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Preventive repair policy and replacement policy of repairable system taking non-zero preventive repair time*

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Abstract: The repairable system with preventive maintenance is one of the typical systems with wide useful applications in engineering. If the system can be made as good as new by preventive maintenance, both the life stochastic variable of different periods and fault correction time stochastic variable form monotonous stochastic process. Based on the above assumption and the available results, in this paper we discuss the maintenance and replacement policy of the repairable system with preventive maintenance. The intervals of preventive maintenance, *T*, and the times of system failure, *N*, are introduced and the vector Markov process method is used. The formulation of steady state average profit rate can be deduced to solve the optimization problem of the maintenance and replacement policy.

Key words: Preventive repair, Monotone process, Vector Markov process method, Preventive repair policy and replacement policy

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INTRODUCTION

In early research of system reliability, it was usually assumed that the systems could be repaired to be as good as new. However, due to the obvious differences between the assumption and reality, in the past 40 years many new models that cannot be repaired to be as good as new have been presented by many investigators (Barlow and Hunter, 1960; Brown and Proschan, 1983; Yeh, 1988a; 1988b; Stadje and Zuckerman, 1990). In recent years, the maintenance and replacement policy of the systems have become more popular.

The maintenance and replacement policy was visited by using the renewal process method (Zhang, 1995). The research was carried out on the assumption that the system life and repair are geometrical processes. However, they confused the operating time when the system's life is less than the time between preventive maintenances with the life after preventive maintenance. In 1997, by using the above two distinguished parameters, the exact mathematical expectation of the operating time was not addressed when the system's life was less than the time between preventive maintenances (Jia and Zhang, 1997). In the late researches on this problem, further improvements were developed (Zhang, 2002; Kim *et al.*, 2003; Wang and Zhang, 2006). However, in the proofs of Lemma 2, Theorem 1, Theorem 2 and Theorem 3, two inappropriate assumptions are employed as follows: (1) The time-distribution of each preventive maintenance time in one period is the same as that of the total time of preventive maintenance in the same period; (2) The distribution of the operating time after each preventive repair in each period is the same as the total work time in the same period. In addition, the exact mathematical expectation of the operating time was not addressed when the system's life is less than the time between preventive maintenances.

In this paper, considering its wide application in

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practice, based on the assumption that both the life stochastic variable sequence and the repair time stochastic variable sequence form the monotonous stochastic process, this model is visited by using vector Markov process method.

DEFINITION AND ASSUMPTION

Definition

Definition 1 X and Y are random variables, whose cumulative distribution functions (abbreviated c.d.f.) are F and H, $\bar{F}=1-F, \bar{H}=1-H$, if for all real t, $\bar{F}(t)\le\bar{H}(t)$, then X is called stochastically smaller than Y, denoted by $X\le_{st}Y$; otherwise X is called stochastically larger than Y, and denoted by $X\ge_{st}Y$;

Definition 2 Let $\{X_n, n=1, 2, \ldots\}$ be a sequence of non-negative and independent random variables. If $X_n\le_{st}X_{n+1}$ (n=1, 2, …), then $\{X_n, n=1, 2, \ldots\}$ is called a monotonously increasing stochastic process. If $X_n\ge_{st}X_{n+1}$ (n=1, 2, …), then $\{X_n, n=1, 2, \ldots\}$ is called a monotonously decreasing stochastic process.

The lifetime of actual repairable system is a monotonously decreasing stochastic process; the repair time of actual repairable system is a monotonously increasing stochastic process. Obviously the renewal process and geometry process are both special monotone stochastic processes.

Assumptions

(1) At t=0, a new system is installed and begins to work. When the operating time of the system reaches T, the system stops working to accept the first preventive repair. When the first preventive repair is finished, the system begins to work. When the operating time of the system reaches T, it stops working to accept the second preventive repair. When the system fails before the working time reaches T, it accepts the first failure repair, … The time interval between the beginning for the new system to work and the completion of the first failure repair in the system is called the 1st cycle. After the completion of the first failure repair, the system goes into the 2nd cycle to begin working. When the working time of the system reaches T, it stops working to accept the first preventive repair in this cycle. After the first preventive repair has finished, the system begins to work. When the working time of the system reaches T, it stops working to accept the second preventive repair … When the system fails before the working time reaches T, it accepts the second failure repair, … The time interval between the completion of the first failure repair and the completion of the 2nd failure repair in the system is called the 2nd cycle …

(2) In the same cycle, the preventive repair keeps the system as good as new, but the failure repair cannot. Suppose lifetime $X_i^{(n)}$ of the system in cycle i after n times preventive repair, n=0, 1, 2, …, are independent and identically distributed, their c.d.f. is $H_i(x)$, their density function is $h_i(x)$, their failure rate function is $a_i(x)$, and their mean is $E[X_i^{(n)}]=\lambda_i$ (i=1, 2, …). If preventive repair time $Z_i^{(n)}$ in cycle i after n times preventive repair, n=0, 1, 2, …, are independently and identically distributed, their c.d.f. is $F_i(z)$, their density function is $f_i(z)$, repair rate function is $b_i(z)$, their mean is $E[Z_i^{(n)}]=b_i$ (i=1, 2, …). Y_i is failure repair time in cycle i, whose c.d.f. is $G_i(x)$, its density function is $g_i(x)$, repair rate function is $\mu_i(x)$, its mean is $E[Y_i]=\mu_i$ (i=1, 2, …). $\{X_i^{(0)}, i=1,2,\ldots\}$ is a monotonously decreasing stochastic process; $\{Z_i^{(0)}, i=1, 2, \ldots\}$ and $\{Y_i, i=1, 2, \ldots\}$ are monotonously increasing stochastic process.

(3) $X_i^{(n)}$ (n=0, 1, 2, …), $Z_i^{(n)}$ (n=0, 1, 2, …) and Y_i (i=1, 2, …) are independent.

(4) When it fails in cycle N, the system is not repairable any more, and a new and identical one replaces it. The replacement time is negligible (Because it is far shorter than the other time of the system, it is reasonable to handle it the way in the former research (Zhang, 2002)). The system replacement is completed this way.

(5) The working reward per unit time, failure repair cost per unit time, preventive repair cost each time, and replacement cost each time are C_1, C_2, C_3, and C, respectively.

The state of the system is defined as follows. State (i, 0, n) means that the system is working after n times preventive repair in cycle i. State (i, 1, n) means that the system is under preventive repair after n times preventive repair in cycle i (n=0, 1, 2, …; i=1, 2, …, N). State (i,2) means the system is under failure repair in cycle i (i=1, 2, …, N−1).

Let $S(t)$ be the state of the system at time t. Define supplement variables as follows. $X_{i0n}(t)$ denotes working time of the system at time t when $S(t)=(i, 0, n)$; $Z_{i1z}(t)$ denotes preventive repair time of the system at time t when $S(t)=(i, 1, n)$ (n=0, 1, 2, …; i=1, 2, …, N). $Y_{i2}(t)$ denotes failure repair time of the system at time t when $S(t)=(i, 2)$ (i=1, 2, …, N−1). After introducing supplement variables, the process is a vector Markov one.

State probability density of the system is defined as follows:

$$P_{i0n}(t,x)\mathrm{d}x=P\{S(t)=(i,0,n),\ x\le X_{i0n}(t)<x+\mathrm{d}x\},$$
$$n=0, 1, 2, \ldots;\ i=1, 2, \ldots, N;$$
$$P_{i1n}(t,x)\mathrm{d}z=P\{S(t)=(i,1,n),\ z\le Z_{i1n}(t)<z+\mathrm{d}z\},$$
$$n=0, 1, 2, \ldots;\ i=1, 2, \ldots, N;$$
$$P_{i2}(t,y)\mathrm{d}y=P\{S(t)=(i,2),\ y\le Y_{i2}(t)<y+\mathrm{d}y\},$$
$$i=1, 2, \ldots, N-1.$$

STATE PROBABILITY DENSITY EQUATION OF SYSTEM

From the probability analysis of system state, the following differential equations can be derived readily:

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+a_i(x)\right]P_{i0n}(t,x)=0,\quad n=0,\ 1,\ 2,\ \ldots;\ i=1,\ 2,\ \ldots,\ N, \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+b_i(z)\right]P_{i1n}(t,z)=0,\quad n=0,\ 1,\ 2,\ \ldots;\ i=1,\ 2,\ \ldots,\ N, \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_i(y)\right]P_{i2}(t,y)=0,\ i=1, 2, \ldots, N-1 \tag{3}$$

The boundary conditions are

$$P_{100}(t,0)=\delta(t)+\sum_{n=0}^{\infty}\int_0^T a_N(x)P_{N0n}(t,x)\mathrm{d}x, \tag{4}$$

$$P_{i00}(t,0)=\int_0^{\infty}\mu_{i-1}(y)P_{i-12}(t,y)\mathrm{d}y,\ i=2,\ \ldots,\ N, \tag{5}$$

$$P_{i1n}(t,0)=P_{i0n}(t,T),\quad i=1,\ 2,\ \ldots,\ N;\ n=0,\ 1,\ 2,\ \ldots, \tag{6}$$

$$P_{i0n}(t,0)=\int_0^{\infty}b_i(z)P_{i1n-1}(t,z)\ \mathrm{d}z,\quad i=1,\ 2,\ \ldots,\ N;\ n=1,\ 2,\ \ldots, \tag{7}$$

$$P_{i2}(t,0)=\sum_{n=0}^{\infty}\int_0^T a_i(x)P_{i0n}(t,x)\mathrm{d}x,\ i=1,2,\ldots,N-1, \tag{8}$$

and the initial conditions are assumed as

$$P_{100}(0,x)=\delta(x),\ \text{others are zero.} \tag{9}$$

The Laplace transform of the above equation solutions can be obtained after letting $f^*(s)=\int_0^{\infty}\mathrm{e}^{-st}f(t)\mathrm{d}t$ be the Laplace transform of $f(t)$, $\bar{f}(x)=1-f(x)$.

$$P_{10n}^*(s,x)=C(s)[\bar{H}_1(T)\mathrm{e}^{-sT}f_1^*(s)]^n\bar{H}_1(x)\mathrm{e}^{-sx},\quad n=0,\ 1,\ 2,\ \ldots, \tag{10}$$

$$P_{11n}^*(s,z)=C(s)[\bar{H}_1(T)\mathrm{e}^{-sT}]^{n+1}[f_1^*(s)]^n\bar{F}_1(z)\mathrm{e}^{-sz},\quad n=0,\ 1,\ 2,\ \ldots, \tag{11}$$

$$P_{i0n}^*(s,x)=C(s)\frac{\prod_{k=1}^{i-1}g_k^*(s)\prod_{k=1}^{i-1}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x}{\prod_{k=1}^{i-1}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\cdot[\bar{H}_i(T)\mathrm{e}^{-sT}f_i^*(s)]^n\bar{H}_i(x)\mathrm{e}^{-sx},\quad i=2,\ \ldots,\ N;\ n=0,\ 1,\ 2\ \ldots, \tag{12}$$

$$P_{i1n}^*(s,z)=C(s)\frac{\prod_{k=1}^{i-1}g_k^*(s)\prod_{k=1}^{i-1}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x}{\prod_{k=1}^{i-1}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\cdot[\bar{H}_i(T)\mathrm{e}^{-sT}]^{n+1}[f_i^*(s)]^n\bar{F}_i(z)\mathrm{e}^{-sz},\quad i=2,\ \ldots,\ N;\ n=0,\ 1,\ 2\ \ldots, \tag{13}$$

$$P_{12}^*(s,y)=C(s)\frac{\int_0^T h_1(x)\mathrm{e}^{-sx}\mathrm{d}x}{1-\bar{H}_1(T)f_1^*(s)\mathrm{e}^{-sT}}\bar{G}_1(y)\mathrm{e}^{-sy}, \tag{14}$$

$$P_{i2}^*(s,y)=C(s)\frac{\prod_{k=1}^{i-1}g_k^*(s)\prod_{k=1}^{i}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x}{\prod_{k=1}^{i}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\bar{G}_i(y)\mathrm{e}^{-sy},\quad i=2,\ \ldots,\ N-1, \tag{15}$$

where

$$C(s)=\left\{1-\frac{\prod_{k=1}^{N-1}g_k^*(s)\prod_{k=1}^{N}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x}{\prod_{k=1}^{N}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\right\}^{-1}. \tag{16}$$

PREVENTIVE REPAIR POLICY AND REPLACEMENT POLICY

Replacement frequency

If the replacement frequency at time t is written as follows:

$$W_f(t)=\sum_{n=0}^{\infty}\int_0^T a_N(x)P_{N0n}(t,x)\mathrm{d}x, \tag{17}$$

the replacement frequency $M(t)$ in $(0,t)$ an be obtained readily (Shi, 1999). It is

$$M(t)=\int_0^t W_f(x)\mathrm{d}x. \tag{18}$$

Using Eq.(12), the Laplace transform of $M(t)$ is given by

$$W_f^*(s)=C(s)\frac{\prod_{k=1}^{N-1}g_k^*(s)\prod_{k=1}^{N}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x}{\prod_{k=1}^{N}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}. \tag{19}$$

Letting M be the steady state replacement frequency, we get

$$M=\lim_{t\to\infty}\frac{M(t)}{t}=\lim_{s\to 0}sW_f^*(s)$$
$$=\frac{1}{\sum_{i=1}^{N}\frac{\int_0^T\bar{H}_i(x)\mathrm{d}x}{H_i(T)}+\sum_{i=1}^{N}\frac{b_i\bar{H}_i(T)}{H_i(T)}+\sum_{i=1}^{N-1}\mu_i}. \tag{20}$$

Availability

The system availability at time t is defined by

$$A(t)=P\,\{\text{the system is working at time } t\}$$
$$=\sum_{i=1}^{N}\sum_{n=0}^{\infty}\int_0^T P_{i0n}(t,x)\mathrm{d}x. \tag{21}$$

Using Eqs.(10) and (12), the Laplace transform of $A(t)$ is given by

$$A^*(s)=C(s)\left[\frac{\int_0^T\bar{H}_1(x)\mathrm{e}^{-sx}\mathrm{d}x}{1-\bar{H}_1(T)f_1^*(s)\mathrm{e}^{-sT}}\right.$$
$$\left.+\sum_{i=2}^{N}\frac{\prod_{k=1}^{i-1}g_k^*(s)\prod_{k=1}^{i-1}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x\int_0^T\bar{H}_i(x)\mathrm{e}^{-sx}\mathrm{d}x}{\prod_{k=1}^{i}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\right]. \tag{22}$$

Hence the steady state availability of the system is written as follows:

$$A=M\sum_{i=1}^{N}\frac{\int_0^T\bar{H}_i(x)\mathrm{d}x}{H_i(T)}. \tag{23}$$

Preventive repair frequency

Assuming the preventive repair frequency at time t

$$W_{1f}(t)=\sum_{i=1}^{N}\sum_{n=0}^{\infty}P_{i0n}(t,T), \tag{24}$$

the preventive repair frequency $M_1(t)$ in $(0, t)$ can be expressed as follows (Shi, 1999):

$$M_1(t)=\int_0^t W_{1f}(x)\mathrm{d}x. \tag{25}$$

Using Eqs.(10) and (12), the Laplace transform of $W_{1f}(t)$ is obtained readily as:

$$W_{1f}^*(s)=C(s)\left[\frac{\bar{H}_1(T)\mathrm{e}^{-sT}}{1-\bar{H}_1(T)f_1^*(s)\mathrm{e}^{-sT}}\right.$$
$$\left.+\sum_{i=2}^{N}\frac{\prod_{k=1}^{i-1}g_k^*(s)\prod_{k=1}^{i-1}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x\bar{H}_i(T)\mathrm{e}^{-sT}}{\prod_{k=1}^{i}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\right]. \tag{26}$$

Therefore, the steady state preventive repair frequency is formulated as follows:

$$M_1=M\sum_{i=1}^{N}\frac{\bar{H}_i(T)}{H_i(T)}. \tag{27}$$

Failure repair probability

If the failure repair probability at time t is

$$P(t)=\sum_{i=1}^{N-1}\int_0^{\infty}P_{i2}(t,y)\mathrm{d}y, \tag{28}$$

using Eqs.(14) and (15), the Laplace transform of $P(t)$ can be written as

$$P^*(s)=C(s)\left[\frac{\int_0^T h_1(x)\mathrm{e}^{-sx}\mathrm{d}x\bar{G}_1^*(s)}{1-\bar{H}_1(T)f_1^*(s)\mathrm{e}^{-sT}}+\sum_{i=2}^{N-1}\frac{\prod_{k=1}^{i-1}g_k^*(s)\prod_{k=1}^{i}\int_0^T h_k(x)\mathrm{e}^{-sx}\mathrm{d}x\bar{G}_i^*(s)}{\prod_{k=1}^{i}[1-\bar{H}_k(T)f_k^*(s)\mathrm{e}^{-sT}]}\right]. \quad (29)$$

Therefore, the steady state failure repair probability is derived readily as

$$P=M\sum_{i=1}^{N-1}\mu_i. \quad (30)$$

Preventive repair policy and replacement policy

The expected total profits generated by the system in the interval $(0,t)$ are

$$R(t)=C_1\int_0^t A(x)\mathrm{d}x-C_2\int_0^t P(x)\mathrm{d}x-C_3M_1(t)-CM(t). \quad (31)$$

Using Eqs.(20), (23), (27) and (30), the steady state average profits rate can be derived readily as:

$$C(N,T)=\lim_{t\to\infty}\frac{R(t)}{t}=\frac{C_1\sum_{i=1}^{N}\frac{\int_0^T\bar{H}_i(x)\mathrm{d}x}{H_i(T)}-C_2\sum_{i=1}^{N-1}\mu_i-C_3\sum_{i=1}^{N}\frac{\bar{H}_i(T)}{H_i(T)}-C}{\sum_{i=1}^{N}\frac{\int_0^T\bar{H}_i(x)\mathrm{d}x}{H_i(T)}+\sum_{i=1}^{N}\frac{b_i\bar{H}_i(T)}{H_i(T)}+\sum_{i=1}^{N-1}\mu_i}. \quad (32)$$

Here $C(N,T)$ is the explicit expression of (N,T), so we can find (N^*,T^*) and let $C(N^*,T^*)$ maximize.

By simple calculation, we can get the following important result:

$$C(N,\infty)=\frac{C_1\sum_{i=1}^{N}\lambda_i-C_2\sum_{i=1}^{N-1}\mu_i-C}{\sum_{i=1}^{N}\lambda_i+\sum_{i=1}^{N-1}\mu_i}. \quad (33)$$

$C(N,\infty)$ is the steady state profits rate without preventive repair. If $C(N^*,T^*)>C(N,\infty)$, the preventive repair is worth doing. If the lifetime $X_i^{(0)}$ (i=1, 2, …) is exponential distribution, $C(N,T)$ becomes

$$C(N,T)=\frac{C_1\sum_{i=1}^{N}\lambda_i-C_2\sum_{i=1}^{N-1}\mu_i-C_3\sum_{i=1}^{N}\frac{\mathrm{e}^{-\frac{1}{\lambda_i}T}}{1-\mathrm{e}^{-\frac{1}{\lambda_i}T}}-C}{\sum_{i=1}^{N}\lambda_i+\sum_{i=1}^{N}\frac{b_i\mathrm{e}^{-\frac{1}{\lambda_i}T}}{1-\mathrm{e}^{-\frac{1}{\lambda_i}T}}+\sum_{i=1}^{N-1}\mu_i}. \quad (34)$$

Obviously, $C(N,T)<C(N,\infty)$, so that when the lifetime is exponential distribution, it is better not to adopt preventive repair. This conclusion accords with non-memory of exponential distribution. When the lifetime is not exponential distribution, preventive repair can prolong the lifetime of system and perhaps can improve economic benefit. In order to get maximized economic benefit of the system, we should take (N,T), not only N, as policy. In detailed calculation, we may find T_N^* for every N and let $C(N,T)$ reach maximum $C(N,T_N^*)$ (N=1, 2, …), maximum $C(N^*, T_N^*)$ among $C(N,T_N^*)$ (N=1, 2, …) is the maximal steady state average profits rate. So (N^*,T_N^*) is the optimal policy (Zhang, 1994).

NUMERICAL EXAMPLES

The parameters of some systems are as follows: C_1=4900, C_2=2100, C_3=20000, and C=2200000; $H_i(x)=1-\mathrm{e}^{-(0.0001\times1.04^{i-1}x)^2}$, $x\geq0$, i=1, 2, …, N; $b_i=5\times1.05^{i-1}$, i=1, 2, …, N; $\{Y_i, i=1, 2, …, N-1\}$ is the geometric process; $\mu_i=150\times1.1^{i-1}$ (i=1, 2, …, $N-1$). For these parameters,

$$C(N,T)=4900-A/B, \quad (35)$$

where

$$A=\sum_{i=1}^{N}\frac{(24500\times1.05^{i-1}+20000)\mathrm{e}^{-(0.0001\times1.04^{i-1}T)^2}}{1-\mathrm{e}^{-(0.0001\times1.04^{i-1}T)^2}}+10500000\,(1.1^{N-1}-1)+2200000,$$

$$B=\sum_{i=1}^{N}\frac{\int_0^T\mathrm{e}^{-(0.0001\times1.04^{i-1}x)^2}\mathrm{d}x}{1-\mathrm{e}^{-(0.0001\times1.04^{i-1}T)^2}}+\sum_{i=1}^{N}\frac{5\times1.05^{i-1}\mathrm{e}^{-(0.0001\times1.04^{i-1}T)^2}}{1-\mathrm{e}^{-(0.0001\times1.04^{i-1}T)^2}}+1500\,(1.1^{N-1}-1).$$

The curves of function with N=2 to 7 are shown in Fig.1a. By numerical calculation, when N^*=3, T^*=1727.343, the steady state profits rate is maximal, and is 4847.148 per unit time. The curves of function with N=2 to 7 in larger range of T are shown in Fig.1b. It is seen that for fixed N, when T becomes infinite, the steady state profits rate rises to $C(N,\infty)$. Fig.1c shows that for the curve of $C(N,T_N^*)$ about N. when N^{**}=4, the steady state benefits rate $C(N,\infty)$ without preventive repair is maximal and is 4732.839 per unit time. Because 4847.148 is 114.309 greater than 4732.839, and the steady state benefits rate $C(N,T)$ cannot be greater than C_1=4900, so the optimal policy with preventive repair is better than that without preventive repair and the effect is remarkable.

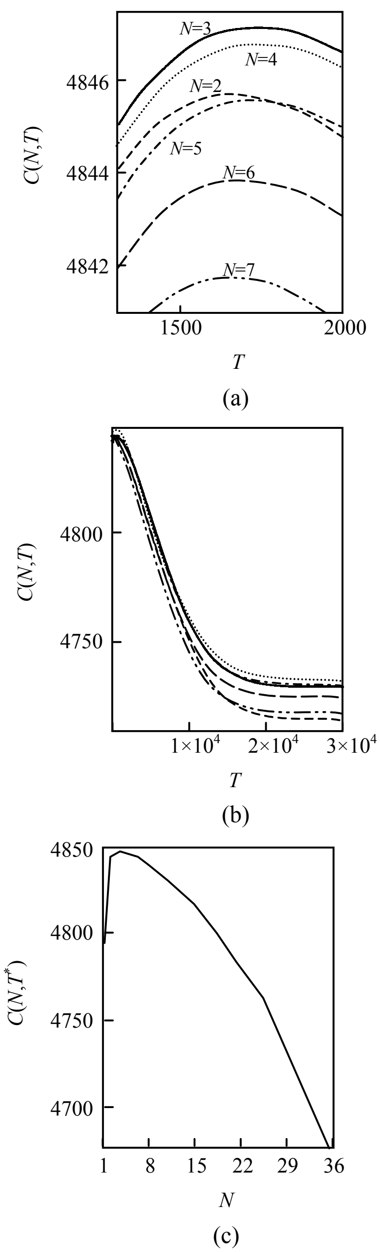

Fig.1 (a) The curves of function with N=2 to 7; (b) The curves of function with N=2 to 7 in larger range of T; (c) The curve of $C(N,T_N^*)$ about N

CONCLUSION

In this paper, the preventive repair policy and replacement policy of a repairable system is visited. The explicit expression of the steady state profits rate is derived by vector Markov process method. By using the expression, the optimal preventive policy and replacement policy can be determined. Next, a criterion judging the worth of preventive repair is given. Numerical examples make it clear that the optimal preventive repair policy and replacement policy can improve the steady state profits rate when lifetime is a non-exponentially and monotonously decreasing process and the criterion is true. Due to the generality of the assumptions, the results have both theoretical value and extensive practical applications.

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