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On the hydrodynamic stability of a particle-laden flow in growing flat plate boundary layer*

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Abstract: The parabolized stability equation (PSE) was derived to study the linear stability of particle-laden flow in growing Blasius boundary layer. The stability characteristics for various Stokes numbers and particle concentrations were analyzed after solving the equation numerically using the perturbation method and finite difference. The inclusion of the nonparallel terms produces a reduction in the values of the critical Reynolds number compared with the parallel flow. There is a critical value for the effect of Stokes number, and the critical Stokes number being about unit, and the most efficient instability suppression takes place when Stokes number is of order 10. But the presence of the nonparallel terms does not affect the role of the particles in gas. That is, the addition of fine particles (Stokes number is much smaller than 1) reduces the critical Reynolds number while the addition of coarse particles (Stokes number is much larger than 1) enhances it. Qualitatively the effect of nonparallel mean flow is the same as that for the case of plane parallel flows.

Key words: Hydrodynamic stability, Blasius boundary layer, Particle-laden nonparallel flow, Numerical simulation
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INTRODUCTION

Turbulent particulate flows occur in many industrial applications, such as in pneumatic transport of particulates, cyclone separators and chemical reactors. The particles suspended in fluid play a role in the turbulence modulation, which has been known for several years. The observation that adding dust to air flowing in turbulent motion through a pipe can appreciably reduce the resistance coefficient was reported by Sproull (1961). The observation can be expressed as saying that the pressure difference required to maintain a given volume rate of flow is reduced by the addition of dust. Torobin and Gauvin (1961) reported that the wall drag in pipes as well as rates of heat transfer and chemical reaction are changed by particles through modifying the fluid turbulence. Since then, the issue of whether fluid turbulence is enhanced or reduced by the particles has been an important subject in the research of turbulent particulate flows.

The issue of turbulence modulation is related to the question of the stability and evolution of two-phase laminar flows. Saffman (1962) presented the first analytical formulation on this subject. He derived a modified Orr-Sommerfeld equation, under the assumptions of dilute monodisperse suspension of particles and uniform initial particle concentration. The momentum coupling is handled by a force term proportional to the local interphase velocity slip and particle concentration. The presence of particles introduces two additional parameters to the stability problem, namely, the bulk mass loading and the particle Stokes number. Saffman concluded that the ad-

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dition of fine particles tends to destabilize the flow, while the addition of coarse particles stabilizes the flow. Michael (1965) investigated the Kelvin Helmholtz instability of an inviscid dusty gas by applying a step mean flow velocity profile and showed that the presence of the particles always stabilizes the flow.

With the development of modern computers, direct numerical simulations provide an alternative tool for studying turbulence modulation by particles. The development in the numerical investigation of transition to turbulence in wall bounded and free shear flows during the past decade was reviewed by Rempfer (2003). Squires and Eaton (1990) reported that the particles enhanced the turbulence kinetic energy at high wave numbers while decreasing the turbulent kinetic energy at low wave numbers in the forced isotopic, stationary turbulent flow. Isakov and Rudnyak (1995) investigated the neutral stability curves of a dusty channel flow, showing similar results given by Saffman (1962). Dimas and Kiger (1998) studied numerically the linear, inviscid, spatial instability of a mixing layer uniformly laden with a dilute concentration of heavy particles. The behavior of the linear instability depends on two dimensionless parameters: the inverse Stokes number and mass loading. The fully coupled character of the instability reveals three important aspects of the particle effect on the flow structure. Tong and Wang (1999) solved the modified Orr-Sommerfeld equation of dust-laden mixing layer, with the results following the asymptotic relations proposed by Saffman (1962). Wan *et al.*(2005) analyzed the instability in the Taylor-Couette flow of fiber suspensions with respect to the non-axisymmetric disturbances. The generalized eigenvalue equation governing the hydrodynamic stability of the system was solved using a direct numerical procedure. The results showed that the fiber additives can suppress the instability of the flow.

The boundary layer represents a typical shear flow and thus provides a building block for many practical inhomogeneous flows. For the flows near wall, the turbulence is generated primarily by velocity gradient. The neutral stability curve for the 2D laminar boundary layer on a flat plate under zero pressure gradient was calculated by Kurtz and Crandall (1962), Jordinson (1970), Grosch and Orszag (1977), Zebib (1984), and these results, obtained by slightly different methods, sufficiently justify the view that the neutral curve eigenvalues of the Orr-Sommerfeld equation for this flow are well established. Barry and Ross (1970) found that the theoretical and experimental results are in close agreement when the Reynolds numbers are larger than 1000, but are different when the Reynolds number is lower. The possible reasons may result from the experiment errors and the approximation of parallel mean flow. Their results showed that the boundary layer flow is slightly less stable when extra nonparallel terms are included. In growing boundary layer, Gaster (1974)'s result showed that the approximation of parallel mean flow leads to a valid solution at very large Reynolds numbers. Fasel and Konzelmann (1990) investigated nonparallel effects in the growing boundary layer by direct numerical simulation of the complete Navier-Stokes equations for incompressible flows, with their results clearly indicating that the nonparallel effects are the strongest in the area closer to the wall inside the boundary layer and decrease with increasing distance from the wall. Bertolotti *et al.*(1992) studied the linear and nonlinear instability with parabolic stability equation (PSE) and Navier-Stokes equation in growing boundary layer, and the effect of nonparallelism is confirmed to be weak and not responsible for the discrepancies between measurements and theoretical results for parallel flow. Bhaganagar *et al.*(2002) developed a highly accurate algorithm to study the process of spatial transition to turbulence in a boundary layer. The algorithmic program is based on a formulation in terms of vertical velocity and vertical vorticity in conjunction with parabolization of the Navier-Stokes equations; the validation of results for the approach is made both for linear and weakly nonlinear cases. For the single phase flow, Govindarajan and Narasimha (1999) formulated a lowest order parabolic theory for investigating the stability of spatially developing boundary layer, and derived a minimal composite equation which is shown to give results close to the full nonparallel theory, and is the highest-order stability theory that is justifiable with the lowest-order mean velocity profiles for the boundary layer. Govindarajan and Narasimha (2005) also showed that, to the order in the reciprocal of the local flow Reynolds number, the amplitude ratio of perturbation growth does not depend on the difference in shape between the eigenfunctions of the full non-parallel and the lowest order minimal composite theory.

These approaches based on the assumption of locally parallel or weakly nonparallel basic flow could fail if a wave length of any perturbation is larger than a characteristic length of the spatial inhomogeneity of the base flow. Consequently a more general eigenvalue problem was developed by some authors as Lin and Malik (1997), Theofilis (2003). Ehrenstein and Gaillaire (2005) researched 2D temporal modes in spatially evolving boundary layer flows; the spatial structure of each individual temporally stable mode is shown to be reminiscent of the spatial exponential growth of perturbations along the flat plate, as predicted by local analyses; the spatially localized wave packet is in qualitative agreement with the convectively unstable disturbance. Alizard and Robinet (2006) discovered that a convective stability of a flat plate boundary layer could be captured by a 2D stability analysis. Results gave quite good similarities between the two approaches compared with linear stability analysis.

Up to now there are relatively few studies of two-way coupled, particle-laden Blasius flow. Asmolov and Manuilovich (1998) investigated the stability of a dusty-gas laminar boundary layer on a flat plate, using two approaches: orthonormalization method and perturbation method. The results showed that the dust suppresses the instability waves for a wide range of particle size. The most efficient suppression takes place when the relaxation length of the particle velocity is close to the wavelength of Tollmien-Schlicting (TS) wave.

However, Asmolov and Manuilovich (1998) only studied the stability of particle-laden Blasius flow within the framework of the quasi-parallel and quasi-homogeneous approach and the two limiting cases of coarse and fine particles. In this study, therefore, we will address the stability equation for particle-laden Blasius flow using Saffman's formulation. Numerical simulations were done to study the linear instability of viscous, 2D, nonparallel, particle-laden Blasius boundary layer flow. A finite difference method is used to determine the temporal growth rate of the imposed disturbance.

MATHEMATICAL DESCRIPTION

Governing equation

The effects of particle concentrations on the continuous phase viscosity can be neglected because the bulk concentration of the particle is assumed to be very low. The particles are uniform in size with diameter much smaller than any characteristic length scales in the flow, and the velocity and number density of particles can be described by flow and concentration fields. The particle density is much larger than that of the gas so that the bulk mass loading of the particulate phase is in the order of unity. The particle Reynolds number is in the Stokes flow region so that the formula of linear Stokes drag is used. The governing equations for gas and particles (subscript p) in the Blasius boundary layer are:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) =-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+KN(u_{\mathrm{p}}-u), \quad (1)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) =-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+KN(v_{\mathrm{p}}-v), \quad (2)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (3)$$

$$mN\left(\frac{\partial u_{\mathrm{p}}}{\partial t}+u_{\mathrm{p}}\frac{\partial u_{\mathrm{p}}}{\partial x}+v_{\mathrm{p}}\frac{\partial u_{\mathrm{p}}}{\partial y}\right)=KN(u-u_{\mathrm{p}}), \quad (4)$$

$$mN\left(\frac{\partial v_{\mathrm{p}}}{\partial t}+u_{\mathrm{p}}\frac{\partial v_{\mathrm{p}}}{\partial x}+v_{\mathrm{p}}\frac{\partial v_{\mathrm{p}}}{\partial y}\right)=KN(v-v_{\mathrm{p}}), \quad (5)$$

$$\frac{\partial N}{\partial t}+\frac{\partial (Nu_{\mathrm{p}})}{\partial x}+\frac{\partial (Nv_{\mathrm{p}})}{\partial y}=0, \quad (6)$$

where u, v and u_{p}, v_{p} are the gas and particle velocities, respectively, p is the gas pressure, N is the number density of particle, mN is the mass of particle per unit volume, ρ and μ are the density and viscosity of the clean gas, $K=3\pi d\mu$ with d being the diameter of particles according to the Stokes drag formula.

We consider a steady laminar flow for flat plate boundary layer. For sufficiently small particles, the sedimentation velocity of particle is small compared with the characteristic velocity of the flow and can be neglected. Then in a steady state, the inertia terms in the equations of motion vanish identically and the particles move along the streamlines with the velocity of the gas, i.e. $u_p=u=U$, $v_p=v=V$. The number density N of particles has the constant value N_0 everywhere. The equations are then reduced to the Prandtl form:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0, \tag{7}$$

$$\partial P/\partial y=0, \tag{8}$$

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\mu\frac{\partial^2 U}{\partial y^2}. \tag{9}$$

Following the method of Schlichting (1954), the Prandtl equations are reduced to the Blasius equation:

$$2F'''(\eta)+F(\eta)F''(\eta)=0, \tag{10}$$

where the prime denotes the derivative with η, and the boundary conditions are:

$$\eta=0:\ F(\eta)=0,\ F'(\eta)=0;\quad \eta=\infty:\ F'(\eta)=1. \tag{11}$$

Using the non-dimensional variable $\eta=y\sqrt{U_0/\upsilon x}$ and stream function $\Psi=\sqrt{\upsilon x U_0}F(\eta)$, the numerical integration of Eq.(10) with boundary conditions Eq.(11) leads to the following results:

$$\delta=\sqrt{\upsilon x/U_0}=x\big/\sqrt{Re_x}\ \text{ and }\ Re_x=U_0x/\upsilon,$$

$$\delta_1=\delta\lim[\eta-F(\eta)]=m\delta,\ \text{where}\ m\text{=1.7208},$$

$$Re=U_0\delta/\upsilon=\sqrt{Re_x},\ U=U_0F'(\eta),$$

$$V=\frac{1}{2}\sqrt{\upsilon U_0/x}\ [\eta F'(\eta)-F(\eta)],$$

$$\lim_{\eta\to\infty}V=V_\infty\to\frac{m}{2}\sqrt{\frac{\upsilon U_0}{x}}=\frac{mU_0}{2Re},$$

where δ_1 is the displacement thickness of the boundary layer.

In order to analyze the linear instability of the flow, a 2D perturbation should be introduced to the flow. Therefore, the velocity, pressure and particle number density are represented by the base-state profile plus a small perturbation:

$$u=U+u',\ v=V+v',\ p=P+p', \tag{12a}$$

$$u_\text{p}=U+u_\text{p}',\ v_\text{p}=V+v_\text{p}',\ N=N_0+N', \tag{12b}$$

where U, V are the base-state streamwise and transverse velocities, respectively, P is the base-state pressure. Substituting Eq.(12) into Eqs.(1)~(6) with nonlinear terms of the perturbation neglected, we have the following linear stability equations:

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0, \tag{13}$$

$$\frac{\partial u'}{\partial t}+u'\frac{\partial U}{\partial x}+v'\frac{\partial U}{\partial y}+U\frac{\partial u'}{\partial x}+V\frac{\partial u'}{\partial y}=-\frac{\partial p'}{\partial x}+\upsilon\left(\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}\right)+s(u_\text{p}'-u'), \tag{14}$$

$$\frac{\partial v'}{\partial t}+u'\frac{\partial V}{\partial x}+v'\frac{\partial V}{\partial y}+U\frac{\partial v'}{\partial x}+V\frac{\partial v'}{\partial y}=-\frac{\partial p'}{\partial y}+\upsilon\left(\frac{\partial^2 v'}{\partial x^2}+\frac{\partial^2 v'}{\partial y^2}\right)+s(v_\text{p}'-v'), \tag{15}$$

$$\frac{\partial N'}{\partial t}+\frac{\partial(N'U+N_0u_\text{p}')}{\partial x}+\frac{\partial(N'V+N_0v_\text{p}')}{\partial y}=0, \tag{16}$$

$$\frac{\partial u_\text{p}'}{\partial t}+u_\text{p}'\frac{\partial U}{\partial x}+v_\text{p}'\frac{\partial U}{\partial y}+U\frac{\partial u_\text{p}'}{\partial x}+V\frac{\partial u_\text{p}'}{\partial y}=\frac{u'-u_\text{p}'}{\tau}, \tag{17}$$

$$\frac{\partial v_\text{p}'}{\partial t}+u_\text{p}'\frac{\partial V}{\partial x}+v_\text{p}'\frac{\partial V}{\partial y}+U\frac{\partial v_\text{p}'}{\partial x}+V\frac{\partial v_\text{p}'}{\partial y}=\frac{v'-v_\text{p}'}{\tau}, \tag{18}$$

where $s=KN_0/\rho$ has the dimension of frequency, and $\tau=m/K$ is the relaxation time of the particles, which is defined as:

$$\tau=\rho_\text{p}d^2/(18\upsilon\rho), \tag{19}$$

where ρ_p is the particle density, then the particle concentration f is:

$$f=mN_0/\rho=s\tau. \tag{20}$$

Consider a Fourier component of wave-number α which propagates with complex velocity c along the x-axis. Then, the basic flow has velocities $U=U(y)$, $V=V(y)$, and the small disturbance terms have the form of $e^{i\alpha(x-ct)}$ or $e^{i(\alpha x-\beta t)}$, that is,

$$u'=u'(y)e^{i(\alpha x-\beta t)}=u'(y)e^{i\alpha(x-ct)}, \tag{21a}$$

$$v'=v'(y)e^{i(\alpha x-\beta t)}=v'(y)e^{i\alpha(x-ct)}, \tag{21b}$$

$$p'=p'(y)e^{i(\alpha x-\beta t)}=p'(y)e^{i\alpha(x-ct)}, \tag{21c}$$

$$u'_{\rm p}=u'_{\rm p}(y){\rm e}^{{\rm i}(\alpha x-\beta t)}=u'_{\rm p}(y){\rm e}^{{\rm i}\alpha(x-ct)}, \quad (21d)$$

$$v'_{\rm p}=v'_{\rm p}(y){\rm e}^{{\rm i}(\alpha x-\beta t)}=v'_{\rm p}(y){\rm e}^{{\rm i}\alpha(x-ct)}, \quad (21e)$$

$$N'=N'(y){\rm e}^{{\rm i}(\alpha x-\beta t)}=N'(y){\rm e}^{{\rm i}\alpha(x-ct)}. \quad (21f)$$

Substituting Eq.(21) into Eqs.(13)~(18), we have (For brevity, the prime is dropped henceforth):

$$\begin{aligned}&{\rm i}\alpha(U-c)u+vDU-uDV+VDu\\&\quad=-{\rm i}\alpha p+\upsilon(D^2-\alpha^2)u-s(Du-Du_{\rm p}),\end{aligned} \quad (22)$$

$$\begin{aligned}&{\rm i}\alpha(U-c)v+vDV+VDv\\&\quad=-Dp+\upsilon(D^2-\alpha^2)v-{\rm i}\alpha s(v-v_{\rm p}),\end{aligned} \quad (23)$$

$${\rm i}\alpha u+Dv=0, \quad (24)$$

$${\rm i}\alpha(U-c)u_{\rm p}+v_{\rm p}DU-u_{\rm p}DV+VDu_{\rm p}=(u-u_{\rm p})/\tau, \quad (25)$$

$${\rm i}\alpha(U-c)v_{\rm p}+v_{\rm p}DV+VDv_{\rm p}=(v-v_{\rm p})/\tau, \quad (26)$$

$$-{\rm i}\alpha cN=-{\rm i}\alpha N_0u_{\rm p}-N_0Dv_{\rm p}-{\rm i}\alpha UN-{\rm i}\alpha UN-VDN, \quad (27)$$

where $D\equiv{\rm d}/{\rm d}y$, and the term $-\partial^2U/(\partial x\partial y)$ has been replaced by $\partial^2V/\partial y^2$ based on the continuity equation. Eq.(24) is satisfied by a stream function ψ, which has the periodic form:

$$\psi=\phi(y){\rm e}^{{\rm i}(\alpha x-\beta t)}, \quad (28)$$

then, we have

$$u=\partial\psi/\partial y, \quad (29)$$

$$v=-\partial\psi/\partial x. \quad (30)$$

For numerical analysis, the resulting equation is changed to a non-dimensional form similar to the Orr-Sommerfeld equation, using the characteristic parameters U_0, δ and υ. The equation to be integrated is then

$$\begin{aligned}&{\rm i}\alpha(U-c)(D^2-\alpha^2)\phi+VD(D^2-\alpha^2)\phi-D^2VD\phi-{\rm i}\alpha\phi D^2U\\&=\frac{1}{Re}(D^2-\alpha^2)^2\phi+\frac{f}{St}[(Du_{\rm p}-i\alpha v_{\rm p})-(D^2-\alpha^2)\phi],\end{aligned} \quad (31)$$

$${\rm i}\alpha(U-c)u_{\rm p}+v_{\rm p}DU-u_{\rm p}DV+VDu_{\rm p}=(D\phi-u_{\rm p})/St, \quad (32)$$

$${\rm i}\alpha(U-c)v_{\rm p}+v_{\rm p}DV+VDv_{\rm p}=(-{\rm i}\alpha\phi-v_{\rm p})/St, \quad (33)$$

$$-{\rm i}\alpha cN=-{\rm i}\alpha N_0u_{\rm p}-N_0Dv_{\rm p}-{\rm i}\alpha UN-{\rm i}\alpha UN-VDN. \quad (34)$$

In order to reduce Eqs.(31)~(34) to the Saffman's form, the assumption V=0 is required, with the consequent relation $\partial U/\partial x$=0, implying parallel mean flow in the boundary layer.

$$\frac{(D^2-\alpha^2)^2\phi}{{\rm i}\alpha Re}=(\bar u-c)(D^2-\alpha^2)\phi-(D^2\bar u)\phi, \quad (35)$$

where

$$\bar u(y)=U(y)+\frac{[U(y)-c]f}{1+{\rm i}\alpha St[U(y)-c]}. \quad (36)$$

For the Blasius boundary layer, the boundary conditions express the requirement that the perturbation velocities vanish at y=0 and y=∞, that is, for the wall,

$$\phi=D\phi=u_{\rm p}=v_{\rm p}=0,\ \text{at}\ y=0, \quad (37)$$

and for large values of y, we take the form:

$$\begin{aligned}&{\rm i}(\alpha-\beta+f/St)(D^2-\alpha^2)\phi+V_\infty D(D^2-\alpha^2)\phi\\&\quad=(D^2-\alpha^2)^2\phi/Re.\end{aligned} \quad (38)$$

The solution fitting the outer boundary conditions is

$$\phi=A{\rm e}^{-\alpha y}+B{\rm e}^{-\xi y}, \quad (39)$$

where A and B are arbitrary constants, and

$$\xi^2-\xi ReV_\infty-\gamma^2=0, \quad (40)$$

where

$$\gamma^2=\alpha^2+{\rm i}Re(\alpha-\beta+f/St). \quad (41)$$

Since $ReV_\infty=m/2$, then $|\xi|\sim|\gamma|$, and for $y>0$, $|{\rm e}^{-\xi y}|\sim|{\rm e}^{-\gamma y}|<<|{\rm e}^{-\alpha y}|$. The required outer boundary condition may therefore be expressed in the form of $\phi\sim{\rm e}^{-\alpha y}$ for large y. The presence of V does not affect the form of the boundary conditions, and the calculations were carried out on this assumption. Thus the boundary condition for large values of y is:

$$\phi=D\phi=u_{\rm p}=v_{\rm p}=0,\ \text{at}\ y=\infty. \quad (42)$$

We take y/ξ=20 in the present work.

Numerical procedure

The differential Eqs.(31)~(34) are replaced by a set of difference equations which are referred to as the algebraic model. Define h as the step length used in the numerical integration, then $D^2\phi$ and $D^4\phi$ can be expressed by Eq.(43) and Eq.(44) with the truncation errors being $O(h^4)$ and $O(h^6)$, respectively:

$$(D^2\phi)_j = \frac{\phi_{j+1} - 2\phi_j + \phi_{j-1}}{h^2}, \tag{43}$$

$$(D^4\phi)_j = \frac{\phi_{j+2} - 4\phi_{j+1} + 6\phi_j - 4\phi_{j-1} + \phi_{j-2}}{h^4}. \tag{44}$$

The algebraic model can be expressed in the matrix form. An iterative technique is used to find the eigenvalues of the matrix. The computer program is written in MATLAB to perform the iteration.

RESULTS AND DISCUSIONS

It is well known that the solution of the modified equations with certain boundary conditions possesses an eigenvalues program. In the time amplified case considered here it is assumed that α, St, f and Re are real and given, and the problem is that of finding a complex eigenvalue c with a corresponding eigenvalue ϕ. The result of such a calculation for a prescribed laminar flow can be represented graphically in an α-Re diagram because every point of this plane corresponds to a pair of values of Re(c_r) and Im(c_i). In particular, the locus c_i=0 separates the stable region from that of unstable disturbances. This locus is called the curve of neutral stability. The point on this curve at which the Reynolds number has its smallest value is of greatest interest since it indicates that value of the Reynolds number below which all individual oscillations decay, whereas above that value at least some are amplified. This smallest Reynolds number is the critical Reynolds number (Re_{crit}) or limit of stability with respect to the type of laminar flow under consideration.

Verification of the numerical method

In order to verify the code of the computation program used in the present study, the ordinary Orr-Sommerfeld equation for the Blasius boundary layer is solved.

Table 1 shows the comparisons of eigenvalues using various methods for the case of Re=580, α=0.179. The eigenvalues are given for the single unstable mode (c_i>0). The comparison of the eigenvalues in Table 1 indicates that the method adopted here is accurate.

Table 1 Eigenvalues of the Orr-Sommerfeld equation for Blasius flow, *Re*=580, *α*=0.179

Methods	Eigenvalues
Present	0.36455+0.0077793i
Kurtz and Crandall (1962)	0.364+0.0077i
Jordinson (1970)	0.3641+0.0079i
Grosch and Orszag (1977)	0.364557+0.007773i
Zebib (1984)	0.364143+0.007959i

Effect of nonparallel flow

Fig.1 shows the neutral stability curves of parallel and nonparallel particle-laden flow. It can be seen that the presence of the V terms does not affect the role of the particles in gas. That is, the addition of fine particles (Stokes number is much smaller than 1) reduces the critical Reynolds number while the addition of coarse particles (Stokes number is much larger than 1) enhances it. Qualitatively the effect of particles is the same as that for the case of parallel flow. For fine particles, the larger the concentration is, the lower the critical Reynolds number is. For coarse particles, the result is the reverse.

The effect of particle concentration given in Fig.2 shows that the inclusion of the V terms produces a reduction in the values of the critical Reynolds number.

Fig.3 shows the effect of Stokes numbers for parallel and nonparallel flows. The value of critical Reynolds number of nonparallel flow is larger than that of the parallel flow at the same concentration and Stokes numbers. It also can be seen that there is a critical value for the effect of Stokes number, and that the critical Stokes number is about unity, and that the most efficient instability suppression takes place when Stokes number is of order 10. Fig.4 shows the comparison of neutral stability curves in Re-α plane between parallel and nonparallel flows. The new neutral stability curve therefore lies outside the curve obtained by Jordinson (1970) and Asmolov and Manuilovich (1998), and the critical Reynolds numbers is lower than that given by them.

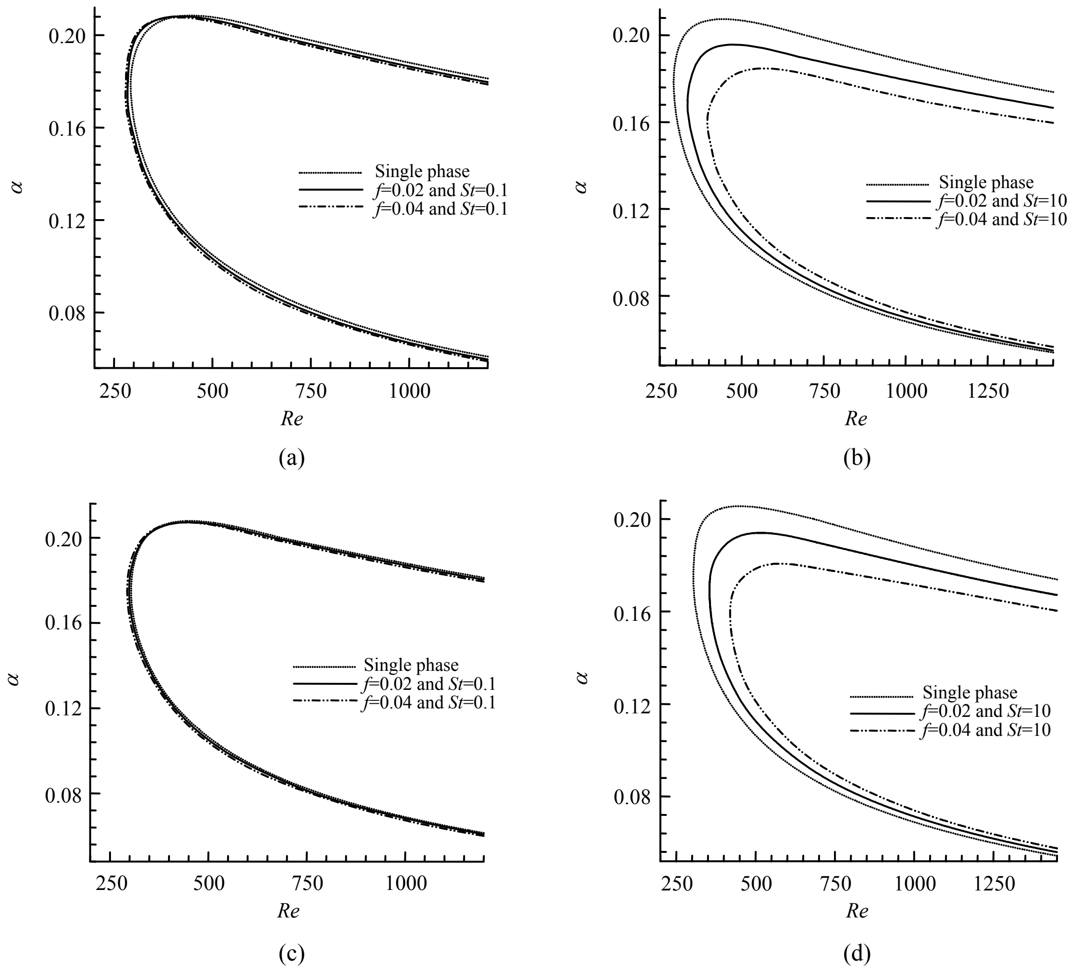

Fig.1 The neutral stability curves of Blasius flow. (a) Nonparallel flow and *St*=0.1; (b) Nonparallel flow and *St*=10; (c) Parallel flow and *St*=0.1; (d) Parallel flow and *St*=10

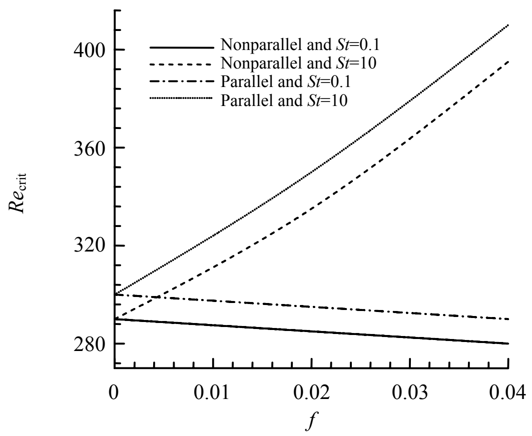

Fig.2 Relationship between particle concentration and critical Reynolds number

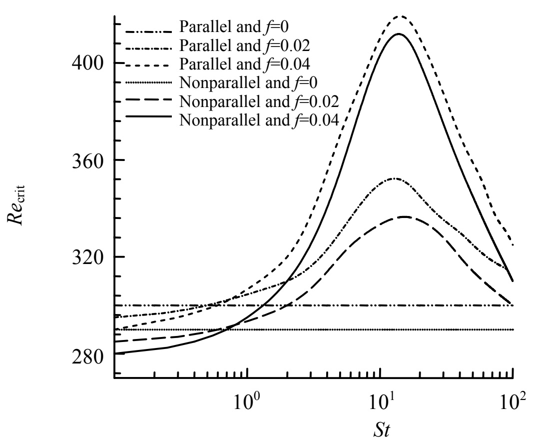

Fig.3 Relationship between Stokes number and critical Reynolds number

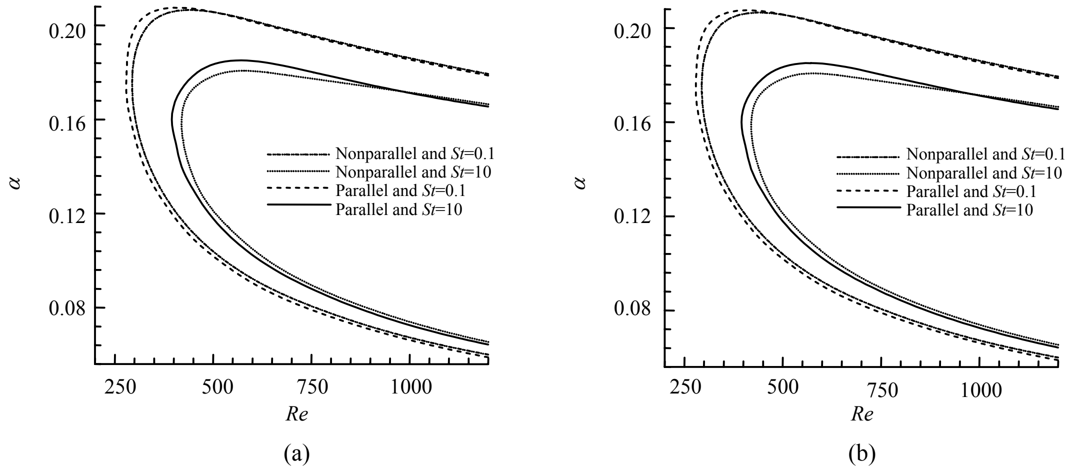

Fig.4 Comparison of neutral stability curves of Blasius flow between parallel and nonparallel flow for different concentration. (a) f=0.02; (b) f=0.04

Distribution of stress resulting from perturbation

The Reynolds stress, as a main interest quantity, is $-\rho\overline{uv}$ which interacts with the mean velocity gradient dU/dy to increase or decrease the energy of the perturbation. The perturbation grows in amplitude if $-\rho\overline{uv}$ and dU/dy are of the same sign over a dominant part of the flow. The Reynolds stress can be calculated by:

$$-\rho\overline{uv} = \frac{1}{T}\int_0^T uv\mathrm{d}t, \tag{45}$$

where u, v are the instantaneous components of the perturbation velocity, which are parallel and perpendicular to the plate, respectively, and T=2π/β is the period of one oscillation.

Taking u, v as the form defined in Eqs.(29) and (30) and substituting them into the right hand side of Eq.(45) yields:

$$-\rho\overline{uv} = \mathrm{i}\alpha(\phi'\phi^* - \phi^{*\prime}\phi), \tag{46}$$

where ϕ^* is the conjugation of the eigenfunctions ϕ, and the primes indicate differential with respect to y. This function is plotted in Fig.5. The curves show how the Reynolds stress distribution varies with y. All distributions show that the energy transfer is virtually restricted to the total thickness of the boundary layer. Each distribution shows a peak close to the critical layer. In general, the amplitude of stress of nonparallel flow is larger than that of parallel flow at the same conditions, which means that the interacting effect of the mean velocity gradient with the energy of the perturbation for nonparallel flow is larger than that for parallel flow.

Table 2 shows the comparison of eigenvalues of unstable mode for parallel and nonparallel flow. We can find that all the image parts of eigenvalues of unstable mode of nonparallel flow are larger than those of parallel flow, which means that the energy of perturbation grows more rapidly for nonparallel flow, that is, nonparallel flow is more unstable.

Table 2 Eigenvalues of unstable mode of parallel and nonparallel flows at Re=580, α=0.179

Parameters	Eigenvalues	
	Parallel	Nonparallel
f=0	0.3645+0.007779i	0.3662+0.008128i
f=0.02, St=0.1	0.3626+0.007686i	0.3653+0.008078i
f=0.02, St=10	0.3637+0.004065i	0.3654+0.004403i
f=0.04, St=0.1	0.3636+0.007737i	0.3644+0.008018i
f=0.04, St=10	0.3628+0.000398i	0.3646+0.000724i

CONCLUSION

The PSE was derived to study the linear stability of particle-laden flow in growing Blasius boundary layer. The stability characteristics for various Stokes numbers and particle concentrations were analyzed after solving the equation numerically using pertur-

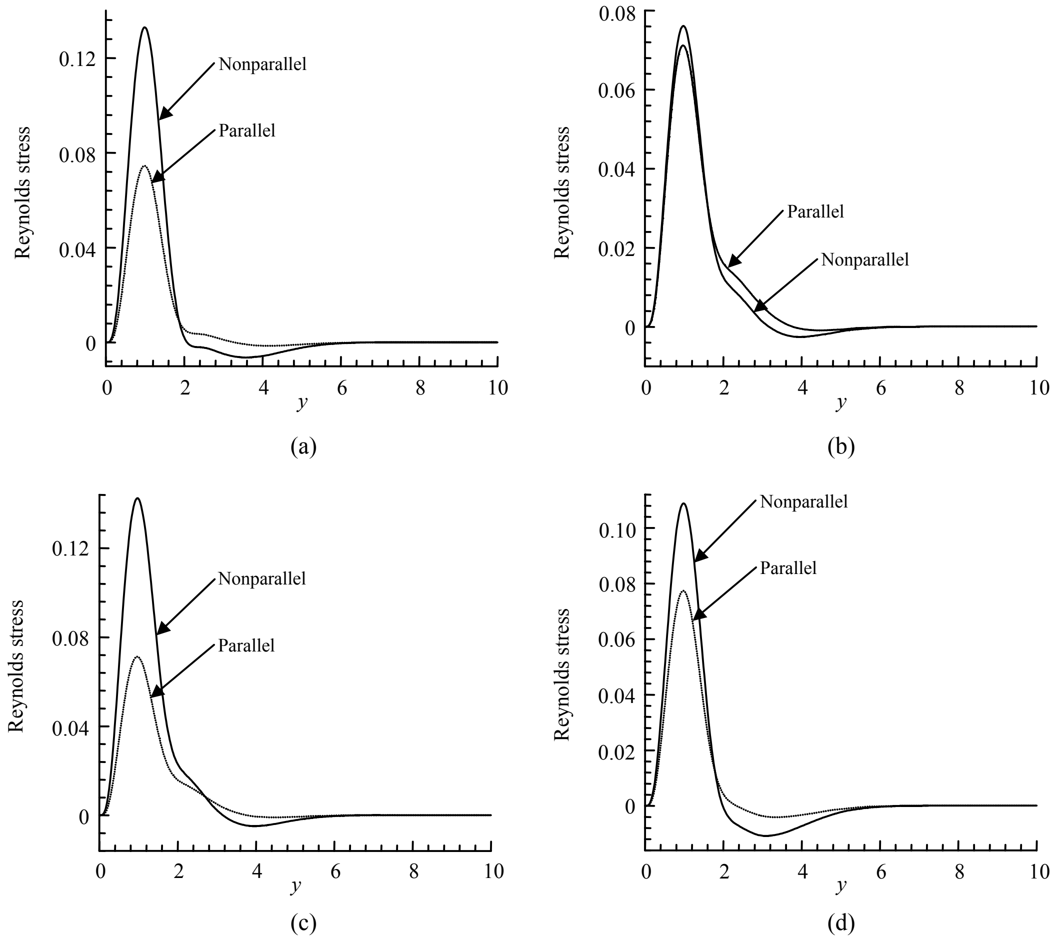

Fig.5 Distribution of Reynolds stress at various conditions for *Re*=580, *α*=0.179. (a) *f*=0.02, *St*=10; (b) *f*=0.02, *St*=0.1; (c) *f*=0.04, *St*=0.1; (d) *f*=0.04, *St*=10

bation method and finite difference. The results in the clean gas flow agree with the calculations given by other authors. Two general asymptotic relations proposed by Saffman have been confirmed numerically for the flow of growing Blasius boundary layer. The addition of fine particles reduces the critical Reynolds number while the addition of coarse particles enhances it. The stabilizing and destabilizing effect of particles depends monotonously on the particle concentration, but not monotonously on the Stokes number. In addition to the stabilizing effect of particles on the gas flow at large Stokes numbers, the higher the concentration, the larger the critical Reynolds number is, and the results are the reverse for the case at small Stokes numbers. There exists an intermediate Stokes number at which the flow is most stable. We have shown that this Stokes number is in the order of 10. The inclusion of the nonparallel terms reduces the values of the critical Reynolds number compared with the parallel flow. But the presence of the nonparallel terms does not affect the role of the particles in gas. Qualitatively the effect of nonparallel mean flow is the same as that for the case of plane parallel flows.

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