Journal of Zhejiang University SCIENCE A
ISSN 1009-3095 (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Analysis of moving load induced ground vibrations based on thin-layer method*

BIAN Xue-cheng†, CHEN Yun-min
(*Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China*)
†E-mail: xcbian@gmail.com
Received Mar. 12, 2006; revision accepted June 13, 2006

Abstract: A time-domain solution of layered ground vibration due to moving load has been developed based on the thin layer method. Fourier-Laplace transforms are applied to derive the transformed domain solution that satisfies the boundary conditions of horizontal infinities. The eigen-decomposition approach is used with respect to the Laplace parameter, and the final ground response solution is constructed with the mode superposition method. The reliability and computation accuracy of the solution are proved by comparison with a closed-form solution. A single soil stratum on rigid bedrock is used to reveal the vibration features induced by a rectangular load moving at speeds below or above ground Rayleigh wave velocity.

Key words: Ground vibration, Moving load, Laplace transform, Eigen-decomposition
doi:10.1631/jzus.2006.AS0309 **Document code:** A **CLC number:** TU4; U41

INTRODUCTION

Cole and Huth (1958) and Fryba (1972) analyzed the responses of a 2D elastic body subjected to a moving point load using the technique of triple Fourier integral transform. Eason (1965) studied the 3D steady-state problem of a uniform half-space subjected to moving forces distributed over a rectangular area at uniform speeds. Lefeuve-Mesgouez *et al*.(2000) gave a semi-analytical solution for the half-space ground responses due to a moving strip load. The layering effect is of importance in interpreting the wave field induced by external excitations. de Barros and Luco (1994) proposed a procedure for obtaining the steady-state displacements and stresses within a multilayered viscoelastic half-space generated by a buried or surface point load moving along a horizontal straight line at subsonic, transonic or supersonic speeds. Grundmann *et al*.(1999) studied the response of a layered half-space subjected to a single moving periodic load, and critical velocities of the moving force were identified. Lieb and Sudret (1998) used the decomposition in wavelets to perform the inverse transformation for the ground responses due to moving loads. Andersen and Nielsen (2003) studied with the boundary element method the steady-state wave propagation through an elastic medium due to a source moving with constant velocity. Lefeuve-Mesgouez *et al*.(2002) investigated the 3D vibrations of a multilayered ground for loads moving with speeds up to and beyond the Rayleigh wave speed of the half-space.

This paper is aimed at developing an explicit time domain solution of ground responses due to moving loads to replace the traditional frequency-domain solutions. The thin layer method originally proposed by Tassoulas and Kausel (1983) in frequency domain is a high-efficiency semi-analytical solution for dynamic analysis of laminated media. The Fourier transform technique is traditionally applied to obtain the algebraic governing equations in the wavenumber and frequency domain. Instead, the Laplace transform with respect to time

* Project (No. 50538010) supported by the National Natural Science Foundation of China

variable is used in this study to express the governing equations with the Laplace parameter, and the explicit time domain formulations for ground vibrations are obtained. Some numerical studies for the ground responses due to moving load acting on ground surface have been conducted. The accuracy of the proposed approach has been checked with a closed-form solution.

PHYSICAL MODEL AND MATHEMATICAL FORMULATIONS

Physical model and governing equations

The ground model adopted in this study is shown in Fig.1. The positive z direction is downward and at the ground surface, z=0 m.

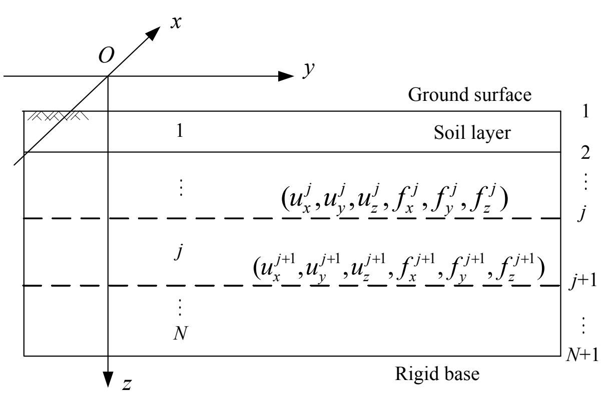

Fig.1 Layered discretization model of ground on rigid bedrock

In this study, the following Laplace transform with respect to time t and Fourier transform pairs with respect to x and y coordinates are used to simplify the governing equation,

$$\tilde{\bar{f}}(\xi_x,\xi_y,z,p)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}f(x,y,z,t)\mathrm{e}^{\mathrm{i}\xi_x x+\mathrm{i}\xi_y y-pt}\mathrm{d}t\mathrm{d}x\mathrm{d}y, \tag{1}$$

$$f(x,y,z,t)=\frac{1}{8\pi^3\mathrm{i}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\gamma-\mathrm{i}\infty}^{\gamma+\mathrm{i}\infty}\tilde{\bar{f}}(\xi_x,\xi_y,z,s)\mathrm{e}^{pt-\mathrm{i}\xi_x x-\mathrm{i}\xi_y y}\mathrm{d}p\mathrm{d}\xi_x\mathrm{d}\xi_y. \tag{2}$$

Variables with bar '-' and tilde '~' represent the components in frequency domain and wave number domain respectively.

Applying the predefined transforms to the Navier equations describing ground motions yields,

$$\rho V_\mathrm{s}^2\left(-\xi^2\tilde{\bar{u}}_x+\frac{\mathrm{d}^2\tilde{\bar{u}}_x}{\mathrm{d}z^2}\right)-\mathrm{i}\xi_x\rho\left(V_\mathrm{p}^2-V_\mathrm{s}^2\right)\tilde{\bar{\Delta}}+\tilde{\bar{f}}_x=\rho p^2\tilde{\bar{u}}_x, \tag{3}$$

$$\rho V_\mathrm{s}^2\left(-\xi^2\tilde{\bar{u}}_y+\frac{\mathrm{d}^2\tilde{\bar{u}}_y}{\mathrm{d}z^2}\right)-\mathrm{i}\xi_y\rho\left(V_\mathrm{p}^2-V_\mathrm{s}^2\right)\tilde{\bar{\Delta}}+\tilde{\bar{f}}_y=\rho p^2\tilde{\bar{u}}_y, \tag{4}$$

$$\rho V_\mathrm{s}^2\left(-\xi^2\tilde{\bar{u}}_z+\frac{\mathrm{d}^2\tilde{\bar{u}}_z}{\mathrm{d}z^2}\right)+\rho\left(V_\mathrm{p}^2-V_\mathrm{s}^2\right)\frac{\partial\tilde{\bar{\Delta}}}{\partial z}+\tilde{\bar{f}}_z=\rho p^2\tilde{\bar{u}}_z, \tag{5}$$

where

$$\tilde{\bar{\Delta}}=\partial\tilde{\bar{u}}_z/\partial z-\mathrm{i}(\xi_x\tilde{\bar{u}}_x+\xi_y\tilde{\bar{u}}_y),\ \xi^2=\xi_x^2+\xi_y^2.$$

V_p and V_s are dilatational velocity and shear velocity of soil material, $V_\mathrm{p}=\sqrt{(\lambda+2\mu)/\rho}$ and $V_\mathrm{s}=\sqrt{\mu/\rho}$. u_j (j=x, y, z) represent displacement components in three directions. λ and μ are the Lame constants of ground material, ρ is the density, f_j (j=x, y, z) are body forces.

The coordinate transform is also applied to change the governing equations of ground motions from the Cartesian coordinate system to the cylindrical coordinate system. The radial, circular and vertical components in the new system are denoted with subscript '1', '2' and '3' respectively. To solve the wave equations in the transformed domain, the medium is divided into several sublayers that are thin in finite element sense, and the physical interfaces of natural soil layers are also used as sublayer interfaces. The thin layers are labelled consecutively as 1, 2, …, N from the ground top, and the layer interfaces are also labelled as 1, 2, …, N+1, with the interface 1 for the ground surface and N+1 for the rigid ground bottom. The ground model with layered discretization is depicted in Fig.1.

By introducing the linear interpolation function for the jth layer, the governing equations of inplane and antiplane motions can be expressed in the compact matrix forms from the virtual work principle,

Inplane motions (with 'p-sv' notation),

$$(\boldsymbol{A}_j^{\text{p-sv}}\xi^2+\boldsymbol{B}_j^{\text{p-sv}}\xi+\boldsymbol{C}_j^{\text{p-sv}}+p^2\boldsymbol{M}_j^{\text{p-sv}})\boldsymbol{U}_j^{\text{p-sv}}=\boldsymbol{F}_j^{\text{p-sv}}. \tag{6}$$

Antiplane motions (with 'sh' notation),

$$(\boldsymbol{A}_j^{\rm sh}\xi^2+\boldsymbol{C}_j^{\rm sh}+p^2\boldsymbol{M}_j^{\rm sh})\boldsymbol{U}_j^{\rm sh}=\boldsymbol{F}_j^{\rm sh}, \qquad (7)$$

in which, the displacement and force vectors for the jth layer are given by,

$$\boldsymbol{U}_j^{\rm p\text{-}sv}=\{u_1^j\ u_2^j\ u_1^{j+1}\ u_2^{j+1}\}^{\rm T},\ \boldsymbol{U}_j^{\rm sh}=\{u_3^j\ u_3^{j+1}\}^{\rm T}, \qquad (8)$$

$$\boldsymbol{F}_j^{\rm p\text{-}sv}=\{f_1^j\ f_2^j\ f_1^{j+1}\ f_2^{j+1}\}^{\rm T},\ \boldsymbol{F}_j^{\rm sh}=\{f_3^j\ f_3^{j+1}\}^{\rm T}. \qquad (9)$$

The detailed expressions of the coefficient matrices in the above equations are given by Tassoulas and Kausel (1983).

Engen-decomposition and mode superposition

The global system matrix with N layers is generated by overlapping the results of a single layer with those of all other layers like the element matrices in a finite element formulation. To solve the system equations of ground motions, the eigen decomposition procedure is applied to enable integration with respect to the Laplace parameter p in closed form by using the residue theory.

If we denote

$$\hat{\boldsymbol{K}}^{\rm p\text{-}sv,sh}=\boldsymbol{A}^{\rm p\text{-}sv,sh}\xi^2+\boldsymbol{B}^{\rm p\text{-}sv,sh}\xi+\boldsymbol{C}^{\rm p\text{-}sv,sh},$$

in which, superscript 'p-sv,sh' is used for either inplane or antiplane wave motion. The uniform expressions of the global system equations for inplane and antiplane motions can be given by,

$$\{\hat{\boldsymbol{K}}^{\rm p\text{-}sv,sh}+(p^2)\boldsymbol{M}^{\rm p\text{-}sv,sh}\}\phi^{\rm p\text{-}sv,sh}=0. \qquad (10)$$

Since the ground is divided into N layers, $2N$ and N eigenvalues can be obtained from Eq.(10) for inplane and antiplane motions individually. The eigenvectors satisfy the standard orthogonality conditions, and can be assumed to be normalized with respect to the mass matrix for both inplane and antiplane motions, hence the eigen-matrices $\boldsymbol{\Phi}^{\rm p\text{-}sv}$, $\boldsymbol{\Phi}^{\rm sh}$ can be given as,

$$\boldsymbol{\Phi}^{\rm p\text{-}sv}=[\phi_1^{\rm p\text{-}sv}\ \phi_2^{\rm p\text{-}sv}\cdots\phi_{2N}^{\rm p\text{-}sv}], \qquad (11)$$

$$\boldsymbol{\Phi}^{\rm sh}=[\phi_1^{\rm sh}\ \phi_2^{\rm sh}\cdots\phi_N^{\rm sh}]. \qquad (12)$$

The displacement solution can be obtained from the conventional mode superposition method as,

$$\begin{Bmatrix}\tilde{\tilde{\boldsymbol{u}}}_1\\ \tilde{\tilde{\boldsymbol{u}}}_2\\ \tilde{\tilde{\boldsymbol{u}}}_3\end{Bmatrix}=\begin{bmatrix}\boldsymbol{\Phi}_1\boldsymbol{\Omega}^{\rm p\text{-}sv}\boldsymbol{\Phi}_1^{\rm T} & \boldsymbol{\Phi}_1\boldsymbol{\Omega}^{\rm p\text{-}sv}\boldsymbol{\Phi}_2^{\rm T} & \boldsymbol{0}\\ \boldsymbol{\Phi}_2\boldsymbol{\Omega}^{\rm p\text{-}sv}\boldsymbol{\Phi}_1^{\rm T} & \boldsymbol{\Phi}_2\boldsymbol{\Omega}^{\rm p\text{-}sv}\boldsymbol{\Phi}_2^{\rm T} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Phi}_3\boldsymbol{\Omega}^{\rm sh}\boldsymbol{\Phi}_3^{\rm T}\end{bmatrix}\begin{Bmatrix}\tilde{\tilde{\boldsymbol{F}}}_1\\ \tilde{\tilde{\boldsymbol{F}}}_2\\ \tilde{\tilde{\boldsymbol{F}}}_3\end{Bmatrix}$$

$$=[\boldsymbol{\Phi\Omega\Phi}]\begin{Bmatrix}\tilde{\tilde{\boldsymbol{F}}}_1\\ \tilde{\tilde{\boldsymbol{F}}}_2\\ \tilde{\tilde{\boldsymbol{F}}}_3\end{Bmatrix}, \qquad (13)$$

where

$$\boldsymbol{\Omega}^{\rm p\text{-}sv}={\rm diag}\left(\frac{1}{p^2+(p_i^{\rm p\text{-}sv})^2}\right),\ i=1,\ 2,\ \cdots,\ 2N, \qquad (14)$$

$$\boldsymbol{\Omega}^{\rm sh}={\rm diag}\left(\frac{1}{p^2+(p_i^{\rm sh})^2}\right),\ i=1,\ 2,\ \cdots,\ N. \qquad (15)$$

The solution of ground motions in the original Cartesian system can be obtained by the inverse coordinate transform. Now the governing equations of ground motions for an elasto-dynamic problem are formulated in the wavenumber domain with the Laplace parameter p, which will be eliminated from Eq.(13) by the analytical inverse Laplace transform.

Analytical solution with rectangular moving load

A unit load acting on ground surface is distributed in a square area whose sizes in x and y directions are $2a\times2b$. The load is assumed to move in the positive x direction at a constant speed c. The mathematical description of the load distribution can be given by,

$$F(x,y,t)=\begin{cases}1, & -a\le x\le a,\ -b\le y\le b,\\ 0, & \text{otherwise},\end{cases} \qquad (16)$$

and the corresponding Laplace transform of Eq.(16) in wavenumber domain is given by,

$$\tilde{F}(\xi_x,\xi_y,p)=\frac{1}{4}\frac{\sin(a\xi_x)}{a\xi_x}\frac{\sin(b\xi_y)}{b\xi_y}\frac{1}{p-{\rm i}\xi_x c}. \qquad (17)$$

Since the eigen-matrices $\boldsymbol{\Phi}^{\rm p\text{-}sv}$ and $\boldsymbol{\Phi}^{\rm sh}$ are independent of the Laplace parameter p, the transient responses of the decomposed modes can be obtained by applying the inverse Laplace transforms to Eqs. (14) and (15) combining Eq.(17),

$$\boldsymbol{\Omega}^{\text{p-sv,sh}} = \frac{1}{2\pi \mathrm{i}} \int_{\gamma \mathrm{i}-\infty}^{\gamma \mathrm{i}+\infty} \mathrm{diag}\left\{ \frac{\tilde{F}(\xi_x, \xi_y, p)}{-(p_j^{\text{p-sv,sh}})^2 + p^2} \right\} \mathrm{e}^{pt} \mathrm{d}p, \tag{18}$$

which is evaluated by the residue theory. The damping effect is taken into account by introducing the damping ratio β in each decomposed mode. The time domain solution can be given in an explicit way as,

$$\boldsymbol{\Omega}^{\text{p-sv,sh}} = \frac{1}{4} \frac{\sin(a\xi_x)}{a\xi_x} \frac{\sin(b\xi_y)}{b\xi_y} \times \mathrm{diag}\{\omega_j^{\text{p-sv,sh}}\}, \tag{19}$$

in which the diagonal elements can be given by,

$$\omega_j^{\text{p-sv,sh}} = \frac{2}{(p_j^{\text{p-sv,sh}})^2 + (\beta p_j^{\text{p-sv,sh}} + \xi_x c)^2} \cdot \left\{ \begin{array}{l} -\mathrm{e}^{-\beta p_j^{\text{p-sv,sh}} t} \{\beta \sin(p_j^{\text{p-sv,sh}} t) + \cos(p_j^{\text{p-sv,sh}} t)\} \\ + \cos(\xi_x ct) - \mathrm{i}\sin(\xi_x ct) \\ -\mathrm{i} \left\{ \dfrac{\xi_x c}{p_j^{\text{p-sv,sh}}} \mathrm{e}^{-\beta p_j^{\text{p-sv,sh}} t} \sin(p_j^{\text{p-sv,sh}} t) \right\} \end{array} \right\}. \tag{20}$$

The formulations of ground responses in the time-wavenumber domain can be achieved by substitution of Eq.(20) into Eq.(13). While the inverse Fourier transforms with respect to wavenumbers ξ_x and ξ_y will be performed with the discrete wavenumber convolution method (Bouchon and Aki, 1977) to get the moving load induced ground responses in the space and time domain.

NUMERICAL STUDIES

A single soil stratum on rigid bedrock is used to check the reliability and applicability of the proposed approach for ground responses under moving load actions. The moving load has a rectangular distribution of 2 m×2 m with total load of 1.0 N as shown in Fig.2. In the same figure, properties of ground material including density ρ, shear velocity V_s, Poisson ratio υ and damping ratio β are also given.

The ground of half-space is approximated with a homogeneous soil stratum of 20 m height on the rigid bedrock. In the numerical computations, two kinds of soil sublayer thickness are used. For the load with moving speed below 150 m/s, the soil stratum is divided into 40 sublayers with height of 0.5 m, while for high speed cases, the soil stratum is divided into 80 sublayers with height of 0.25 m. These two kinds of thin-layer sizes give satisfying numerical results.

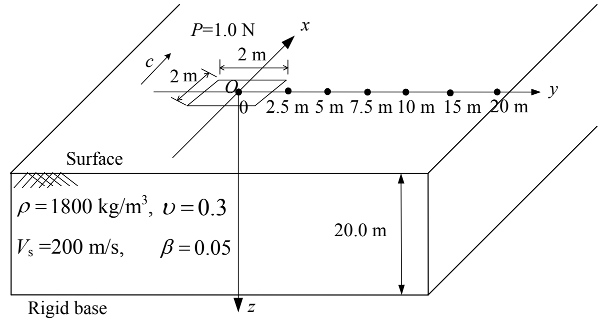

Fig.2 Parameters used in numerical computation

The transient ground responses due to a uniform moving load at constant speed c=100 m/s are investigated, and the computed results of vertical and longitudinal ground responses at y=0 m are presented in Fig.3. In the same figures, results given by Eason (1965)'s analytical solution for a homogeneous elastic half-space are also depicted for the comparison purpose. A good agreement is attained between these two solutions.

Totally 7 observation points denoted in Fig.2 are used to inspect the time histories of ground dynamic responses due to moving loads' passages. The time histories of responses at specific observation points are shown in Fig.4. Here, two speeds of 100 m/s and 250 m/s are used to represent moving speed below and above the ground's Rayleigh velocity. From the computation results, it is found that two horizontal components of ground responses at nearby ground surface increase significantly, while the vertical one only has a little increase in the amplitude. For the low-speed moving load case, the wave forms are almost symmetric about the time when the load center passes the origin point (x=0 m and y=0 m), while for high-speed case, the amplitudes of ground responses are very small before the moving load reaches the origin point, and become very significant when the load passes.

The dynamics responses of the overall ground surface are studied for the moving loads at different speeds. The ratio of the load's moving load speed to

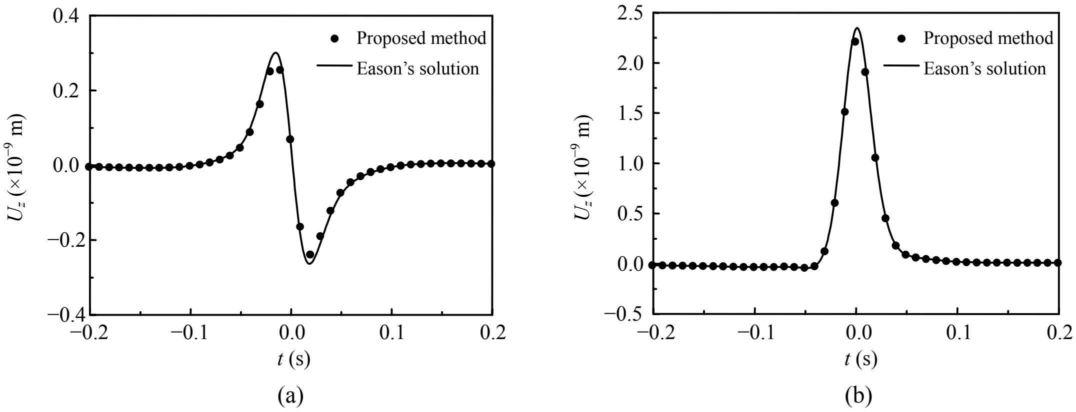

Fig.3 Ground surface responses at y=0 m due to moving rectangular load. (a) Longitudinal displacement U_x; (b) Vertical displacement U_z

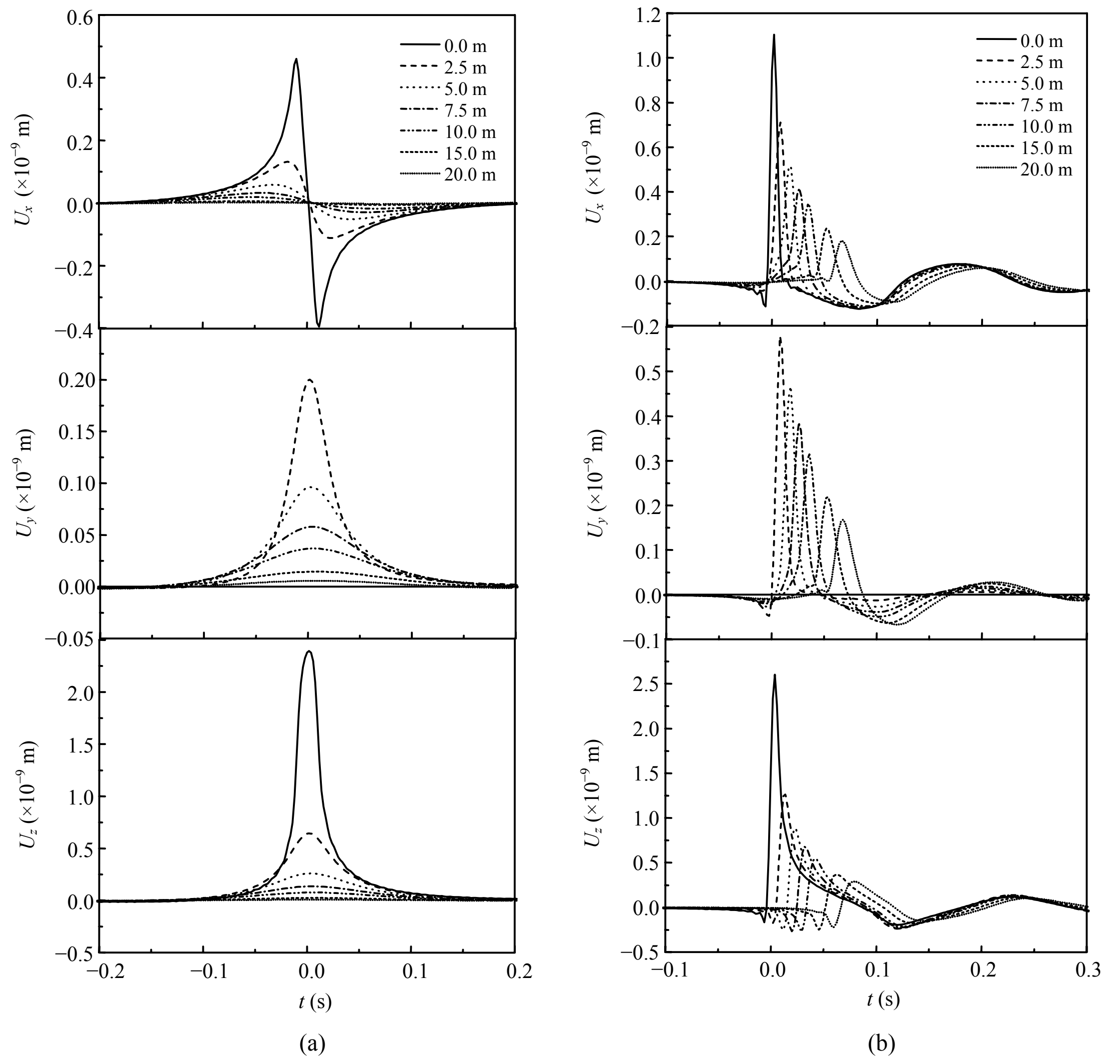

Fig.4 Time histories of ground responses due to moving loads at different speeds. (a) c=100 m/s; (b) c=250 m/s

the Rayleigh velocity (V_R) of ground is defined as Mach number $M_g=c/V_R$. The contour lines of ground response amplitudes due to the moving loads with different Mach numbers are presented in Fig.5. It is found that ground vibrations are mostly confined in a local area when the load's Mach number is small (i.e. M_g<<1.0). The wave propagation phenomena in the ground are not obvious and the vibration amplitudes on the ground surface are almost symmetric about the load center. With the increasing of the moving speed, wave propagation in the surrounding ground grows. When M_g approaches or exceeds 1.0, ground vibrations increase dramatically. Ground deformations in front of the load position become very small, while at rear part become very significant as the load passes.

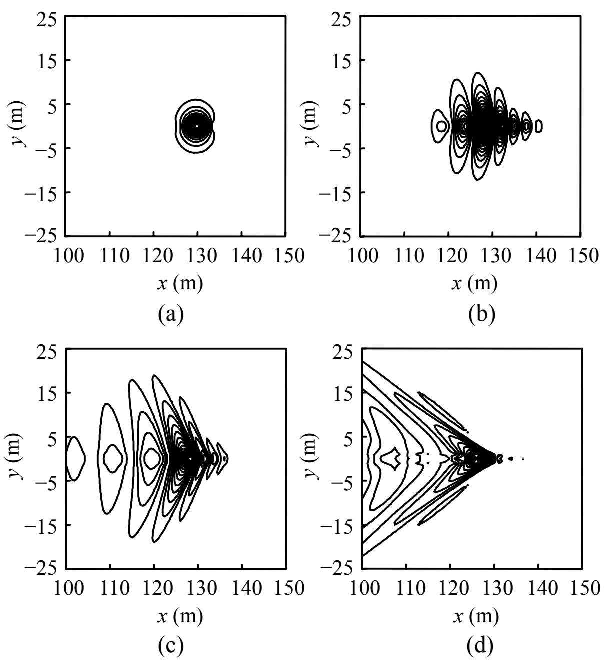

Fig.5 Vertical responses on ground surface due to moving loads with different Mach numbers
(a) M_g=0.5; (b) M_g=0.9; (c) M_g=1.0; (d) M_g=1.5

CONCLUSION

An explicit time domain solution base on the thin layer method for ground responses due to moving load has been developed in this study. The Laplace transform with respect to time variable instead of the traditional Fourier transform is applied to simplify the governing equations, and the explicit time domain solution is derived by using the eigen-decomposition technique in the transformed domain. The accuracy of the proposed approach is validated by comparison with a closed-form solution. From the numerical computations, it is found that the Mach number of the moving load is critical in determining the ground responses due to moving loads. The ground vibrations are confined in a local area if the Mach number is small. When the Mach number approaches or exceeds 1.0, ground vibration amplitudes and affected area increase dramatically.

References

Andersen, L., Nielsen, S.R.K., 2003. Boundary element analysis of the steady-state response of an elastic half-space to a moving force on its surface. *Engineering Analysis with Boundary Elements*, **27**(1):23-38. [doi:10.1016/S0955-7997(02)00096-6]

Bouchon, M., Aki, K., 1977. Discrete wave number representation of seismic source wave fields. *Bull. Seis. Soc. Am.*, **67**(2):259-277.

Cole, J., Huth, J., 1958. Stresses produced in a half plane by moving loads. *J. Appl. Mech., ASME*, **25**:433-436.

de Barros, F.C.P., Luco, J.E., 1994. Response of a layered viscoelastic half-space to a moving point load. *Wave Motion*, **19**(2):189-210. [doi:10.1016/0165-2125(94)90066-3]

Eason, G., 1965. The stresses produced in a semi-infinite solid by a moving surface force. *Int. J. Eng. Sci.*, **2**(6):581-609. [doi:10.1016/0020-7225(65)90038-8]

Fryba, L., 1972. Vibration of Solids and Structures under Moving Loads. Noordhoff International Publishing, Groningen, The Netherlands.

Grundmann, H., Lieb, M., Trommer, E., 1999. The response of a layered half-space to traffic loads moving along its surface. *Archive Appl. Mech.*, **69**(1):55-67. [doi:10.1007/s004190050204]

Lefeuve-Mesgouez, G., Le Houédec, D., Peplow, A.T., 2000. Ground vibration in the vicinity of a high-speed moving harmonic strip load. *Journal of Sound and Vibration*, **231**(5):1289-1309. [doi:10.1006/jsvi.1999.2731]

Lefeuve-Mesgouez, G., Peplow, A.T., Le Houédec, D., 2002. Surface vibration due to a sequence of high speed moving harmonic rectangular loads. *Soil Dynamics and Earthquake Engineering*, **22**(6):459-473. [doi:10.1016/S0267-7261(02)00034-9]

Lieb, M., Sudret, B., 1998. A fast algorithm for soil dynamics calculations by wavelet decomposition. *Archive Appl. Mech.*, **68**(3-4):147-157. [doi:10.1007/s004190050152]

Tassoulas, J.L., Kausel, E., 1983. Elements for the numerical analysis of wave motion in layered strata. *International Journal for Numerical Methods in Engineering*, **19**(7):1005-1032. [doi:10.1002/nme.1620190706]