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A high performance frequency offset estimator for OFDM*

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Abstract: This paper proposes a simple method to enlarge the estimation range of conventional carrier frequency offset (CFO) estimation methods based on correlations among the identical parts of the preamble. A novel preamble is designed, which is composed of one regular OFDM training block with even numbers of identical parts and one irregular OFDM training block with odd numbers of identical parts. The initial estimates obtained over the two training blocks are next exploited to jointly estimate the CFO. By elaborately selecting the numbers of identical parts for the two training blocks, the proposed CFO estimator can estimate frequency offset over tens of the subcarrier spacing. Simulation results showed that the proposed CFO estimator satisfies the estimate range requirement for the practical OFDM systems, while achieving a very good estimate performance.

Key words: OFDM, Frequency offset estimation, Preamble, Chinese residues theorem
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INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become a promising technology for future wireless communication systems (Kim *et al*., 2003; 3GPP TSG-RAN-1.TR, 2004) due to its ability to transform a wideband frequency selective channel to a set of parallel flat-fading narrow-band channels, which substantially simplifies the channel equalization problem. However, a critical weakness of OFDM is its sensitivity to carrier frequency offset (CFO) since it can only tolerate offsets which are a fraction of the spacing between the subcarriers without a large degradation in system performance (Rugini and Banelli, 2005). Therefore, estimation of the CFO at the receiver must be performed very accurately.

The CFOs in the OFDM systems are usually larger than the subcarrier spacing. To perform CFO estimation for these systems, several methods have been proposed. Tureli *et al*.(2004) proposes a high efficiency estimation algorithm in which only virtual subcarriers are used. However, this method has high computational complexity and suffers from the lack of identifiability when it is used in single-input single-output OFDM systems. Schmidl and Cox (1997) proposes a scheme to estimate CFO in two steps: coarse estimation and fine estimation. The fine estimation is performed in the time domain and the coarse estimation is accomplished by taking the cross-correlation between two consecutive OFDM blocks in the frequency domain. However, this method necessitates much high computational load to find the correlation values for all possible CFOs. To reduce the computational complexity of the coarse estimation, Seo *et al*.(2002) proposes an estimate method by using the phase difference between two adjacent subcarriers of the received signal. However, the training blocks designed by them suffer from high PAPR (Peak-to-Average Power Ratio). Among other things, several identical parts within a single OFDM training block can be used to estimate CFO larger than the subcarrier spacing (Morelli and Mengali,

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1999; Wu and Zhu, 2005; Kuo and Chang, 2005). However, the available estimate range is limited to $[-L/2, L/2)$ with L being the number of the identical parts. It can be verified that the larger the value of L is, the smaller the number of the switched-on subcarriers would be. For practical systems, L is usually chosen as a relatively small integer, i.e., $L=2^{\alpha}$, $\alpha\in\{1,2,3\}$ (Schmidl and Cox, 1997; Wu and Zhu, 2005; Morelli and Mengali, 1999), because sufficient switched-on subcarriers of the preamble are required to transmit the system configuration information, since they are not required to be known. Therefore, only a relatively small CFO can be corrected based on the conventional preamble with a reasonable L.

For practical applications of OFDM, the accuracy of the oscillator is in the range of a few tens of parts per million (ppm, 10^{-6}), which results in a relatively large CFO (Seo *et al*., 2002). For example, the oscillator with accuracy of $\pm 20\times 10^{-6}$ in the OFDM system (3GPP TSG-RAN-1.TR, 2004), which contains about 1024 subcarriers in a 6.528 MHz channel bandwidth, yields CFO in the range of −6.28 to 6.28 if it operates at 2 GHz frequency band. On the other hand, the important factor for the coarse frequency offset estimator is not only the accuracy or/and wide acquisition range but also the estimation speed at practical signal-to-noise (SNR) level in a high speed communication system. In this environment, the CFO estimate methods in the time domain are preferable compared to the methods in the frequency domain, which suffer from large delay or high computational complexity introduced by the discrete Fourier transform (DFT).

In this work, we design a new preamble composed of one regular OFDM training block with even numbers of identical parts and one irregular OFDM training block with odd numbers of identical parts. Furthermore, the numbers of identical parts in the two OFDM training blocks are relatively prime. Based on the designed preamble, we present a method to jointly estimate the CFO by utilizing the initial estimates obtained over the two OFDM training blocks. Simulation results showed that the estimate range of the proposed frequency offset estimator, which is several times larger than that of estimators based on the conventional preamble, satisfies the requirements for practical OFDM systems. Moreover, it has a very good estimate performance while maintaining a low computational complexity.

SYSTEM DESCRIPTION

System model

Time-domain complex base-band samples $\{x_i(n)\}$ of ith OFDM signal are generated by taking the N-point inverse discrete Fourier transform (IDFT) of a block of information symbols $\{u_i(k)\}$ belonging to a QAM or PSK constellation as

$$x_i(n)=\frac{1}{\sqrt{N}}\sum_{k=-N_{\mathrm{u}}}^{N_{\mathrm{u}}} u_i(k)\exp(\mathrm{j}2\pi kn/N), \qquad (1)$$

where $n=-N_{gi},\ldots,N-1$, the number of used subcarriers is $2N_{\mathrm{u}}+1\le N$. The useful part of each OFDM symbol block has duration of T corresponding to N sample points. N_{gi} is the length of cyclic prefix (CP) that is longer than the channel impulse response in order to avoid inter-symbol interference (ISI). The samples $\{x_i(n)\}$ are then transmitted through the possibly unknown frequency-selective channel, which in discrete-time equivalent form has finite impulse response $h(l)$ of length N_{L}.

Assuming that the timing synchronization eliminates the ISI, the received complex base-band signal $r_i(t)$ sampled with period $T_{\mathrm{s}}=T/N$ can be expressed as

$$r_i(n)=\mathrm{e}^{\mathrm{j}\phi}\mathrm{e}^{\mathrm{j}2\pi(n+iN_{\mathrm{sym}})\varepsilon/N}\sum_{m=0}^{N_{\mathrm{L}}-1} h(m)x_i(n-m)+v_i(n), \qquad (2)$$

where N_{sym} is the length of OFDM block including the CP, ε is the frequency offset normalized to the subcarrier spacing, ϕ is an arbitrary carrier phase factor, and $v_i(n)$ is the sample of zero-mean complex Gaussian noise process with variance σ_v^2.

FREQUENCY OFFSET ESTIMATION

The proposed preamble

The conventional CFO estimators based on preamble with L identical parts can estimate CFO within $[-L/2, L/2)$. To extend the CFO estimate range, a new preamble is constructed as shown in Fig.1. The preamble is composed of a regular OFDM block T_1 and an irregular OFDM block T_2. The CP of T_i is denoted as C_i and the length of C_i is N_{gi}. The training block T_1 with N_1=N complex samples is composed of L_1 identical parts, where L_1=2^{β}, $\beta\in\mathbb{N}$. The training block T_2 with N_2 complex samples is composed of L_2 identical parts, where L_2 is relatively prime to L_1 and

$$N_2=\arg\min_{\mu}\left\{|\mu-N|\,\middle|\,[\mu]_{L_2}=0\right\}. \qquad (3)$$

C_1	T_1	C_2	T_2

Fig.1 The proposed preamble structure

To obtain training block T_i satisfying the requirement of the spectrum, we first generate time domain signal $\bar{x}_i(n)$ as

$$\bar{x}_i(n)=\frac{1}{\sqrt{M_i}}\sum_{k=-N_{ui}}^{N_{ui}}u_i(k)\exp(\mathrm{j}2\pi kn/M_i), \qquad (4)$$

where n=0, 1, …, M_i−1, $M_i\triangleq N_i/L_i$ and $N_{ui}\triangleq\lfloor N_u/L_i\rfloor$. Then the time signal of T_i is obtained from

$$x_i(n)=\bar{x}_i([n]_{M_i}), \qquad (5)$$

where n=$-N_{gi}$,…,N_i−1 and N_{g1}+N_1=N_{g2}+N_2.

The CFO estimate method

Based on the preamble with L identical parts, several methods can be used to estimate CFO. Among them, the CFO estimator proposed by Morelli and Mengali (1999) has high performance, which is referred to as MM in the sequel. The MM exploits the correlations of the samples from the receiver filter

$$R_i(m)=\frac{1}{N_i-mM_i}\sum_{k=mM_i}^{N_i-1}r_i(k)r_i^*(k-mM_i), \qquad (6)$$

where '*' stands for conjugation.

Based on the MM, an initial CFO estimate from T_i (i=1,2) can be expressed as

$$\hat{\xi}_i=\frac{L_i}{2\pi}\sum_{m=1}^{H_i}w_i(m)\varphi_i(m), \qquad (7)$$

where H_i≤L_i/2 and

$$w_i(m)\triangleq\frac{3(L_i-m)(L_i-m+1)-H_i(L_i-H_i)}{H_i(4H_i^2-6L_iH_i+3L_i^2-1)}, \qquad (8)$$

$$\varphi_i(m)\triangleq[\arg\{R_i(m)\}-\arg\{R_i(m-1)\}]_{2\pi}. \qquad (9)$$

It should be noted that the length of the block T_2 is N_2, therefore the valid number of identical parts of block T_2 actually is $L_2'\triangleq NL_2/N_2$. Substituting L_2' for L_2 in Eq.(7), we can obtain a more precise estimation of ξ_2. Moreover, the ranges of $\hat{\xi}_1$ and $\hat{\xi}_2$ are $[-L_1/2,L_1/2)$ and $[-L_2'/2,L_2'/2)$, respectively. The periods of $\hat{\xi}_1$ and $\hat{\xi}_2$ are L_1 and L_2', respectively, since the period of arg(·) is 2π. Then the true CFO can be represented as

$$\varepsilon\approx\varepsilon_1=L_1P_1+\hat{\xi}_1, \qquad (10)$$

$$\varepsilon\approx\varepsilon_2=L_2'P_2+\hat{\xi}_2. \qquad (11)$$

Subtracting $\hat{\xi}_1$ from Eqs.(10) and (11), we have

$$\bar{\varepsilon}\approx\bar{\varepsilon}_1=L_1P_1, \qquad (12)$$

$$\bar{\varepsilon}\approx\bar{\varepsilon}_2=L_2'P_2+\hat{\xi}_2-\hat{\xi}_1, \qquad (13)$$

when N is very close to N_2, $L_2'\approx L_2$, $\hat{\xi}_2-\hat{\xi}_1$ is an integer. Based on the Chinese residual theory, $\bar{\varepsilon}$ can be solved from Eqs.(12) and (13). Then, CFO within $[-L_x/2,L_x/2)$ can be estimated, $L_x\triangleq L_1L_2$.

ε_1 and ε_2 can be estimated from

$$(\hat{\varepsilon}_1,\hat{\varepsilon}_2)=\arg\min_{\varepsilon_1,\varepsilon_2}\left\{|\varepsilon_1-\varepsilon_2|\,\middle|\,\varepsilon_1\in\Omega_1,\varepsilon_2\in\Omega_2\right\}, \qquad (14)$$

where

$$\Omega_1=\left\{\varepsilon_1\,\middle|\,\varepsilon_1=L_1P+\hat{\xi}_1\,\middle|\,P\in\mathbb{Z},\ |\varepsilon_1|<L_x/2\right\}, \qquad (15)$$

$$\Omega_2=\left\{\varepsilon_2\,\middle|\,\varepsilon_2=L_2'P+\hat{\xi}_2\,\middle|\,P\in\mathbb{Z},\ |\varepsilon_2|<L_x/2\right\}. \qquad (16)$$

Eq.(14) can be solved by enumerating method. Then the true CFO can be estimated from

$$\hat{\varepsilon}=\rho\hat{\varepsilon}_1+(1-\rho)\hat{\varepsilon}_2. \qquad (17)$$

The ρ can be determined by minimizing the estimate variance. Since the final variance of the estimate error is mainly due to the estimate errors of ξ_1 and ξ_2 at high SNRs, it can be expressed as

$$var(\hat{\varepsilon}) = \rho^2 var(\hat{\xi}_1) + (1-\rho)^2 var(\hat{\xi}_2)\,. \tag{18}$$

To obtain the minimum value of Eq.(18), we have

$$\rho = \frac{var(\hat{\xi}_2)}{var(\hat{\xi}_1) + var(\hat{\xi}_2)}. \tag{19}$$

The variance of ξ_i (i=1,2) can be calculated from (Morelli and Mengali, 1999)

$$var\{\hat{\xi}_i\} = \frac{1}{4\pi^2}\frac{3L_i^2(SNR)^{-1}}{M_iH_i(4H_i^2 - 6L_iH_i + 3L_i^2 - 1)}\,. \tag{20}$$

The optimum of the CFO estimate algorithm

Solving Eq.(14) by enumerating method has high complexity. According to the properties of Eqs.(12) and (13), we give two simplified algorithms.

Algorithm 1:
Step 1: Estimate ε_2 from

$$\hat{\varepsilon}_2 = \arg\min_{\varepsilon}\left\{\left\|\frac{(\varepsilon-\hat{\xi}_1)}{L_1} - round\left(\frac{(\varepsilon-\hat{\xi}_1)}{L_1}\right)\right\| \varepsilon\in\Omega_2\right\}, \tag{21}$$

where $round(x)$ denotes rounding the elements of x to the nearest integers.

Step 2: Estimate P_1 from

$$\hat{P}_1 = round((\hat{\varepsilon}_2 - \hat{\xi}_1)/L_1). \tag{22}$$

Step 3: Estimate ε_1 from

$$\hat{\varepsilon}_1 = L_1\hat{P}_1 + \hat{\xi}_1. \tag{23}$$

Step 4: Obtain the final estimation of ε from Eq.(17).

Furthermore, when $|L_2-L_1|$=1, such as L_1=8, L_2=7 and L_1=4, L_2=5, the CFO estimate algorithm can be simplified further. Let $F_{\text{sig}} \triangleq (L_2 - L_1)$, the CFO can be estimated using the algorithm summarized as follows.

Algorithm 2:
Step 1: Estimate an initial value of P_2

$$\hat{P}_2 = F_{\text{sig}} round(\hat{\xi}_1 - \hat{\xi}_2). \tag{24}$$

Step 2: Obtain the estimate of ε_2 from

$$\hat{\varepsilon}_2 = L_2'\hat{P}_2 + \hat{\xi}_2. \tag{25}$$

If $|\hat{\varepsilon}_2| \le L_x/2$, $\hat{P}_1 = \hat{P}_2$, go to Step 4.

Step 3: Estimate P_1 from

$$\hat{P}_1 = \begin{cases} F_{\text{sig}} round(\hat{\xi}_1 - \hat{\xi}_2 - L_2'), \text{when } \hat{\xi}_1 > \hat{\xi}_2; \\ F_{\text{sig}} round(\hat{\xi}_1 - \hat{\xi}_2 + L_2'), \text{when } \hat{\xi}_1 < \hat{\xi}_2. \end{cases} \tag{26}$$

Then we have

$$\hat{\varepsilon}_2 = \begin{cases} L_2'(\hat{P}_1 + 1) + \hat{\xi}_2, \text{ when } \hat{\xi}_1 > \hat{\xi}_2; \\ L_2'(\hat{P}_1 - 1) + \hat{\xi}_2, \text{ when } \hat{\xi}_1 < \hat{\xi}_2. \end{cases} \tag{27}$$

Step 4: Estimate ε_1 from

$$\hat{\varepsilon}_1 = L_1(\hat{P}_1) + \hat{\xi}_1. \tag{28}$$

Step 5: Obtain the final estimation of ε from Eq.(17).

DISCUSSION

It turned out that L_1 and L_2' should be relatively prime to estimate a large CFO within $[-L_x/2, L_x/2)$. However, the approximation of $L_2' \approx L_2$ will reduce the estimate range. For example, when L_1=4, L_2' = 16/3≈5.333, the CFO estimate range is not [−10,10) but [−8,8). Therefore, the closer L_2' is to L_2, the closer the estimate range is to $[-L_x/2, L_x/2)$. We define the error introduced by the approximation as

$$err_{\max} = \left|1 - N/N_2\right|. \tag{29}$$

When N_2 is very close to N, the introduced error is trivial, whose effect on the CFO estimate can be ignored. When N is a small integer, it is relatively difficult to find an integer very close to it containing an odd prime divisor. However, when N is a large number, this can be easily satisfied. For example, when N=1024, L_1=4, N_2=1025, L_2=5, the introduced error will be $err_{\max}$=0.00098. This small error has almost no effect on the decisions of P_1 and P_2. However, the CFO estimate range is enlarged to $[-10,10)$ at the price of slightly increased computational complexity. Whereas, based on L_1=L_2=4 identical parts, the CFO estimate range of the conventional estimator is limited to [−2,2).

The parameters of L_1 and L_2 can be selected according to the practical requirement. The partial parameters suitable for practical OFDM systems with N=1024 subcarriers and the resulting errors are given in Table 1.

Table 1 The partial parameters suitable for practical systems

L_1	L_2	N_2	M_2	Est. rag.	$err_{\max}$
2	3	1023	341	[−3,3)	0.00098
4	5	1025	205	[−10,10)	0.00098
8	3	1023	341	[−12,12)	0.00098
8	5	1025	205	[−20,20)	0.00098
4	3	1023	341	[−6 6)	0.00098
8	7	1022	146	[−28,28)	0.00195

We next compare the computational complexity of the proposed estimator with that of the estimators in (Seo *et al*., 2002) and (Schmidl and Cox, 1997), which will be referred to as SEO and SC, respectively. Since the complexities of Algorithm 1 and Algorithm 2 are very low, the complexity of the proposed method is mainly due to the estimations of ξ_1 and ξ_2. The complex multiplications required by the proposed estimator are

$$\Pi = \sum_{i=1}^{2}\sum_{u=1}^{H_i}(N-uM_i). \tag{30}$$

The SC and the SEO are composed of fine estimate, fine CFO compensation and coarse estimation. Therefore, the computational complexities of the SC and SEO are

$$\Pi_{\text{SC}} = N/2+N+N\log_2(N)/2+(N/2)(L_x/2), \tag{31}$$

$$\Pi_{\text{SEO}} = 2\times(N/2+N+N\log_2(N)/2+N/2). \tag{32}$$

Letting $L_1' = L_1$, we have $H_i \le L_i'/2$. Considering $L_i' \le 8$ in the practical OFDM systems, the computational complexity of the proposed method satisfies $\Pi<5.5N$. Moreover, the complexity of the proposed estimator can be reduced by choosing small H_i. However, the SEO with greatly reduced complexity over SC requires $10N$ complex multiplications even for OFDM systems with small number of subcarriers, N=64. When used in OFDM systems with large number of subcarriers, the low complexity advantage of the proposed estimator is more evident. Although Fast Fourier transform (FFT) can be implemented by utilizing the intrinsic structure in the OFDM systems, the FFT process causes considerable delay. Therefore, the proposed method outperforms the frequency-domain CFO estimators.

SIMULATION RESULTS

Simulations were conducted to test the performance of the proposed CFO estimation method. The simulation parameters are set according to 3GPP (3GPP TSG-RAN-1.TR, 2004). The entire channel bandwidth is 6.528 MHz, and is divided into 1024 subcarriers (or tones). The symbol block duration is 156.85 μs and the carrier frequency is 2 GHz. An additional 9.038 μs guard interval is used to provide protection from ISI due to channel delay-spread. The channel has 25 paths, with path delays of 0,1,2,…,24 samples. The amplitude A_i of the ith path varies independently of the others according to a Rayleigh distribution with exponential power delay profile, i.e. $E(A_i)$=exp(−i/5). The phase of each path is uniformly distributed on the interval [0, 2π]. The other parameters are N_2=1025, L_1=4 and L_2=5.

Fig.2 shows the mean square estimation error (MSE) as a function of SNR for ε=1.4. It also includes comparisons with the MSE of several conventional estimators. Compared to the SC, the proposed estimator guarantees a gain of 4 dB, because one block is used by the SC to perform the fine estimate or the coarse estimate, while two blocks are utilized together by the proposed estimator to estimate the whole CFO. Fig.2 also shows that the performance of the proposed estimator has about 0.7 dB improvement over the SEO. Furthermore the proposed estimator has almost the same estimate performance as the estimator based on the conventional preamble, which is composed of two training blocks each with 4 identical parts. We further compare the proposed estimator with the Cramer-Rao bound (CRB) (Morelli and Mengali, 1999). Since two blocks are used for the CFO estimate, the length of the preamble is 2*N*, thus the CRB is

$$CRB(\hat{\varepsilon})=\frac{1}{4\pi^2}\frac{3(SNR)^{-1}}{N(1-1/4N^2)}. \quad (33)$$

We can see that the MSE performance of the proposed estimator is closer to the CRB than that of other estimators.

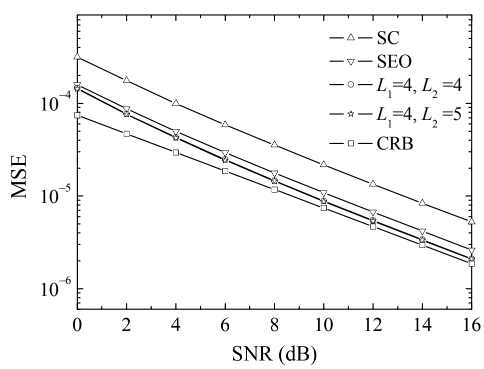

Fig.2 MSE vs SNR for the proposed estimator and the conventional estimators

Fig.3 illustrates the MSE as a function of the true offset ε for SNR=0 dB. It shows that the estimate range of the estimator with $L_2'=16/3\approx5.333$ is only [−8,8). However, the estimator with $L_2'=5N/N_2\approx5$ has constant estimate performance over [−10,10). This demonstrates that the closer L_2' is to L_2, the closer the estimate range is to $[-L_x/2, L_x/2)$.

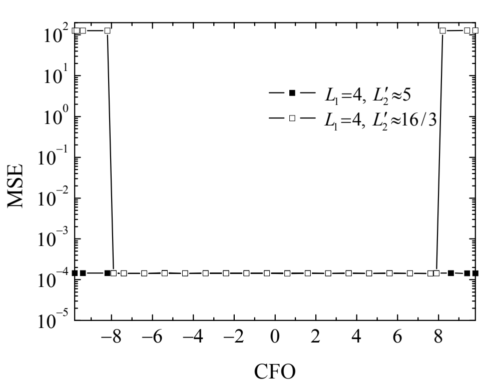

Fig.3 MSE vs CFO for the proposed estimator

CONCLUSION

In this work, a novel preamble composed of two training blocks with different numbers of identical parts was designed. The CFO estimates obtained over the two OFDM blocks by means of the MM algorithm are next exploited to enlarge the CFO estimation range. Simulation results showed that the proposed method has a larger frequency offset estimation range while achieving approximately the same estimate performance compared to the estimation method based on the conventional preamble with several identical parts. Moreover, it has the advantages of fast estimation speed and low computational complexity over the frequency-domain estimators. Considering the structure-property of the designed preamble, the proposed method is especially suitable for estimating a large CFO for OFDM systems with large number of subcarriers, such as beyond 3 G systems.

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