



## Adaptive synchronization of chaotic Colpitts circuits against parameter mismatches and channel distortions<sup>\*</sup>

SHEN Cheng<sup>†</sup>, SHI Zhi-guo<sup>†‡</sup>, RAN Li-xin

(Department of Information and Electronic Engineering, Zhejiang University, Hangzhou 310027, China)

<sup>†</sup>E-mail: sc\_zju@hotmail.com; scottszg@hotmail.com

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**Abstract:** In this work, synchronization of chaotic Colpitts circuits, using adaptive controllers to combat circuit parameter mismatches and channel distortions, is studied by numerical simulations. Synchronization errors caused by different main circuit components are compared, and compensation for time-constant and time-varying circuit parameter mismatches is demonstrated. Different kinds of channel distortions, including time-constant and time-varying channel attenuation, Additive White Gaussian Noise (AWGN) are all investigated by numerical simulations and discussed. Simulation results indicated that the synchronization performance of chaotic Colpitts circuits can be greatly improved by applying adaptive controllers when parameter mismatches and channel attenuation are considered as time-constant or time-varying, but have no obvious enhancements regarding the effect on AWGN channel.

**Key words:** Chaos, Synchronization, Adaptive control, Colpitts circuit

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### INTRODUCTION

There has been considerable interest in chaos communication over the past several years. Since Pecora and Carroll (1990) discovered that chaotic systems can be synchronized, research into applications of chaos communication has been greatly motivated, as synchronized systems could offer potential advantage over non-coherent detection in terms of noise performance and data rate when the information is recovered from a noisy distorted received signal (Kolumban *et al.*, 1998). The use of synchronized chaotic systems for communications relies largely on the robustness of synchronization within the transmitter-receiver pair.

The choice of Colpitts circuit, first studied by Kennedy (1994), as the drive and response system in the chaotic communication was motivated by the observation of Ababei and Marculescu (2000) on the

simple implementation and reduced power consumption of chaotic Colpitts circuits. Synchronization of Colpitts circuits was discussed by Baziliauskas *et al.*(2001), Shi *et al.*(2003; 2004), Shi and Ran (2004), Rubezic and Ostojic (1999), Rubezic *et al.*(2002), etc. But in most of previous work, the drive and the response systems were assumed to be identical and their parameters were assumed to be time-constant. The channel between the two systems was also assumed to be perfect. However, both parameter mismatches and channel distortions exist in the actual circumstance. In real-world communications, channel distortions including fading, additive noise and the like are unavoidable. The parameters of the transmitter (drive system) and the receiver (response system) also cannot be identical and time-constant. These channel distortions and parameter mismatches will cause considerable synchronization mismatch between the transmitter and the receiver, and make difficult the recovery of transmitted signals in the receiver (Shi *et al.*, 2004).

To maintain synchronization in such circum-

<sup>‡</sup> Corresponding author

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stances, one good approach (to utilize adaptive controllers to offset the effects of parameter mismatches and channel distortions on transmitted signals) shows tremendous potential for developing practical chaotic spread-spectrum communication systems. Chua *et al.* (1996) successfully applied this method in Chua's circuits. Enhanced synchronization performances were obtained in their simulations and experiments.

In this work we studied the adaptive controllers and their capabilities for synchronizing chaotic Colpitts circuits when channel distortions and parameter mismatches were taken into account; and used error feedback synchronization scheme as Shi *et al.* (2004) found that it outperformed other schemes like the Pecora-Carroll synchronization scheme. We compared the influence exerted by each of the three main parameters on the synchronization performance, and found the most pivotal parameter whose mismatch had the most negative influence on the synchronization performance. Then we used adaptive controllers to compensate for the time-varying parameters of the transmitter. Also, we studied the synchronization performance under two main channel distortions, namely channel fading and Additive White Gaussian Noise (AWGN), and employed adaptive controllers to combat such distortions.

## SYSTEM CONFIGURATION

Generally a chaotic communication system consists of three parts: a transmitter based on chaotic generator, a channel and a receiver based on a corresponding chaotic circuit of the generator. In the system for considered, we used two identical chaotic Copitts circuits as the transmitter and the receiver, and adopted the error feedback synchronization scheme, with the resistor  $R_1$  coupling received signals into the receiver. The signal  $V_{C2}$ , serving as the transmitted signal, passes through a time-varying channel between the transmitter and the receiver, and becomes a distorted signal defined as  $\tilde{V}_{Tr}$ . To implement adaptive synchronization in the system, we construct an adaptive controller before the resistor  $R_1$ . The received signal  $\tilde{V}_{Tr}$  is processed by the adaptive controller and is then coupled into the receiver through resistor  $R_1$ . The whole system configuration is shown in Fig.1.

The state equations for the Colpitts circuit (the transmitter in Fig.1) are as Eq.(1):

$$\begin{cases} \frac{dV_{C1}}{dt} = \frac{1}{C_1}(-f(-V_{C2}) + I_L), \\ \frac{dV_{C2}}{dt} = \frac{1}{C_2}(I_L - I_0), \\ \frac{dI_L}{dt} = \frac{1}{L}(-V_{C1} - V_{C2} - I_L R + V_{CC}), \end{cases} \quad (1)$$

where  $f(\cdot)$  is the driving-point characteristic of the nonlinear resistor of the BJT, given by Maggio *et al.* (1999).

In most literatures it was described by:

$$f(x) = I_S \left[ \exp\left(\frac{x}{V_T}\right) - 1 \right] \approx I_S \left[ \exp\left(\frac{x}{V_T}\right) \right], \text{ if } x \gg V_T. \quad (2)$$

In Colpitts circuits, this characteristic can be expressed as  $I_E = f(V_{C2}) = f(-V_{BE})$  and from Eq.(1) it follows that

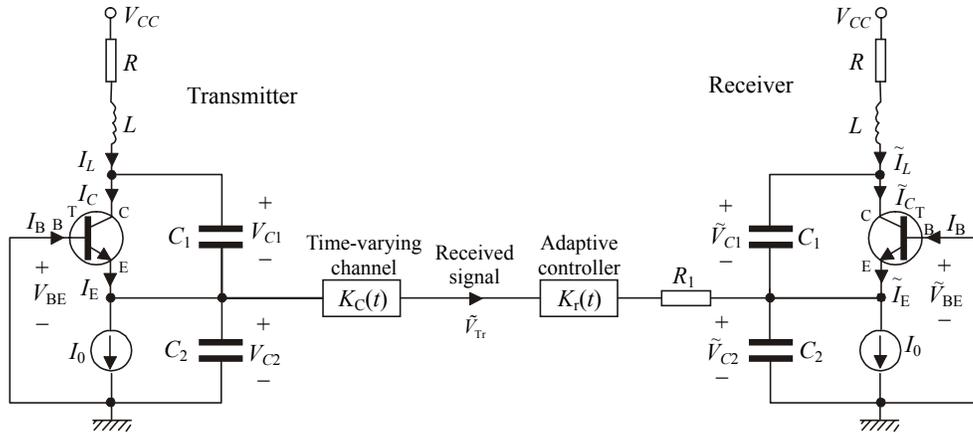
$$f(V_{C2}) = I_S \exp\left(-\frac{V_{C2}}{V_T}\right). \quad (3)$$

When no adaptive controller is employed in the system, the resistor  $R_1$  couples the received signal  $\tilde{V}_{Tr}$  directly into the receiver, and the state equations of the receiver can be written as:

$$\begin{cases} \frac{d\tilde{V}_{C1}}{dt} = \frac{1}{C_1}(-f(-\tilde{V}_{C2}) + \tilde{I}_L), \\ \frac{d\tilde{V}_{C2}}{dt} = \frac{1}{C_2}(\tilde{I}_L - I_0) + \frac{1}{R_1 C_2}(\tilde{V}_{Tr} - \tilde{V}_{C2}), \\ \frac{d\tilde{I}_L}{dt} = \frac{1}{L}(-\tilde{V}_{C1} - \tilde{V}_{C2} - \tilde{I}_L R + V_{CC}), \end{cases} \quad (4)$$

where  $\tilde{V}_{Tr}$  is a distorted signal from the non-ideal channel, as shown in Fig.1.

In our simulations, the parameters of Colpitts circuits are:  $C_1=C_2=237$  nF,  $L=2.1$  mH,  $R=74.5$   $\Omega$ ,  $V_{CC}=5$  V,  $I_0=2.5$  mA. With these parameters, both the transmitter and the receiver exhibit chaotic oscillation separately. While the circuit parameters are time-



**Fig.1 Synchronization scheme of two chaotic Colpitts circuits with an adaptive controller over a time-varying channel**

constant and the channel has no distortion, that is  $\tilde{V}_{Tr} = V_{C2}$ , the synchronization between systems Eqs.(1) and (4) can be achieved by properly selecting the value of coupling resistor  $R_1$ . This was demonstrated by Baziliauskas *et al.*(2001) by numerical simulations and experimental investigation.

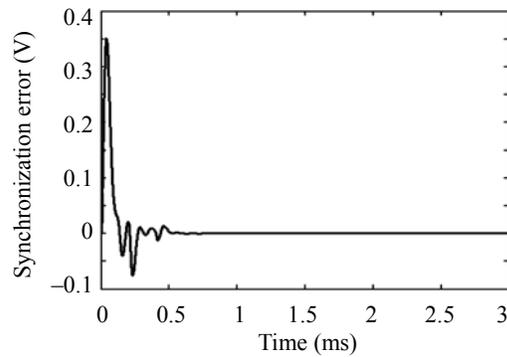
In our simulations, we use the initial conditions of the transmitter and the receiver as  $(V_{C1}(0), V_{C2}(0), I_L(0))=(4.9\text{ V}, -0.4\text{ V}, 0\text{ mA})$ ,  $(\tilde{V}_{C1}(0), \tilde{V}_{C2}(0), \tilde{I}_L(0))=(5.9\text{ V}, -0.4\text{ V}, 0\text{ mA})$ . The fourth order Runge-Kutta method with fixed step-size  $h=10^{-6}\text{ s}$  is used to simulate the system.

With the coupling resistor  $R_1=100\ \Omega$ , perfect synchronization between the transmitter and the receiver could be easily achieved in simulations. Fig.2 plots the synchronization error  $(V_{C2} - \tilde{V}_{C2})$  of the chaotic Colpitts circuit system, under the above given conditions, without parameter mismatches and channel distortions. Although the system is initially desynchronized, the synchronization is rapidly achieved with a settling time of about 0.8 ms.

However, as the parameter mismatches or channel distortions are exerted, the synchronization performance will undoubtedly be degraded. To compare the synchronization performance, we define the average attractor distance between the transmitter and the receiver as:

$$D = \lim_{t_s \rightarrow \infty} \frac{\int_{t_0}^{t_s} \sqrt{e_1^2 + e_2^2 + e_3^2} dt}{t_s - t_0}, \quad (5)$$

where  $e_1 = V_{C1} - \tilde{V}_{C1}$ ,  $e_2 = V_{C2} - \tilde{V}_{C2}$ ,  $e_3 = I_L - \tilde{I}_L$ , and



**Fig.2 The synchronization error  $(V_{C2} - \tilde{V}_{C2})$  without parameter mismatches and channel distortions**

$t_0$  denotes the settling time when the transient parts of the signals have passed. When the transmitter and the receiver are in perfect synchronization state,  $D$  equals zero. A bigger value of  $D$  means worse synchronization performance.

### ADAPTIVE CONTROL FOR TIME-VARYING PARAMETER COMPENSATION

Generally the corresponding circuit parameters in the transmitter and the receiver of a chaotic communication system cannot be exactly the same. It is necessary to consider the case of parameter mismatches between the transmitter and the receiver, and examine their effects on the recovery of a transmitted signal in a chaotic communication system.

The three main passive circuit components, namely the inductor  $L$ , the capacitors  $C_1$  and  $C_2$ , compose the Colpitts circuit together with BJT, and

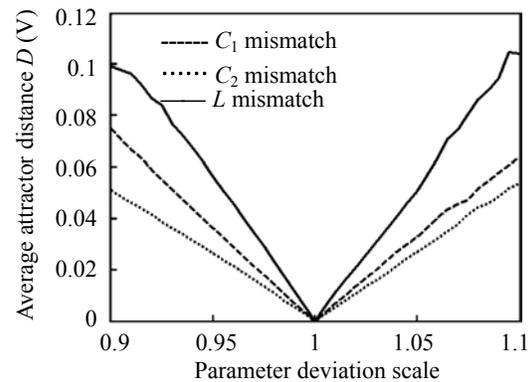
generate the chaotic waveforms studied by Kennedy (1994) and Baziliauskas *et al.*(2001). Thus, the stability of these three circuit components plays a significant role in the synchronization performance of chaotic Colpitts circuits. In this section, we examine the effects of parameter mismatches (of these three circuit components) between the transmitter and the receiver on the synchronization performance. By comparing the performance degradation caused by each of these three parameter mismatches, we found the most pivotal circuit parameter from transmitter to receiver having the most negative influence on the synchronization performance. Then, we studied the adaptive control for time-varying parameter compensation. Note that no channel distortion is considered in this section.

### Comparison between synchronization performances resulting from each time-constant parameter mismatch

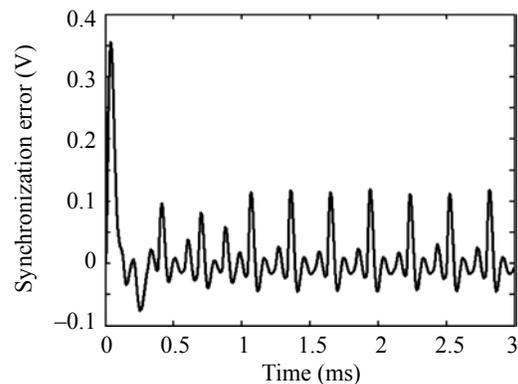
To study the degradation of synchronization performance caused by a certain circuit parameter mismatch, taking the inductor  $L$  for example, we use the following method. We set a certain mismatch of the inductor  $L$  between the drive system and the response system and fix the values of all other circuit parameters in the drive system to the corresponding values in the response system in each simulation step. To compare effects of parameter mismatches of different degrees on the synchronization, we vary the value of  $L$  in the drive system in the range from 0.9 to 1.1 (Parameter deviation scale) times that of its corresponding value in the response system, and record the average attractor distance  $D$  for each value of  $L$ . Then we employ the same method on parameters  $C_1$  and  $C_2$ , respectively, and record the average attractor distance  $D$  each time.

Fig.3a plots the average attractor distance  $D$  versus the transmitter circuit parameter deviation scale. This figure shows effects of parameter mismatches of the three main circuit components, namely  $L$ ,  $C_1$  and  $C_2$  on the synchronization of Colpitts circuit system. Comparing the average attractor distances  $D$  of the three curves, one can see that the mismatch of inductor  $L$  (solid line) causes the greatest degradation on synchronization performance. Thus, we conclude that the synchronization of Colpitts circuit system is more sensitive to the mismatch of  $L$  than that of  $C_1$

(dashed line) and  $C_2$  (dotted line), or in other words, the stability of parameter  $L$  has the most significant influence on the synchronization.



(a)



(b)

### Fig.3 Synchronization of chaotic Colpitts circuits with time-constant parameter mismatches

(a) Average attractor distance between the transmitter and the receiver when three transmitter circuit parameters  $L$ ,  $C_1$  and  $C_2$  deviate respectively from 0.9 to 1.1 times of their corresponding values in the receiver; (b) Synchronization error ( $V_{C_2} - \tilde{V}_{C_2}$ ) when the capacitor  $C_2$  in the transmitter is 0.9 times of its corresponding value in the receiver

Furthermore, when the value of  $L$ ,  $C_2$  or  $C_1$  in the transmitter deviates to 1.1 or 0.9 times that of their corresponding values in the receiver, corresponding to 10% parameter mismatch, the average attractor distance  $D$  in all cases rises above 0.05, which stands for bad synchronization state. The synchronization error ( $V_{C_2} - \tilde{V}_{C_2}$ ) is shown in Fig.3b, where the value of  $C_2$  in the transmitter is 0.9 times that of its corresponding value in the receiver. Obviously, the synchronization performance is sharply degraded, compared with Fig.2. So we conclude that the tolerance degree of

Colpitts system on parameter mismatches is very low.

**Compensating for time-varying parameter mismatch**

In this case, we consider the parameter mismatch being time-varying. We rewrite the drive system as Eq.(6):

$$\begin{cases} \frac{dV_{C1}}{dt} = \frac{K_{C1}(t)}{C_1}(-f(-V_{C2}) + I_L), \\ \frac{dV_{C2}}{dt} = \frac{K_{C2}(t)}{C_2}(I_L - I_0), \\ \frac{dI_L}{dt} = \frac{K_L(t)}{L}(-V_{C1} - V_{C2} - I_L R + V_{CC}), \end{cases} \quad (6)$$

where  $K_{C1}(t)$ ,  $K_{C2}(t)$  and  $K_L(t)$  are the time-varying factors of circuit parameters  $C_1$ ,  $C_2$  and  $L$ , respectively.

The response system is as Eq.(7):

$$\begin{cases} \frac{d\tilde{V}_{C1}}{dt} = \frac{\tilde{K}_{C1}(t)}{C_1}(-f(-\tilde{V}_{C2}) + \tilde{I}_L), \\ \frac{d\tilde{V}_{C2}}{dt} = \frac{\tilde{K}_{C2}(t)}{C_2}\left(\tilde{I}_L - I_0 + \frac{\tilde{V}_{Tr} - \tilde{V}_{C2}}{R_1}\right), \\ \frac{d\tilde{I}_L}{dt} = \frac{\tilde{K}_L(t)}{L}(-\tilde{V}_{C1} - \tilde{V}_{C2} - \tilde{I}_L R + V_{CC}), \end{cases} \quad (7)$$

where  $\tilde{K}_{C1}(t)$ ,  $\tilde{K}_{C2}(t)$  and  $\tilde{K}_L(t)$  are compensating adjustments of circuit parameters  $C_1$ ,  $C_2$  and  $L$ , respectively, which are adaptively modified by using adaptive controllers.

In this simulation, we consider the case when the transmitter parameter  $L$  mismatches with that of the receiver. We make the value of inductor  $L$  time-varying in the transmitter, with a time-varying factor  $K_L(t)$  as defined by the following sinusoidal function:

$$K_L(t) = 1 - 0.1 \sin(10\pi t). \quad (8)$$

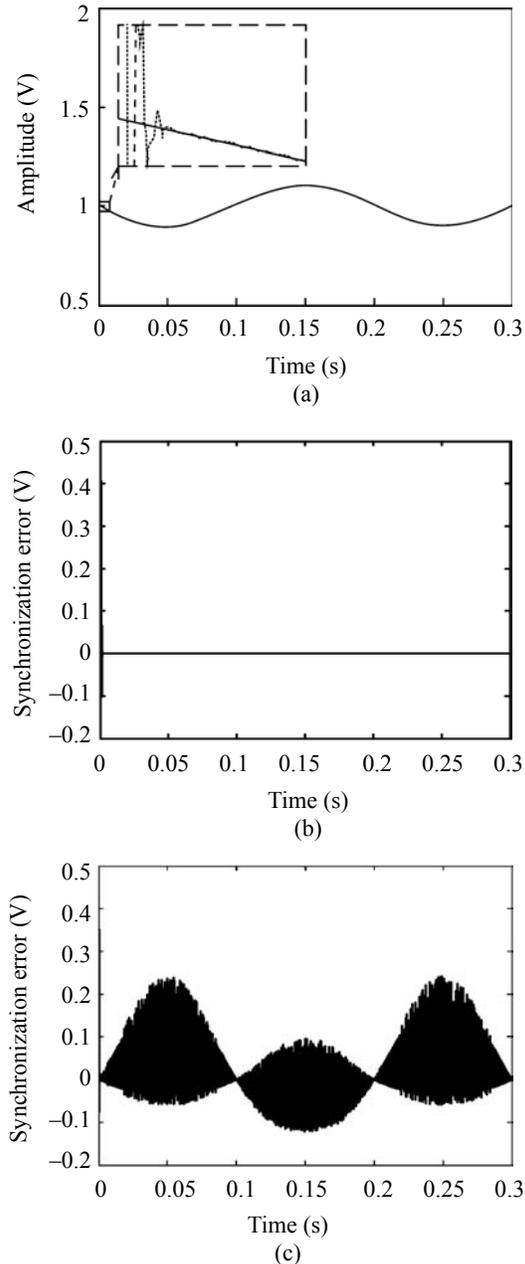
The dynamics of the compensating adjustment  $\tilde{K}_L(t)$  is given as

$$\dot{\tilde{K}}_L(t) = k_1 \operatorname{sgn}\left(\frac{\partial \tilde{I}_L}{\partial \tilde{K}_L}\right)(V_{C2} - \tilde{V}_{C2})$$

$$= k_1 \operatorname{sgn}\left(\frac{1}{L}[-\tilde{V}_{C1} - \tilde{V}_{C2} - \tilde{I}_L R + V_{CC}]\right)(V_{C2} - \tilde{V}_{C2}), \quad (9)$$

where  $k_1 = 5 \times 10^4$ . The simulation time is 0.3 s.

Fig.4 shows the adaptive synchronization of Colpitts circuits when mismatch of  $L$  exists in the sys-



**Fig.4 Synchronization of chaotic Colpitts circuits when the inductor  $L$  is time-varying in the transmitter**  
 (a)  $\tilde{K}_L(t)$  and  $K_L(t)$ ; (b) The synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ) when the adaptive controller is used; (c) The synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ) when no adaptive controller is used

tem, where the  $L$  in the transmitter varies as a sinusoidal function of time. Both  $K_L(t)$  (solid line) and  $\tilde{K}_L(t)$  (dashed line) are plotted in Fig.4a, with magnified details of the foremost simulation time in the rectangle. From this figure, one can see that  $\tilde{K}_L(t)$  asymptotically approaches  $K_L(t)$  after the transient parts of the signal passed with a settling time of about 2 ms. Hence, the parameter  $L$  in the receiver can track the variation of  $L$  in the transmitter, and keeps matching it.

Fig.4b plots the synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ), which keeps to almost zero after the system archives synchronization. For comparison, the synchronization error in the case when no adaptive controller is used is shown in Fig.4c. One can see that the adaptive controller markedly compensates for the de-synchronization caused by the time-varying parameter mismatch, and successfully recovers the transmitted signal in the receiver. Furthermore, since the synchronization of the Colpitts circuit system is more immune to mismatches of  $C_1$  and  $C_2$ , it is safe to say that the adaptive controller can also compensate for mismatches of  $C_1$  and  $C_2$ , which our simulations also verified.

## ADAPTIVE CONTROL FOR TIME-VARYING CHANNEL COMPENSATION

When the system parameters keep identical and time-constant, the channel distortions between the transmitter and the receiver pose the major problems on the synchronization, and are unavoidable in practical chaotic communications. In this section, two major channel distortions, channel fading and Additive White Gaussian Noise (AWGN), are considered. We investigated the impact exerted by each of these distortions on the synchronization, and studied the adaptive control for combating such distortions.

First, we introduced the time-varying gain of the channel  $K_C(t)$  into the system (shown in Fig.1), hence  $\tilde{V}_{Tr} = K_C(t)V_{C2}$ . Constant unit gain channel corresponds to  $K_C(t)=1$ , and  $\tilde{V}_{Tr} = V_{C2}$ . In the receiver, we constructed an adaptive gain  $K_r(t)$  such that  $K_C(t)K_r(t) \rightarrow 1$  as  $t \rightarrow \infty$  to maintain the synchronization. Then the receiver should be rewritten as:

$$\begin{cases} \frac{d\tilde{V}_{C1}}{dt} = \frac{1}{C_1}(-f(-\tilde{V}_{C2}) + \tilde{I}_L), \\ \frac{d\tilde{V}_{C2}}{dt} = \frac{1}{C_2}(\tilde{I}_L - I_0) + \frac{1}{R_1 C_2}(K_r(t)K_C(t)V_{C1} - \tilde{V}_{C2}), \\ \frac{d\tilde{I}_L}{dt} = \frac{1}{L}(-\tilde{V}_{C1} - \tilde{V}_{C2} - \tilde{I}_L R + V_{CC}). \end{cases} \quad (10)$$

The dynamics of  $K_r(t)$  is given by one of the following adaptive controllers:

Controller #1:

$$\dot{K}_r(t) = -k_1(K_r(t)|\tilde{V}_{Tr}| - |\tilde{V}_{C2}|); \quad (11)$$

Controller #2:

$$\dot{K}_r(t) = -k_1(K_r(t)\tilde{V}_{Tr}^2 - \tilde{V}_{Tr}\tilde{V}_{C2}); \quad (12)$$

Controller #3:

$$\dot{K}_r(t) = -k_1 \operatorname{sgn}\left(\frac{\partial \dot{\tilde{V}}_{C2}}{\partial K_r}\right)(K_r(t)\tilde{V}_{Tr}(t) - \tilde{V}_{C2}). \quad (13)$$

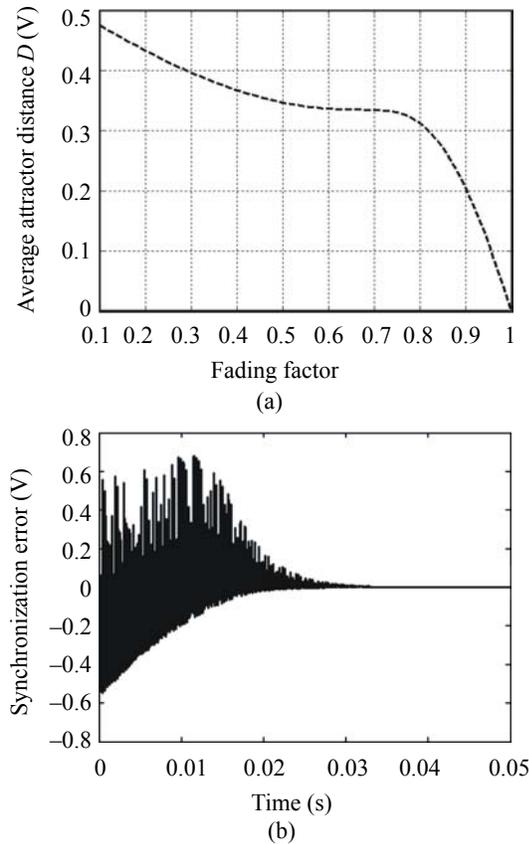
Note that no parameter mismatch is considered in this section.

## Compensating for fading channel

1. Different time-constant fading factors

In this simulation, we considered the time-constant fading as the only channel distortion. The channel gain  $K_C(t)$  is set as a constant in each simulation step. To compare the effects of different fading factors on synchronization performance, we varied the time-constant channel gain  $K_C(t)$  in the range from 0.1 to 1, and recorded the average attractor distance  $D$  for each value of  $K_C(t)$ . The second controller Eq.(12) with  $k_1=5 \times 10^4$  is used to compensate for such time-constant channel fading.

The average attractor distance  $D$  versus the fading factor is plotted in Fig.5a. The dashed line, representing the system without adaptive controller, shows that the synchronization performance sharply declines as the fading factor decreases. Note that even when the fading factor is more than 0.95, or to say, less than 5% of transmitted signals are attenuated, the  $D$  rises up to 0.1, which means a bad synchronization state. So it can be inferred that the Colpitts circuit system can tolerate a very low degree of channel fading in practical communications.



**Fig.5 Synchronization of chaotic Colpitts circuits with time-constant fading in the channel**

(a) Average attractor distance between the transmitter and the receiver when the fading factor varies from 0.1 to 1; (b) The synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ) with adaptive controller is used when the fading factor is 0.1

Fortunately, the adaptive controller can be employed to greatly compensate for such constant channel fading. The solid line, representing the system with the adaptive controller, obviously shows that the synchronization performance is distinctly improved. Even as 90% of the transmitted signal is attenuated, corresponding to the fading factor of 0.1, the  $D$  still keeps almost zero, which means the system is almost perfectly synchronized.

Fig.5b plots the synchronization error  $V_{C2} - \tilde{V}_{C2}$  with fading factor of 0.1 when the adaptive controller is used. One can see that even when 90% of the transmitted signal has been attenuated in channel, the signal can be recovered in the receiver at last. Fig.5b reveals the process of compensating the time-constant channel fading by the adaptive controller. With time proceeding, the adaptive controller calculates the

appropriate adaptive gain  $K_r(t)$  to offset the channel attenuation, gradually reduces the synchronization error, and finally achieves synchronization. It can be inferred that as long as the transmitted signal is not totally submerged in the channel, the adaptive controller can finally recover it in the receiver. Certainly, as the constant channel attenuation worsens, the settling time will be longer. So the adaptive controller is powerful in terms of compensating for the time-constant channel fading.

## 2. Different frequencies of sinusoidal fading channel function

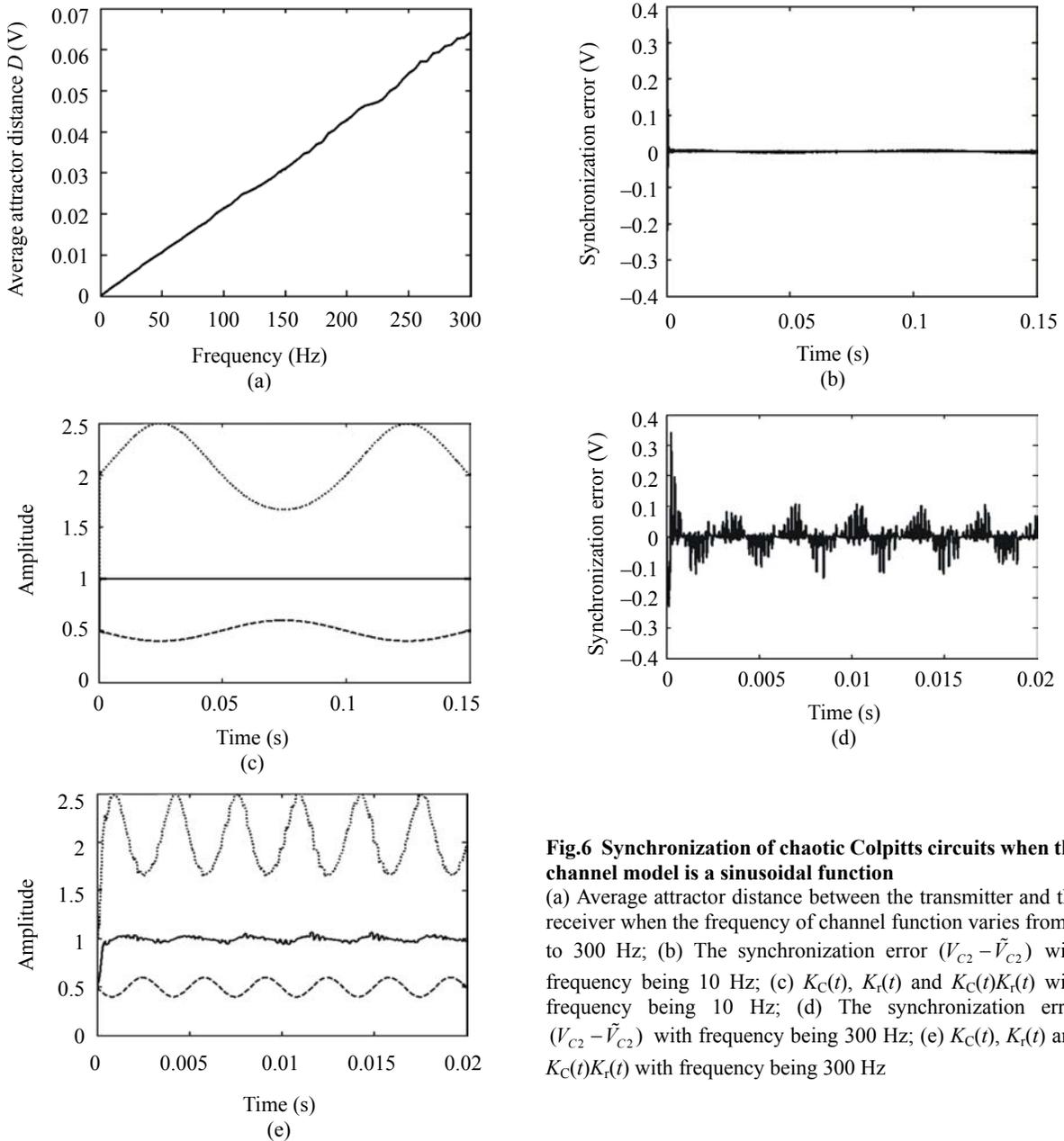
In this simulation, we consider the case when the channel fading is time-varying. We set the channel gain  $K_C(t)$  as a sinusoidal function given by Eq.(14),

$$K_C(t)=0.5-0.1\sin(x\pi t), \quad (14)$$

with a certain frequency in each simulation step. Different frequencies reflect different channel varying rates. To compare effects of different channel varying rates on synchronization performance, we varied the frequency of the sinusoidal function in the range of 0 Hz to 300 Hz, and recorded the average attractor distance  $D$  for each frequency. The second controller Eq.(12) with  $k_1=10^5$  was used to compensate for the sinusoidal varying channel attenuation on the transmitted signals.

Fig.6a plots the average attractor distance  $D$  versus the frequency of the sinusoidal channel function. As the frequency increased, the  $D$  rose almost linearly, which means the synchronization performance linearly declined. When the frequency was as high as 250 Hz,  $D$  rises above 0.05, corresponding to a bad synchronization state.

To get deeper insight into the effects of the increasing frequency on the synchronization performance, we compared the cases of channel function frequency being 10 Hz (Figs.6b~6c) and being 300 Hz (Figs.6d~6e). The synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ) is plotted in Fig.6b and Fig.6d respectively. Also,  $K_C(t)$  (dashed line),  $K_r(t)$  (dotted line) and  $K_C(t)K_r(t)$  (solid line) are respectively plotted in Fig.6c and Fig.6e. Obviously, the synchronization performance differed considerably between the two cases. The synchronization error corresponded periodically with the variation of sinusoidal channel function. Faster channel varying rate led to higher



**Fig.6 Synchronization of chaotic Colpitts circuits when the channel model is a sinusoidal function**

(a) Average attractor distance between the transmitter and the receiver when the frequency of channel function varies from 0 to 300 Hz; (b) The synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ) with frequency being 10 Hz; (c)  $K_C(t)$ ,  $K_r(t)$  and  $K_C(t)K_r(t)$  with frequency being 10 Hz; (d) The synchronization error ( $V_{C2} - \tilde{V}_{C2}$ ) with frequency being 300 Hz; (e)  $K_C(t)$ ,  $K_r(t)$  and  $K_C(t)K_r(t)$  with frequency being 300 Hz

synchronization error as the adaptive controller became incapable of following the fast changing of channel attenuation and hastily calculated an inaccurate adaptive gain  $K_r(t)$ , and so, cannot easily maintain  $K_C(t)K_r(t) \rightarrow 1$ . Figs.5c and 5e show that the  $K_C(t)K_r(t) \rightarrow 1$  cannot keep constant unit gain as the channel varying rate increases to a certain degree. From the simulation results, we concluded that the adaptive controller is capable of compensating for sinusoidal channel fading. But such capability declines with the increasing frequency of sinusoidal channel function. Fortunately,

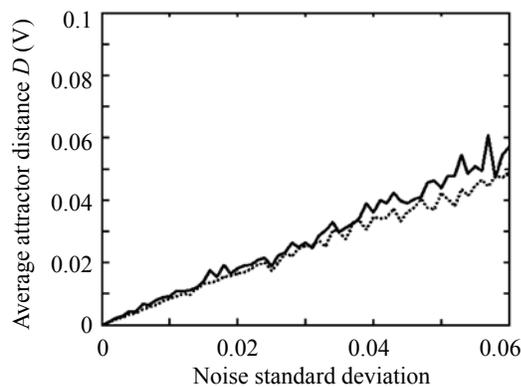
since the channel fading has low-varying rate in most practical environment, the chaotic Colpitts system will definitely benefit from the adoption of adaptive controllers for practical communications.

**Compensating for AWGN channel**

In this simulation, we considered the AWGN as the only channel distortion. The AWGN source here has zero means, and the noise standard deviation was from 0 to 0.06 in the simulation. We used the second controller Eq.(12) with  $k_1=10000$  to offset AWGN,

and recorded the average attractor distance  $D$  for each value of noise standard deviation.

Fig.7 plots the average attractor distance  $D$  versus the noise standard deviation in two cases, one with the adaptive controller (solid line), and one without the adaptive controller (dotted line). It can be seen that the synchronization performance cannot benefit from the adoption of adaptive controllers, but on the contrary, is even a little worse than the case without using the adaptive controller. The reason lies in that the additive noise in the channel is random and unpredictable, unlike the variation of parameters or channel fading simulated above, which changes continuously and predictably. The adaptive controller therefore cannot appropriately calculate the adaptive gain  $K_r(t)$  to maintain  $K_C(t)K_r(t) \rightarrow 1$ . However, since the feedback scheme applied in our system is rather tolerable to the AWGN (Shi *et al.*, 2004), the adaptive controller seems not necessary in practical communications.



**Fig.7** Average attractor distance between the transmitter and the receiver when the noise standard deviation was 0 to 0.06

## CONCLUSION

We have demonstrated that two chaotic Colpitts circuits can be properly synchronized with employment of adaptive controllers while the circuit parameters and the channel are time-varying. The synchronization performance of chaotic Colpitts circuits can be markedly improved by applying the adaptive controller when time-constant and time-varying circuit parameter mismatches and channel attenuation

are considered, but has no obvious enhancement regarding the effect of the AWGN channel. Simulation results indicated that this approach has tremendous potential for developing practical chaotic spread-spectrum communication systems.

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