



## A model of regional economic development with increasing returns\*

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**Abstract:** This paper develops mathematically and empirically tractable regional and interregional model of economic development with increasing returns to scale (IRS) under the neoclassical assumptions. A one-sector, two-region model in which one region exhibits IRS is presented and the whole nation presents constant returns to scale. The development of the local IRS economy is shown to be constrained to a “moving equilibrium” path. The preliminary empirical results are sufficiently supportive of the argument to encourage further research along the lines of the model. In particular, the neoclassical model does not predict negative coefficients on the real rental value of capital in regressions explaining population or employment relative to that in the nation.

**Key words:** Economic development, Increasing returns, Moving equilibrium

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### INTRODUCTION

The phenomenon of increasing returns to scale (IRS) has been discussed frequently in the economic literature (Young A.A., 1928; Romer, 1986; 1990; Young A., 1998; Dinopoulos and Sener, 1999; Jones, 2003; Corrado *et al.*, 2006). However, much theoretical literature on regional economic growth assumed constant returns to scale (CRS) to avoid the technical difficulties presented by IRS. Interest in dynamic models of growth with IRS was sparked by Arrow (1962) on learning by doing. One result of this interest has been an increase in the number of models of regional and interregional economic growth in which IRS is assumed. Notable examples include those of (Dixon and Thirlwall, 1975; Faini, 1984; Krugman, 1991; Li, 2000; 2003; Jones, 2002; 2003; 2004; Esteban and Wright, 2005). Neither Dixon and Thirlwall (1975) nor Faini (1984) introduced distance into their models. Their regions are spaceless ones.

Krugman (1991) tried to explain how an industrialized “core” (populated by IRS firms) and an agricultural “periphery” (populated by CRS firms) can be endogenously formed in a country. Essentially his model uses the assumptions of monopolistic competition, non-zero transportation cost and immobile agricultural workers to make IRS consistent with a non-extreme equilibrium. The resulting model can encompass regions located in space (Krugman, 1991). However, the system is mathematically and empirically intractable. Its solutions are numerical ones obtained using ad-hoc specifications of the parameters, while its empirical implementation is thwarted by a number of unusual normalizations of the units in which variables are measured.

This paper is aimed at developing a mathematically and empirically tractable regional and interregional model that uses the assumption of IRS in one region to introduce distance into the system while still retaining the basic flavor of traditional regional analysis. This development is based on the work by Guccione and Gillen (Guccione and Gillen, 1980; Gillen and Guccione, 1983; Guccione *et al.*, 1995). They assumed two regions producing one homoge-

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nous product under conditions of CRS. We generalize the system, allowing one region to exhibit IRS, while the rest of the country operates under CRS. The paper is arranged as follows: in the next section, a modification of the Guccione and Gillen model that allows IRS, and the existence of a non-extreme equilibrium is discussed; some very preliminary empirical tests are reported in the third section; finally there is a conclusion.

### ONE-SECTOR, TWO-REGION GROWTH MODEL

We assume two regions: one with IRS, hereinafter “the local”; and the other with CRS, hereinafter “the nation”. These regions produce one homogenous product by means of two inputs (capital  $K$  and labor  $E$ ). We distinguish between the short-run and long-run by the flexibility of the capital stock. Let the local and national output be  $X$  and  $\bar{X}$  respectively, and let units of labor and capital utilized be  $E$ ,  $\bar{E}$ ,  $K$  and  $\bar{K}$  respectively. Price is denoted by  $P$  for the local and  $\bar{P}$  for the nation. The production functions are assumed to be Cobb Douglas, the local one being

$$X = \alpha_1 E^{\alpha_2} K^{\alpha_3}, \tag{1}$$

and the national one being

$$\bar{X} = \alpha_1 \bar{E}^{\alpha_2} \bar{K}^{1-\alpha_2}, \tag{2}$$

where  $\alpha_1, \alpha_2, \alpha_3 > 0, 1 - \alpha_2 < \alpha_3 < 1$  and  $\alpha_2 < 1$ .

To retain competitive behavior by the local, we assume that IRS is external to the firm, that means each firm has IRS production function. In particular, it is assumed that there are a large number of firms each having the national production function augmented by a function of the local capital. That is

$$X_i = \alpha_1 E_i^{\alpha_2} K_i^{1-\alpha_2} K^u,$$

where the subscript  $i$  denotes the  $i$ th firm and  $u > 0$ . That is, the productivity of each firm is increased beyond that in the nation by the agglomeration of capital within the confines of the local economy. Each local firm is a price taker and, in maximizing profit, neglects its effect on the total local capital stock. The resulting cost function is

$$C_i = [\alpha_1 \alpha_2^{\alpha_2} (1 - \alpha_2)^{1-\alpha_2}]^{-1} E^{1-\alpha_2} W^{\alpha_2} K^{-u} X_i,$$

which implies the following perfectly elastic supply

$$P = [\alpha_1 \alpha_2^{\alpha_2} (1 - \alpha_2)^{1-\alpha_2}]^{-1} E^{1-\alpha_2} W^{\alpha_2} K^{-u}. \tag{3}$$

Thus the local supply increases (the supply price decreases) as the capital stock increases, i.e., the aggregate local economy exhibits IRS but the individual firms act as perfect competitors. Since all local firms have the same production function and factor prices, we have  $E_i/K_i = E/K$  for all  $i$ 's. Hence, observationally, the aggregate local production function can be obtained as

$$\begin{aligned} X &= \sum X_i = \sum \alpha_1 (E_i / K_i)^{\alpha_2} K_i K^u \\ &= \alpha_1 (E / K)^{\alpha_2} \sum K_i K_i^u = \alpha_1 E^{\alpha_2} K^{1-\alpha_2+u}, \end{aligned}$$

where the summation is over all local firms. Defining  $\alpha_3 = 1 - \alpha_2 + u$  yields the local production function (1).

Without further restrictions, the above model will have an extreme equilibrium. With an IRS production function, the local economy would eventually yield price  $P$  lower than  $\bar{P}$ , and satisfy the whole national demand. Similar to the extreme, however, that the time required to attain this extreme solution is much longer than the “long run” defined in terms of the flexibility of the capital stock. In particular, it is assumed that there is a non zero “iceberg” transport cost  $c$  per dollar of product per unit of distance, paid by the consumers, so that the condition  $P < \bar{P}$  for local monopolization becomes  $P(1+cd) < \bar{P}$  for all distance  $d^1$ . The existence of local IRS insures that, for a sufficiently large local capital stock, this last condition will be met eventually. It is assumed, however, that local growth is not instantaneous but is constrained in finite time. Thus at any point in time (and hence for all empirical observations) there exists a distance  $d_t$  such that  $P(1+cd_t) > \bar{P}$  and the nation has a positive (but declining) share of total production. This kind of “equilibrium” is similar to the “moving equilibria” suggested by Young (1928)<sup>2</sup>. Under this

<sup>1</sup> The right hand side (RHS) of this inequality does not include transport costs since the nation produces under CRS

<sup>2</sup> He also thought “the apparatus which economists have built up for the analysis of supply and demand in their relations to prices does not seem to be particularly helpful for the purposes of an inquiry into these broader aspects of increasing returns” [Young (1928), p.233]

assumption of the temporal existence of non-extreme equilibrium, we revise Guccione and Gillen (1980)'s C-model and consider the growth path of the local region. For this purpose, it is assumed that the real wage rates are equal over regions in both the short and long run. That is  $w/P = \bar{w}/\bar{P}$ , where  $w$ ,  $\bar{w}$  denote the local and national wage rates respectively.

First we consider the short-run (i.e., capital stocks are fixed). Firms maximize their profit with respect to employment. The first order condition (FOC) for the  $i$ th local firm is

$$P\alpha_1\alpha_2E_i^{\alpha_2-1}K_i^{1-\alpha_2}K^u = w,$$

which in the aggregate local economy yields<sup>3</sup>

$$P\alpha_1\alpha_2E^{\alpha_2-1}K^{\alpha_3} = w. \quad (4)$$

Similarly for the nation, we have

$$\bar{P}\alpha_1\alpha_2\bar{E}^{\alpha_2-1}\bar{K}^{1-\alpha_2} = \bar{w}. \quad (5)$$

Eqs.(4) and (5) imply

$$E = \frac{K^{\alpha_3/(1-\alpha_2)}}{\bar{K}}\bar{E} = \theta\bar{E}. \quad (6)$$

Since  $K$  and  $\bar{K}$  are fixed, the ratio  $\theta$  should be treated as fixed too. This relation gives the employment function of the export-base model.

In the long-run, firms can adjust capital as well as labor. For the nation, the FOC for profit maximization subject to Eq.(2) yields the factor proportion

$$\frac{\bar{K}}{\bar{E}} = \frac{1-\alpha_2}{\alpha_2} \frac{\bar{w}}{\bar{r}}, \quad (7)$$

and the factor price frontier

$$\bar{P}\alpha_1\alpha_2\left(\frac{1-\alpha_2}{\alpha_2}\right)^{1-\alpha_2}\left(\frac{\bar{K}}{\bar{w}}\right)^{\alpha_2-1} = \bar{w}, \quad (8)$$

where  $\bar{r}$  denotes the rental value of capital and is assumed to be equal across regions. The FOC for cost minimization by local firm  $i$ , subject to its production function, can be written as  $K_i/E_i = [(1-\alpha_2)/\alpha_2](w/\bar{r})$  since the firm ignores its effect on the term  $K^u$ . Aggregating this last relation over all  $i$ 's yields a local factor proportion equation of the same form as the nation's, specifically,

$$K/E = \frac{1-\alpha_2}{\alpha_2} \cdot \frac{w}{r}. \quad (9)$$

The factor price frontier for the local given by Eq.(3) differs from that for the nation in that it contains the term  $K^u$ . This causes the local optimal price to fall as capital accumulates. Hence the equilibrium distance satisfying  $P(1+cd) = \bar{P}$ , that is

$$d = (\bar{P} - P)/(cP). \quad (10)$$

varies. We now turn to model this variation.

Let the market be located on a straight line between 0 and 1, with the local at point 0. Owners of capital are assumed to locate evenly along  $[0, 1]$  while workers locate at the point of their employment<sup>4</sup>. Since it is assumed that consumers pay the transport cost, and workers will spend their wages at the point of their employment, thus the value of the worker demand for the local output is  $wE$  while that for the nation's output is  $\bar{w}\bar{E}$ . The total expenditure of capital owners ( $\bar{r}K + \bar{r}\bar{K}$ ) will be distributed between the local and the nation in the ratio  $d/(1-d)$ . It follows that the values of demands for local and national outputs are  $PX = wE + d\bar{r}(K + \bar{K})$  and  $\bar{P}\bar{X} = \bar{w}\bar{E} + (1-d)\bar{r}(K + \bar{K})$  respectively. These relations require that

$$(PX - wE)/(\bar{P}\bar{X} - \bar{w}\bar{E}) = d/(1-d).$$

The FOC conditions (3), (7)~(9) imply that the left hand side (LHS) of the latter equality is  $wE/\bar{w}\bar{E}$ , hence, using condition (10), we have

$$wE/\bar{w}\bar{E} = (\bar{P} - P)/[cP - (\bar{P} - P)]. \quad (11)$$

Eq.(11) essentially allows the equilibrium at a point in time to shift as  $\bar{P}/P$  varies. The model that defines this moving equilibrium is completed by the requirement that the real wage rate be equal at all points of production, the assumption of full employment and exponential total population growth,

<sup>3</sup> It is easy to check that the second order condition is satisfied

<sup>4</sup> This assumption is only a slight variation of that frequently used in location theory. See, for example, Hotelling (1929)

and by an appropriate specification of the exogenous variables. Thus we assume

$$w/P = \bar{w}/\bar{P}, \tag{12}$$

$$E = KN, \tag{13}$$

and

$$\bar{E} = \eta \bar{N}, \tag{14}$$

where  $\eta$  is the labor force participation,  $N$  and  $\bar{N}$  are the local and national population respectively. Finally, population grows exponentially, i.e.

$$N + \bar{N} = n_0 e^{\alpha t}. \tag{15}$$

The six-equation system given by Eqs.(1), (3), (9), (11)~(13) is a complete regional model. That is, its solution yields the local variables  $X, P, w, E, K$  and  $N$  in terms of the national ones. The national sector [Eqs.(2), (7), (8) and (14)] does not determine all national variables; instead, two of them must be exogenous. It is assumed that  $\bar{P}$  and  $\bar{r}/\bar{P}$  are exogenous. The former is determined by monetary policy (the model determines only relative national prices), while the latter is determined internationally (i.e., the supply of capital to the local and the nation is perfectly elastic at the international real capital rental value). For the purpose of describing equilibrium at a point in time,  $\bar{N}$  is also exogenous, but relation (15) provides the equation of motion for the interregional system. That is, population growth shifts the equilibrium (recall that the supply of the other input, capital is perfectly elastic) over time.

The national equilibrium at a point in time is

$$\bar{E} = \eta \bar{N}, \tag{16}$$

$$\bar{w}/\bar{P} = [\alpha_1 \alpha_2^{\alpha_2} (1 - \alpha_2)^{1 - \alpha_2}]^A, \quad A = \frac{1}{\alpha_2} (r/P)^{-(1 - \alpha_2)/\alpha_2}, \tag{17}$$

$$\bar{K} = K [\alpha_1 (1 - \alpha_2)]^{\alpha_2^{-1}} (r/P)^{-\alpha_2^{-1}} \bar{N}, \tag{18}$$

$$X = \eta \alpha_1^{\alpha_2^{-1}} (1 - \alpha_2)^{(1 - \alpha_2)/\alpha_2} (\bar{r}/\bar{P})^{-(1 - \alpha_2)/\alpha_2} \bar{N}. \tag{19}$$

The regional model of the local economy can be obtained as follows. Eqs.(3), (8) and (12) yield

$$P/\bar{P} = K^{-u/(1 - \alpha_2)}. \tag{20}$$

While relations (7), (9) and (11) imply that

$$K = (\bar{P}/P - 1)\bar{K}/(1 + c - \bar{P}/P). \tag{21}$$

Eqs.(20) and (21) define (implicitly)  $P$  and  $K$  as function of the national variables  $\bar{P}$  and  $\bar{K}$ . Eq.(12) yields

$$w = (P/\bar{P}) \cdot \bar{w}. \tag{22}$$

and, using relation (9) gives

$$E = \frac{\alpha_2}{1 - \alpha_2} \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right) \frac{\bar{P} \bar{r}}{P \bar{w}} \bar{K}. \tag{23}$$

Finally, the regional model requires that

$$N = \frac{1}{\eta} \frac{\alpha_2}{1 - \alpha_2} \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right) \frac{\bar{P} \bar{r}}{P \bar{w}} \bar{K}, \tag{24}$$

and

$$X = \alpha_1 \left( \frac{\alpha_2}{1 - \alpha_2} \right)^{\alpha_2} \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right)^{\alpha_2 + \alpha_3 K \alpha_2 + \alpha_3} \left( \frac{\bar{P} \bar{r}}{P \bar{w}} \right)^{\alpha_2}. \tag{25}$$

Equilibrium (at a point in time) for the interregional system is obtained by substituting the national one into the regional. This yields the following functions of the exogenous variables  $\bar{P}$ ,  $\bar{r}/\bar{P}$  and  $\bar{N}$ , where  $f(\bar{P}, \bar{K})$  denotes the solution for  $P$  obtained from Eqs.(20) and (21):

$$P = f(\bar{P}, \eta [\alpha_1 (1 - \alpha_2)]^{\alpha_2^{-1}} (\bar{r}/\bar{P})^{-\alpha_2^{-1}} \bar{N}), \tag{26}$$

$$K = \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right) \eta [\alpha_1 (1 - \alpha_2)]^{\alpha_2^{-1}} (\bar{r}/\bar{P})^{-\alpha_2^{-1}} \bar{N}, \tag{27}$$

$$w = P [\alpha_1 \alpha_2^{\alpha_2} (1 - \alpha_2)^{1 - \alpha_2}]^{\alpha_2^{-1}} (\bar{r}/\bar{P})^{-(1 - \alpha_2)/\alpha_2}, \tag{28}$$

$$E = \eta \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right) \frac{\bar{P}}{P} \bar{N}, \tag{29}$$

$$N = \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right) \frac{\bar{P}}{P} \bar{N}, \tag{30}$$

and

$$X = K^{\alpha_2 + \alpha_3} \alpha_1^{(\alpha_2 + \alpha_3)/\alpha_2} \alpha_2^{\alpha_2} (1 - \alpha_2)^{-\alpha_2 + \alpha_3/\alpha_2} \cdot \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right)^{\alpha_2 + \alpha_3} (\bar{r}/\bar{P})^{\alpha_2 - \alpha_3/\alpha_2} (\bar{P}/P) \bar{N}^{-\alpha_3}. \tag{31}$$

Eqs.(16)~(19) and (26)~(31) describe the inter-

regional equilibrium at a point in time.

Finally, the moving equilibrium can be derived by using assumption (15) and relation (30) to obtain

$$\bar{N} = n_0 \left( 1 + \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \frac{\bar{P}}{P} \right)^{-1} e^{n_1 t} \quad (32)$$

and substituting this result into temporal interregional equilibrium.

## PRELIMINARY EMPIRICAL RESULTS

A detailed econometric study of the above model is beyond the scope of this paper. The purpose of this section is to report on a preliminary examination of the two equations that constitute the core of the model's reduced form, i.e., equations specifying that the local population and employment, relative to that in the nation (including the local) increase as the cost of capital decreases and increases (slowly) over time.

To obtain these two equations it is convenient to use a log-linear Taylor approximation of two functions. These are:

$$\ln \left( \frac{\bar{P}/P - 1}{1 + c - \bar{P}/P} \right) \approx \ln c_0 + c_1 \ln(\bar{P}/P),$$

$$\ln(N + \bar{N}) \approx \ln \lambda_0 + (1 - \lambda_1) \ln \bar{N} + \lambda_1 \ln N,$$

where

$$\ln c_0 = \ln \left( \frac{d_0}{1 - d_0} \right) - \left[ \frac{1 + cd_0}{cd_0(1 - d_0)} \right] \ln(1 + cd_0),$$

$$\ln \lambda_0 = \ln \left( \frac{N_0 + \bar{N}_0}{\bar{N}_0} \right) - \frac{\bar{N}_0}{N_0 + \bar{N}_0} \ln(1 + cd_0),$$

$$c_1 = (1 + cd_0) / [(cd_0)(1 - d_0)],$$

$$\lambda_1 = N_0 / (N_0 + \bar{N}_0),$$

$d_0$ ,  $N_0$  and  $\bar{N}_0$  are appropriately chosen points around which the approximation is taken.

These approximations convert the model into a system of log-linear equations and the reduced form equations are then easily obtained. The reduced form equations that are of particular interest here are:

$$\begin{aligned} \ln [N_0 / (N_0 + \bar{N}_0)] &= \beta_0 + \beta_1 t + \beta_2 \ln(\bar{r}/\bar{P}) \\ &= \ln[E/(E + \bar{E})], \end{aligned}$$

where

$$\begin{aligned} \beta_0 &= \frac{1 - \alpha_2 + u}{1 - \alpha_2 + u - (1 - \lambda_1)(1 + c_1)u} [\ln(n_0/\lambda_0) + (1 - \lambda_1) \ln c_0] \\ &\quad - \ln n_0 + \frac{(1 - \lambda_1)(1 + c_1)u}{1 - \alpha_2 + u - (1 - \lambda_1)(1 + c_1)u} \ln[\eta \alpha_1^{\alpha_2^{-1}} (1 - \alpha_2)^{\alpha_2^{-1}}], \\ \beta_1 &= \frac{(1 - \lambda_1)(1 + c_1)u}{1 - \alpha_2 + u - (1 - \lambda_1)(1 + c_1)u} n_1, \\ \beta_2 &= \frac{(\lambda_1 - 1)(1 + c_1)u}{[1 - \alpha_2 + u - (1 - \lambda_1)(1 + c_1)u] \alpha_2}. \end{aligned}$$

Notice first that if  $u=0$ , i.e., the region has CRS, and then these equations collapse to the Borts-Stein-Muth neoclassical model in which  $N/(N + \bar{N})$  and  $E/(E + \bar{E})$  are constant. Then, with appropriate specification of adjustment mechanisms the Guccione and Gillen models are special case of the one formulated here.

Notice second that, except for implausibly large  $\alpha_3$  relative to  $1 - \alpha_2$ , the quantity  $1 - \alpha_2 + u - (1 - \lambda_1)(1 + c_1)u$  will be positive. Hence we restrict  $1 - \alpha_2 + u - (1 - \lambda_1)(1 + c_1)u$  to be positive. With this constraint we have the restrictions  $\beta_1 > 0$ ,  $\beta_2 < 0$ . That is, the region's share of total population and employment increases over time and increases as the rental value of capital decreases. The former implication captures the movement of the static equilibrium as the population increases. The latter restriction arises from the increase in the static equilibrium's capital stock (when the cost of capital declines) combined with the assumption that the region's IRS is caused by capital agglomeration.

Notice finally that from a statistical perspective the model is a touch "suicidal". The restrictions are  $\beta_1 > 0$ ,  $\beta_2 < 0$ , but if  $|\beta_1|$  or  $|\beta_2|$  are very large then the region will quickly dominate the nation—a result that is inconsistent with casual observation and with (the resulting) assumption that the extreme equilibrium solution is not observable in the existing data. Hence statistically we are faced with the problem of distinguishing between small variations of  $\beta_1$  and  $\beta_2$  from zero (i.e., the IRS case) and zero values of both parameters (i.e., the Borts-Stein-Muth case). Given this fact together with the well known difficulties of measuring the rental value of capital, and the compactness of the model, the preliminary empirical results are encouraging (although not sterling).

The model was estimated using data for the Ontario and Canada economies from 1947 to 2000, where Ontario is defined to be “the local” and Canada to be “the nation”. Employment data obtained from Statistics Canada, Annual Review of Employment and Payrolls Population data were obtained from Statistics Canada, Canadian Economic Observer. The exogenous variable  $\bar{r}/\bar{P}$  is measured by a 12-month average of the industrial composite common stock yields obtained from the Historical Statistics of Canada (Statistics Canada, 1991-1992) and the Bank of Canada Review (Bank of Canada, 2001). The resulting O. L. S. estimates, with  $t$ -ratios in brackets, are

$$\begin{aligned} & \ln[N/(N + \bar{N})] \\ &= -1.00860 + 0.0023469t - 0.016645 \ln(\bar{r}/\bar{P}), \\ & \quad (20.377) \quad (17.080) \quad (2.065) \\ & \quad R^2=0.9059, DW=0.28156. \end{aligned}$$

$$\begin{aligned} & \ln[E/(E + \bar{E})] \\ &= -0.86884 + 0.0018075t - 0.024466 \ln(\bar{r}/\bar{P}), \\ & \quad (17.788) \quad (13.330) \quad (3.075) \\ & \quad R^2=0.8679, DW=0.72045. \end{aligned}$$

The positive coefficients of time  $t$  and the negative coefficients of  $\ln(\bar{r}/\bar{P})$  fit the restrictions well. Comparing these two estimated equations, the estimated value of  $\beta_0$  of the population equation is 1.16 times of that of the employment equation, and the estimated values of  $\beta_1$  and  $\beta_2$  are 1.298 times and 0.68 time separately. Clearly it may be required to find some way to measure the degree of these difference and similarity. All the coefficients appear to be significant at the 10% level. The coefficients of determination are reasonable, and the appropriate  $F$  statistics are large enough to indicate the model is structurally stable. However the low Durbin-Watson statistics suggest the presence of serial correlation. Thus we follow Guccione and Gillen and introduce an adjustment mechanism of the form

$$\begin{aligned} & \ln\left(\frac{N_t}{N_t + \bar{N}_t}\right) - \ln\left(\frac{N_{t-1}}{N_{t-1} + \bar{N}_{t-1}}\right) \\ &= \gamma \left[ \ln\left(\frac{N_t^*}{N_t^* + \bar{N}_t^*}\right) - \ln\left(\frac{N_{t-1}}{N_{t-1} + \bar{N}_{t-1}}\right) \right], \end{aligned}$$

where  $0 < \gamma < 1$ , superscript \* denotes the model’s prediction. The resulting equation has the form

$$\ln\left(\frac{N}{N + \bar{N}}\right) = \gamma_0 + \gamma_1 t + \gamma_2 \ln\left(\frac{\bar{r}}{\bar{P}}\right) + \gamma_3 \ln\left(\frac{N_{-1}}{N_{-1} + \bar{N}_{-1}}\right),$$

where  $\gamma_1 > 0$ ,  $\gamma_2 < 0$  and  $0 < \gamma_3 < 1$ , the subscript  $-1$  indicates a one-period lag. The equation for  $\ln[E/(E + \bar{E})]$  has the same form but, since the speeds of adjustment of employment and population may vary, the magnitudes of the coefficients may differ. The regression results for these adjusted equations are

$$\begin{aligned} \ln[N/(N + \bar{N})] &= 0.090386 + 0.00029002t \\ & - 0.0010555 \ln(\bar{r}/\bar{P}) + 0.85913 \ln\left[\frac{N}{(N_{-1} + \bar{N}_{-1})}\right], \\ & \quad (1.263) \quad (1.767) \quad (2.986) \quad (13.454) \\ & \quad R^2=0.9826, DW=1.6536. \\ \ln[E/(E + \bar{E})] &= -0.86884 + 0.0018075t \\ & - 0.024466 \ln(\bar{r}/\bar{P}) + 0.73698 \ln\left[\frac{E}{(E_{-1} + \bar{E}_{-1})}\right], \\ & \quad (2.455) \quad (1.987) \quad (0.743) \quad (6.977) \\ & \quad R^2=0.9396, DW=1.8781. \end{aligned}$$

The coefficients on the term  $\ln(\bar{r}/\bar{P})$  of the employment equation and the constant of the population equation are insignificant at the 10% level, but all remaining coefficients are significant at the 10% level. The signs of the coefficients also satisfy the restrictions.

Since the Durbin Watson test is not valid when there is a lagged dependent variable in the equations, Durbin (1970)’s  $h$ -statistics were tried. The test is carried out by referring  $h=(1-DW/2)[T/(1-TS^2)]^{1/2}$ , where  $S^2$  is the estimated variance of the least squares regression coefficient on the one-period lagged dependent variable and  $T$  is the number of observations, to standard normal tables. Large values of  $h$  lead to rejection of the non-autocorrelation hypothesis. The  $h$ -statistics for the population and employment equations are 1.28583 and 0.57925 respectively, which indicate that there is no autocorrelation problem. Thus after the introduction of an adjustment mechanism, the preliminary empirical results lend mild support to the model developed in the paper.

## CONCLUSION

A one-sector, two-region model in which one region exhibits IRS has been presented in this paper. The development of the local IRS economy is shown to be constrained to a “moving equilibria” path. The preliminary empirical results are sufficiently supportive of the argument to encourage further research along the lines of the model. In particular, the neoclassical model does not predict negative coefficients on the real rental value of capital in regressions explaining population or employment relative to that in the nation. Further, although the estimates of the coefficients on time are not inconsistent with the neoclassical system (at the 5% level the null hypothesis cannot be rejected), their  $\rho$ -values are large enough to raise some doubt. This is especially true since the signs of the estimated coefficients are positive in both equations. This suggests that data covering a longer span of time, and/or in which the region is defined to be a large metropolitan area, may indicate that the CRS model is appropriate only as a first approximation. In other words, the short-run/long-run C-model may be improved by the introduction of a secular drift designed to capture the effects of IRS.

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