices such as laser drivers. The realization of SPICE simulation depends on the transformation from the model equations above into the equivalent circuit representation.

To improve the model convergence property, we introduce some transformations for P_o and N by following equations:

$$P_o = (v_m + \delta)^2, \quad (11)$$

$$N = z_n v_n, \quad (12)$$

where z_n and δ are arbitrary constants. Such transformation makes the simulator converge to a correct numerical solution more quickly. Substituting Eqs.(11)~(12) into Eqs.(7) and (8) and applying proper operations, we obtained the equivalent circuit illustrated in Fig.2.

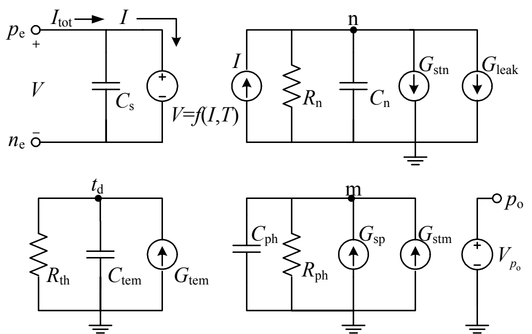

Fig.2 Equivalent-circuit representation of VCSEL thermal model

In Fig.2, p_e and n_e are the VCSEL's electrical interface; p_o is the terminal whose node voltage models the optical output power P_o and t_d is the terminal which models the temperature T defined by Eq.(9). Electrical characteristics of VCSELs are modelled with a nonlinear voltage source V defined by Eq.(10), and C_s models a simple shunting capacitor. The temperature equation Eq.(9) is modelled by a resistor R_{th}, capacitor C_{tem} and a nonlinear current source G_{tem}. From Eq.(9), we can know $C_{tem}=\tau_{th}/R_{th}$ and

$$G_{tem} = T_0 / R_{th} + (I_{tot}V - P_o). \quad (13)$$

The carrier rate Eq.(7) is modelled with the capacitor $C_n=qz_n/\eta_i$, resistor $R_n=\eta_i\tau_n/(qz_n)$, and the nonlinear current sources G_{stn} and G_{leak}, where

$$G_{stn} = \frac{qG_0}{\eta_i \rho} \cdot \frac{a_0 + a_1 V_{td} + a_2 V_{td}^2}{b_0 + b_1 V_{td} + b_2 V_{td}^2} \cdot \frac{z_n v_n - N_{tr0} \cdot (c_0 + c_1 V_{td} + c_2 V_{td}^2)}{\left(1 + (v_m + \delta)^2 \varepsilon / \rho\right)(v_m + \delta)^2}, \quad (14)$$

$$G_{leak} = \frac{I_{l0}}{\eta_i} \exp \frac{\left(-d_0 + d_1 z_n v_n + d_2 z_n v_n V_{td} - \dfrac{d_3}{z_n v_n}\right)}{V_{td}}. \quad (15)$$

In the same way, the photon rate Eq.(8) can be modelled with the capacitor $C_{ph}=2\tau_p$, resistor $R_{ph}=1$, and the nonlinear current sources G_{sp} and G_{stm}, where

$$G_{sp} = \frac{\rho \tau_p \beta z_n v_n}{\tau_p (v_m + \delta)}, \quad (16)$$

$$G_{stm} = \frac{G_0 \tau_p [z_n v_n - N_{tr0} \cdot (c_0 + c_1 V_{td} + c_2 V_{td}^2)]}{\left(1 + (v_m + \delta)^2 \varepsilon / \rho\right)(v_m + \delta)} \cdot \frac{a_0 + a_1 V_{td} + a_2 V_{td}^2}{b_0 + b_1 V_{td} + b_2 V_{td}^2} - \delta. \quad (17)$$

Then V_{p_o} transforms v_m into the optical output power P_o.

SIMULATION AND COMPARISON TO EXPERIMENT

The results presented in the previous section demonstrate that our rate-equation-based thermal VCSEL model can be used to simulate the VCSEL behavior perfectly. Now it is necessary to verify it with experimental data. To accomplish the work, we used the experimental devices data from (Scott *et al*., 1993) and fit our thermal model to the reported operation characteristics.

The device reported in (Scott *et al*., 1993) is an index-guided, vertically-contacted VCSEL. The device has 100 μm^2 area and is composed of GaAs-AlAs DBR mirrors, three $In_{0.2}Ga_{0.8}As$ quantum wells, and $Al_{0.2}Ga_{0.8}As$ confinement layers. In addition to presenting measured LI characteristics at ambient temperatures of 25, 45, 65, and 85 °C, the authors also provided an analytical estimate for the thermal impedance and a formula for the device voltage as a polynomial function of current and temperature. Utilizing these additional equations in conjunction with the measured LI curves, we determined the measured IV data at 25, 45, 65, and 85 °C.

Using the LI and IV data, we determined model parameters for this device (Bruensteiner and Papen, 1999). Furthermore, we obtained the following empirical expression for the voltage V:

$$V = \frac{IR_0}{T - T_1} + V_T \ln\left[1 + \frac{I}{I_0(T - T_1)}\right], \qquad (18)$$

where R_0=13120.4, T_1=190.68, V_T=0.966 and I_0=3.026×10^{-6}. Fig.3 compares the resulting simulated LI data with the measured curves, showing a good match between the two. The biggest discrepancy is found in 25 °C, where the simulation reveals lower output power than that in the measured characteristic. However, the error is still less than ~8%.

Fig.3 shows that with the increase of input current, the output power will rise when the current is less than a certain value. But after that point, the output power will decrease with the current increases. We call the value threshold current of the VCSEL.

Fig.4 comparing the resulting simulated IV data with the measured curves, also shows a good agreement between the two. But the error is bigger below the threshold.

CONCLUSION

In this paper, we erected the VCSEL model based on simple rate-equations which utilize an offset current to account for the thermal effects. The comparison in Section III suggests our model is useful for describing the VCSEL's thermal LI characteristic. Just as mentioned before, such a model is very helpful in the simulation and design of optoelectronic systems with VCSELs. Implementing SPICE-like simulators, we can conveniently combine the VCSELs and electrical components in large-scale EDA designs.

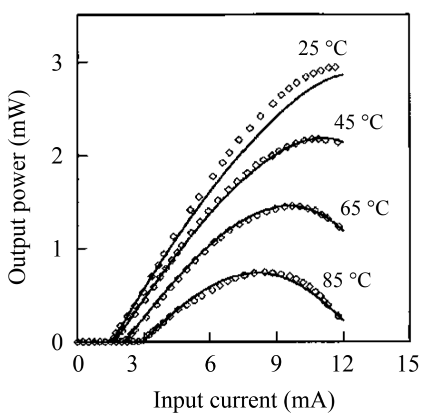

Fig.3 Comparison of measured (data points) and simulated (lines) LI curves for VCSEL in (Scott *et al.*, 1993)

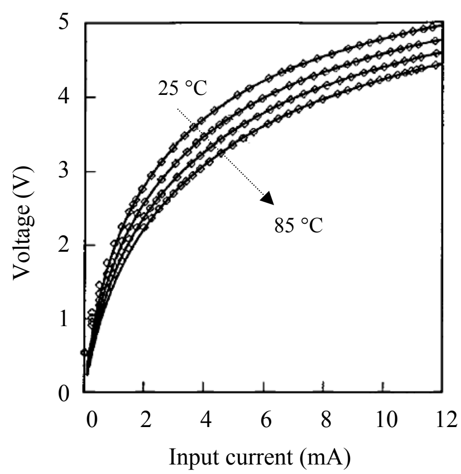

Fig.4 Comparison of measured (data points) and simulated (lines) IV curves for VCSEL in (Scott *et al.*, 1993)

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