



LS-SVM model based nonlinear predictive control for MCFC system*

CHEN Yue-hua[†], CAO Guang-yi, ZHU Xin-jian

(Institute of Fuel Cell, Department of Automation, Shanghai Jiao Tong University, Shanghai 200030, China)

[†]E-mail: chenqi78@sjtu.edu.cn

Received Mar. 23, 2006; revision accepted Oct. 21, 2006

Abstract: This paper describes a nonlinear model predictive controller for regulating a molten carbonate fuel cell (MCFC). In order to improve MCFC's generating performance, prolong its life and guarantee safety, it must be controlled efficiently. First, the output voltage of an MCFC stack is identified by a least squares support vector machine (LS-SVM) method with radial basis function (RBF) kernel so as to implement nonlinear predictive control. And then, the optimal control sequences are obtained by applying genetic algorithm (GA). The model and controller have been realized in the MATLAB environment. Simulation results indicated that the proposed controller exhibits satisfying control effect.

Key words: Molten carbonate fuel cell (MCFC), Least squares support vector machine (LS-SVM), Genetic algorithm (GA), Nonlinear predictive controller

doi:10.1631/jzus.2007.A0748

Document code: A

CLC number: TP273.3; TP183

INTRODUCTION

According to analyses of molten carbonate fuel cells (MCFCs) system, the dynamics of MCFC system is nonlinear with multi-input and multi-output as well as multiple recycling gas flow loops, multiple phase flows and complex chemical and electrochemical reactions. It is very difficult to model and control MCFC system. Performance and availability of MCFC are greatly dependent on its output voltage, which is crucial for improving MCFC performance and life so that the output voltage is controlled in an appropriate range.

The MCFC model must be established in order that an object can be controlled efficiently. A detailed 1D mechanism model of an MCFC system is presented in (Yoshida *et al.*, 2004). Cell voltage distribution can be calculated by this method. However, it takes much time to solve these complex mechanism equations. Neural networks are used to model nonlinear systems in (Shen *et al.*, 2002; Wang *et al.*,

2006). Least squares support vector machine (LS-SVM) method is used to model the operating temperature of a PEMFC (proton exchange membrane fuel cell) stack in (Li *et al.*, 2006) and obtain satisfying effect. The same method is utilized by Vong *et al.*(2006) to predict the automotive engine power and torque. Compared with neural networks, LS-SVM method can offer some advantages. Therefore, in this paper, an LS-SVM model is established to predict the output voltage of an MCFC plant. Nonlinear predictive controllers are proposed by many researchers. Nonlinear predictive model based controllers for controlling PEMFCs are proposed in (Golbert and Lewin, 2004); and Zhu (2002) described a nonlinear predictive control algorithm based on neural network predictive model. A fuzzy logic controller is also presented in (Schumacher *et al.*, 2004) to control miniature PEMFCs. In simulation experiments, these controllers worked well. Genetic algorithm (GA) method for optimization problem is described by (Elliott *et al.*, 2005; Belarbi *et al.*, 2005); simulations showed that the GA is a robust, efficient and fast method for optimization problem.

In this paper, a nonlinear predictive control al-

* Project (No. 2003 AA517020) supported by the Hi-Tech Research and Development Program (863) of China

gorithm based on an LS-SVM predictive model is proposed to control the output voltage of an MCFC system. First, the system configuration of the MCFC system is described briefly. Then LS-SVM formulations for nonlinear function estimation are presented. A predictive model of MCFC is obtained using LS-SVM method. Based on this nonlinear predictive model, GA is used to obtain the optimal output of the predictive controller. Numerical experiments showed the effectiveness of the proposed control algorithm and performance comparison between the proposed controller and a traditional fuzzy controller is also presented.

SYSTEM CONFIGURATION OF AN MCFC PLANT

The fuel cell stacks are MCFC type with external reforming and operating at ambient pressure, with the configuration of the whole plant being presented in Fig.1 (Lunghi *et al.*, 2003).

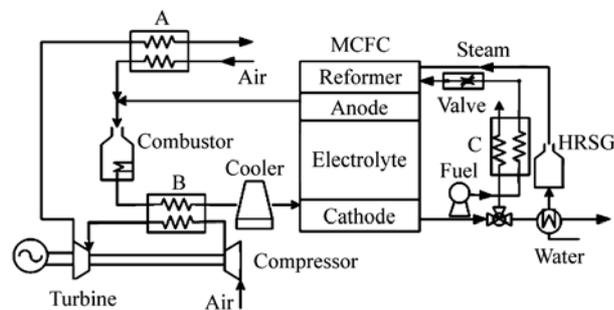
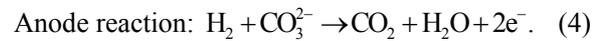
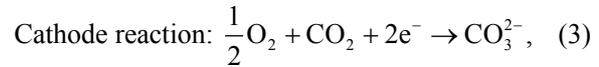
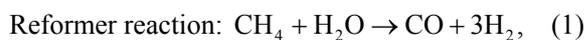


Fig.1 MCFC plant configuration

The fuel used is natural gas. In order to obtain higher cell efficiency, the natural gas is pre-heated using the heat content of the cathodic gas at heat exchanger C. For this purpose a valve is used to split the cathodic gas stream into two parts which are used to produce steam needed for the reforming process in HRSG and pre-heat the fuel. The fuel reacts with the steam provided by the heat recovery steam generator (HRSG) in the reformer. Reactions in MCFC are presented as follows (Yoshida *et al.*, 2004):



The anode exhaust gas exits the stack and the unutilized fuel is burned in a combustor. After combustion and before re-entering the stack, the gas is cooled until the temperature reaches the appropriate value required. The heat released can be utilized by the gas turbine (at heat exchanger B) for the purpose of improving cell efficiency. For the same reason, the exhaust gas from the turbine is sent to the heat exchanger A that increases the temperature of the air supplied to the combustor. The parameters of the MCFC plant are summarized in Table 1 (Ishikawa and Yasue, 2000).

Table 1 The parameters of a 50 kW MCFC plant

Parameters	Values
Current density (mA/cm ²)	50~300
Pressure of the stack (MPa)	0.1
Available area (m ²)	1
Operating temperature (°C)	650
Number of single cells	80
Flux of anode inlet gas (kg/h)	4.2~26
Flux of cathode inlet gas (kg/h)	9~56
Coefficient of fuel utilization	0.7
Coefficient of air utilization	0.3
Components of anode inlet gas	CH ₄ /H ₂ O=1:2
Components of cathode inlet gas	Air/CO ₂ =7:3
Output power of MCFC system (kW)	50
Sampling periodicity (s)	0.2

LS-SVM FORMULATION FOR NONLINEAR FUNCTION ESTIMATION

SVM is one of the methods by which the statistical learning theory can be introduced to nonlinear system identification. Comparing SVM with other nonlinear identification approaches, the advantages of SVM method are presented as follows:

(1) The learning algorithms of traditional nonlinear identification approaches, including neural networks (Shen *et al.*, 2002; Wang *et al.*, 2006), fuzzy modeling, etc., are almost always based on the expectation risk minimization principle. These kinds of algorithms often lead to the problem of over fitting.

That is, less training error may result in poorer generalization performance. Based on statistical learning and the structural risk minimization principle, SVM can give attention to both expectation risk and generalization performance (Li et al., 2006).

(2) SVM formulates the training process as a quadratic programming (QP) problem of minimizing the data fitting error function plus regularization, which produces a global solution instead of many local ones. And another important advantage of SVM over other traditional nonlinear identification methods is its ability to handle high nonlinearity. Similar to nonlinear regression, SVM transforms the low dimensional nonlinear input data space into high-dimensional linear feature space through a nonlinear mapping: $\varphi(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^{n_k}$, m is the dimension of input data space, and n_k is the dimension of the unknown feature space (which can be infinite dimensional). Then linear function estimation over the feature space can be performed (Vong et al., 2006).

On the basis of classical SVM, Suykens and Vandewalle (2000) presented LS-SVM approach, in which the following function is used to approximate the unknown function:

$$y(x) = w^T \varphi(x) + b, \tag{5}$$

where $x \in \mathbb{R}^m$ are the input data, $y \in \mathbb{R}$ are the output data, $\varphi(\cdot): \mathbb{R}^m \rightarrow \mathbb{R}^{n_k}$ is the nonlinear function that maps the input space into a higher dimension feature space.

Given training data $\{x_i, y_i\}_{i=1}^M$ where M denotes the number of training data, LS-SVM approach defines an optimization problem as follows:

$$\begin{aligned} \min_{w,b,e} J(w,e) &= \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^M e_k^2, \quad \gamma > 0, \\ \text{s.t. } y_k &= w^T \varphi(x_k) + b + e_k, \quad k=1, 2, \dots, M, \end{aligned} \tag{6}$$

where e_k are slack variables and γ is the regularization factor.

To solve this optimization problem, one defines the following Lagrange function:

$$L(w,b,e,\alpha) = J(w,e) - \sum_{k=1}^M \alpha_k \{w^T \varphi(x_k) + b + e_k - y_k\}, \tag{7}$$

where α_k are Lagrange multipliers. The conditions for optimality

$$\begin{cases} \frac{\partial L}{\partial w} = 0 \rightarrow w = \sum_{k=1}^M \alpha_k \varphi(x_k), \\ \frac{\partial L}{\partial b} = 0 \rightarrow \sum_{k=1}^M \alpha_k = 0, \\ \frac{\partial L}{\partial e_k} = 0 \rightarrow \alpha_k = \gamma e_k, \quad k=1, \dots, M, \\ \frac{\partial L}{\partial \alpha_k} = 0 \rightarrow w^T \varphi(x_k) + b + e_k - y_k = 0, \quad k=1, \dots, M \end{cases} \tag{8}$$

can be expressed as the solution to the following set of linear equations after elimination of w and e_k :

$$\begin{pmatrix} 0 & 1 & \dots & 1 \\ 1 & \varphi(x_1)^T \varphi(x_1) + \frac{1}{\gamma} & \dots & \varphi(x_1)^T \varphi(x_M) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \varphi(x_M)^T \varphi(x_1) & \dots & \varphi(x_M)^T \varphi(x_M) + \frac{1}{\gamma} \end{pmatrix} \begin{pmatrix} b \\ a_1 \\ \vdots \\ a_M \end{pmatrix} = [0 \quad y_1 \quad \dots \quad y_M]^T, \tag{9}$$

where

$$\varphi(x_k)^T \varphi(x_l) = K(x_k, x_l), \quad k, l=1, \dots, M, \tag{10}$$

according to Mercer's conditions. One has several possibilities for the choice of this kernel function $K(x_k, x_l)$, including linear, polynomial, splines, RBF. In the sequel of this paper we will focus on RBF kernels

$$K(x, x_k) = \exp\{-\|x - x_k\|^2 / \sigma^2\}, \tag{11}$$

where σ is the width parameter of RBF.

The resulting LS-SVM model for function estimation becomes

$$y(x) = \sum_{k=1}^M \alpha_k K(x, x_k) + b = \sum_{k=1}^M \alpha_k \exp(-\|x_k - x\|^2 / \sigma^2) + b, \tag{12}$$

where $\alpha_k, b \in \mathbb{R}$ are the solution of Eq.(9), x_k is training data, x is the new input case.

TRAINING AND PREDICTIVE RESULTS

Practically, output voltage of MCFC is influenced by many input control parameters. For simplicity, the operating temperature and pressure of the MCFC stack are kept constant during the experiments. Then the output voltage is dependent on the current density and flow rates of reactant gases. For the application of LS-SVM approach, the following adjustable parameters are selected to be the input vector:

$$\mathbf{x}_k = \{I_{\text{stack}}(k), F_a(k), F_c(k)\}, \text{ and } y_k = \{V(k)\}, \quad (13)$$

where I_{stack} is the current density of MCFC stack (mA/cm^2), F_a is the flow rates of anode gas (kg/h), F_c the flow rates of cathode gas (kg/h) and V the output voltage of MCFC stack (V).

The following NARMAX model

$$y(k) = f[y(k-1), y(k-2), \dots, y(k-n_y), \mathbf{x}(k-1), \mathbf{x}(k-2), \dots, \mathbf{x}(k-n_x)] + e(k) \quad (14)$$

is employed to denote the output voltage. n_x and n_y denote the input and output order of the system, respectively. The training data set is expressed as $D = \{d_k\}$, for example, when $n_x = n_y = 3$, d_k contains the following parameters:

$$[\mathbf{x}(k-1), \mathbf{x}(k-2), \mathbf{x}(k-3), V(k-1), V(k-2), V(k-3), V(k)]. \quad (15)$$

All the 1000 experimental sampled data points from the power test of the 50 kW MCFC in the Institute of Fuel Cell, Shanghai Jiao Tong University are divided into two sets: training set and testing set, where training set contains 900 data points and testing set containing the remaining 100 data points. These data points contain voltage response values under various current densities of the stack and gas flow rates, which will be used for establishing LS-SVM model of MCFC stack and evaluating the model performance. The identification structure of MCFC system is shown in Fig.2, where TDL is the tapped delay line. The parameters used in the training and testing process are summarized: $n_x = 3, n_y = 3, \gamma = 10, \sigma = 0.1$.

For establishing LS-SVM model and evaluating

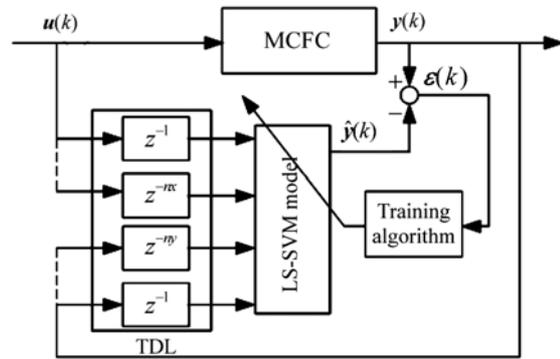


Fig.2 Schematic of the identification structure of MCFC stack with LS-SVM

its performance, 10-fold cross validation method is used, which can be found in LS-SVMlab1.5 (Pelckmans *et al.*, 2003), an MATLAB toolbox under MS Windows XP. All the 1000 data points are randomly divided into 10 disjunct sets. At each iteration, one of these sets (testing set) is used to estimate the MSE performance index of the model trained on the other 9 sets (training set). At last, the 10 different MSE performance indexes are averaged. MSE performance index is defined as follow:

$$E_{\text{MSE}} = \frac{1}{M} \sum_{k=1}^M (y_k - \hat{y}_k)^2, \quad (16)$$

where M is the number of data points in the test set, y_k is the actual output voltage value and \hat{y}_k the output of LS-SVM model at instance k . Then, the accuracy rate is calculated using Eq.(17):

$$\text{Accuracy} = \left(1 - \sqrt{\frac{1}{M} \sum_{k=1}^M [(y_k - \hat{y}_k) / y_k]^2} \right) \times 100\%. \quad (17)$$

Under a 3.0 GHz Pentium IV PC with 1 GB RAM on board, training LS-SVM model and estimating the model performance takes about 43 s. The average MSE obtained is 1.45, and the best MSE is only 0.54. In the best case, comparison between the actual output voltage values with the predicted voltage values by LS-SVM model is shown in Fig.3, with the predicted errors also given. As can be seen from Fig.3, the predicted results are in good agreement with the actual test results, and the maximal predicted error is not beyond 0.24 V. The accuracy calculated by Eq.(17) is 98.17%.

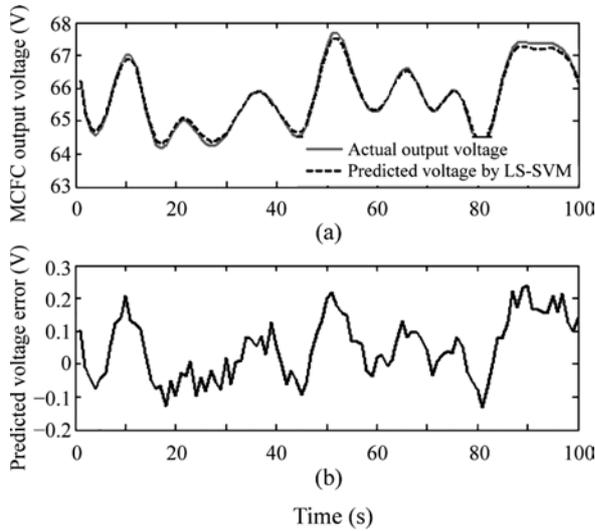


Fig.3 Predicted voltage by LS-SVM method. (a) Comparison between actual voltage and predicted voltage by LS-SVM model; (b) The predicted error

To illustrate the advantages of LS-SVM method, the results are compared with those obtained from training a radial basis function neural network (RBFNN). As neural network is similar to LS-SVM, it is also a well-known universal estimator. Function “newrb” in MATLAB neural network toolbox is chosen to train an RBF network. The same training and testing sets are also used. The parameters used in RBFNN training process and test results are shown in Table 2.

Table 2 Parameters in training process and test results by RBFNN

Items	Values
Number of neurons	34
Width parameter of RBF	0.1
MSE	1.21
Accuracy (%)	97.13
Training time (s)	65

The performance of RBFNN is commendable; it produces less MSE than the average value of LS-SVM, what is more, LS-SVM produces better accuracy and needs less training time. According to the test results, the identification accuracy of LS-SVM is high and the model can be trained fast. Therefore this model can be used to predict the voltage responses that make it possible to design a nonlinear predictive controller of MCFC stack.

NONLINEAR MODEL PREDICTIVE CONTROLLER

The structure of nonlinear predictive control system of MCFC stack is given in Fig.4. In the figure, the predictive output voltage $\hat{V}(k+j)$ for p steps ahead is obtained by LS-SVM model:

$$\hat{V}(k+j) = \sum_{n=1}^N \alpha_n \exp(-\|\mathbf{x}(k+j) - \mathbf{x}_n\|^2 / \sigma^2) + b, \quad 0 \leq j \leq p, \quad (18)$$

where N is the number of data points in the training set. Supposing the actual output voltage at k instance is $V(k)$, the predictive error at k instance is:

$$e(k) = V(k) - \hat{V}(k), \quad (19)$$

then the predictive output voltage of the feedback system for p steps can be defined as:

$$V_p(k+j) = \hat{V}(k+j) + e(k), \quad 0 \leq j \leq p. \quad (20)$$

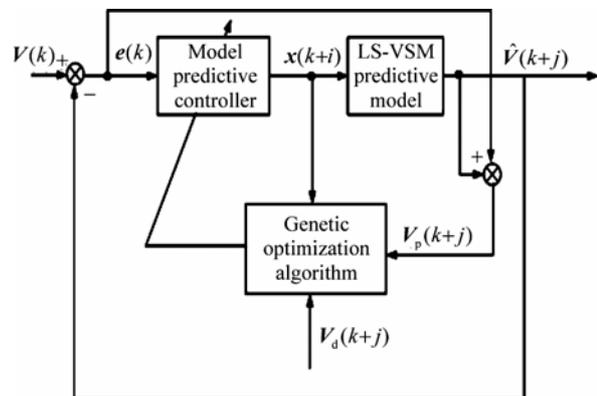


Fig.4 Structure of the nonlinear predictive control system of an MCFC stack

Referenced trajectories of output voltage are introduced to avoid excessive movement of the control input, which are defined as:

$$V_d(k+j) = c^j V(k) + (1 - c^j) V_{sp}, \quad 0 \leq j \leq p, \quad 0 < c < 1, \quad (21)$$

where V_{sp} is the set point of output voltage.

The optimization problem for the predictive controller is the minimization of the sum of squared errors between the referenced trajectory and the predictive output, with an additional penalty imposed on excessive changes in the manipulated variables (Zhu, 2002):

$$J(k) = \sum_{j=1}^p q_j [V_p(k+j) - V_d(k+j)]^2 + \sum_{i=1}^l r_i [\mathbf{x}(k+i) - \mathbf{x}(k+i-1)]^2, \quad 0 \leq j \leq p, 0 \leq i \leq l, q_j \geq 0, r_i > 0. \quad (22)$$

The weight parameters q_j and r_i are used to increase the importance of specific variables at given instances. For example, the weights may increase over time to ensure rapid convergence with no offset (Golbert and Lewin, 2004). p is predictive horizon and l is control horizon. In this case, we choose $p=l=4$.

In order to obtain the solution of the optimization problem, we consider a single objective float number encoded GA. We have opted for this GA because, in general, it is known that real coded GA performs better than binary coded GA for high precision optimization problem (Elliott *et al.*, 2005). The genetic operators and the parameters used for this GA are taken as follows (Belarbi *et al.*, 2005): (1) population size $n_{\text{pop}}=50$; (2) fitness function $F(k)=1/J(k)$; (3) uniform arithmetic two-point crossover, crossover probability $p_c=0.85$; (4) non-uniform mutation probability p_m .

One chromosome is composed of three sub-chromosomes: the first denotes the current density of MCFC stack I_{stack} , the second is the flowrate of anode inlet gas F_a , and the last represents the flowrate of cathode inlet gas F_c . The current density values and the flowrate values of anode inlet gas are normalized within the range [000,999], and the cathode flowrate within the range [0000,9999]. The chromosome v_k is thus structured as $[I_{\text{stack}}, F_a(k), F_c(k)]$.

Fitness values of all chromosomes are calculated and we sort the chromosomes from greater to lower fitness. Truncation selection scheme is used, that is, the 50% best chromosomes are selected and reproduced correspond to their fitness value until the number of offspring is equal to the size of the popu-

lation. The offspring generation replaces the parent population.

A non-uniform mutation operator is used, which enables fine local tuning and is defined as follows: The mutation possibility of each chromosome is in inverse proportion to its fitness:

$$p_m(v_k) = 0.10 - 0.01F(v_k)/n_{\text{pop}}. \quad (23)$$

After searching for several times, the optimal control moves $\mathbf{X}(k)=[\mathbf{x}(k+1), \dots, \mathbf{x}(k+l)]$ can be obtained. The first of the future control moves $\mathbf{x}(k+l)$ is implemented, and the entire optimization is repeated from the next step on, and so on.

SIMULATION RESULTS

In this section, we present simulations of using the predictive control algorithm based on LS-SVM model. Fig.5 shows the performance obtained with the predictive controller. When the current density has several step changes, the output voltage changes suddenly at first, then the predictive controller controls the output voltage to the required level in only 6 s without overshoot.

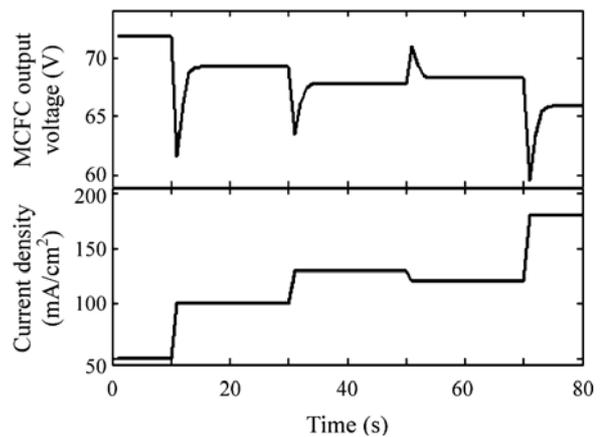


Fig.5 Performance of the predictive controller

When MCFC stack works at rated power, we get the tracking curve of the controlled output voltage shown in Fig.6. A traditional fuzzy controller is also used in the simulation experiment. For detailed description please see (Schumacher *et al.*, 2004).

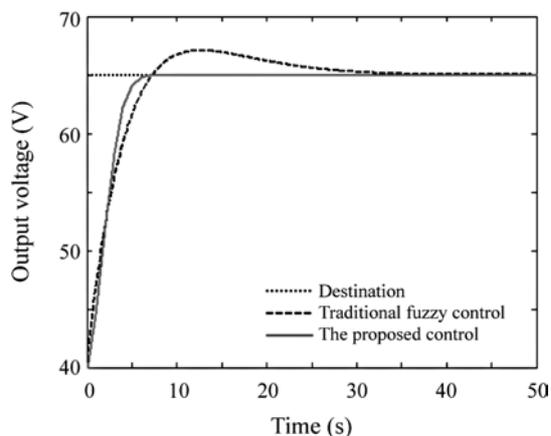


Fig.6 The experimental results with two control methods

The performance of traditional fuzzy controller is commendable, its overshoot is about 2.7 V, and its convergence time is 33 s. Comparatively, the performance of the predictive controller is better than that of the traditional fuzzy controller, it needs only 7 s to reach the steady state with no overshoot.

Figs.5 and 6 show that under various current densities, the proposed controller can regulate and control the MCFC output voltage to change smoothly and quickly to its stable target value. Therefore it is feasible to use this proposed controller for MCFC stack.

CONCLUSION

The output voltage is an important variable controlled in MCFC system. However, the relation of output voltage and current density is nonlinear and complex, indicating that nonlinear control is required to adequately regulate the output voltage in the case of drastic current changes. An LS-SVM predictive model is put forward to study the sampling data. Then a nonlinear predictive control algorithm using GA is proposed. In simulations, use of the nonlinear model predictive control enables accurate control in such a nonlinear and complex system, the performance of proposed controller is satisfying.

References

Belarbi, K., Titel, F., Bourebia, W., Benmahammed, K., 2005. Design of Mamdani fuzzy logic controllers with rule base minimization using genetic algorithm. *Engineering Ap-*

plications of Artificial Intelligence, **18**(7):875-880. [doi:10.1016/j.engappai.2005.03.003]

Elliott, L., Ingham, D.B., Kyne, A.G., Mera, N.S., Pourkashanian, M., Wilson, C.W., 2005. The use of ignition delay time in genetic algorithms optimization of chemical kinetics reaction mechanisms. *Engineering Applications of Artificial Intelligence*, **18**(7):825-831. [doi:10.1016/j.engappai.2005.02.006]

Golbert, J., Lewin, D.R., 2004. Model-based control of fuel cells: (1) Regulatory control. *Journal of Power Sources*, **135**(1-2):135-151. [doi:10.1016/j.jpowsour.2004.04.008]

Ishikawa, T., Yasue, H., 2000. Start-up, testing and operation of 1000 kW class MCFC power plant. *Journal of Power Sources*, **86**(1-2):145-150. [doi:10.1016/S0378-7753(99)00446-2]

Li, X., Cao, G.Y., Zhu, X.J., 2006. Modeling and control of PEMFC based on least squares support vector machines. *Energy Conversion and Management*, **47**(7-8):1032-1050. [doi:10.1016/j.enconman.2005.04.002]

Lunghi, P., Bove, R., Desideri, U., 2003. Analysis and optimization of hybrid MCFC gas turbines plants. *Journal of Power Sources*, **118**(1-2):108-117. [doi:10.1016/S0378-7753(03)00068-5]

Pelckmans, K., Suykens, J., van Gestel, T., de Brabanter, J., Lukas, L., Hamers, B., de Moor, B., Vandewalle, J., 2003. LS-SVMlab: A MATLAB/C toolbox for Least Squares Support Vector Machines. <http://www.esat.kuleuven.ac.be/sista/lssvmlab>

Schumacher, J.O., Gemmar, P., Denne, M., 2004. Control of miniature proton exchange membrane fuel cells based on fuzzy logic. *Journal of Power Sources*, **129**(2):143-151. [doi:10.1016/j.jpowsour.2003.10.010]

Shen, C., Cao, G.Y., Zhu, X.J., 2002. Nonlinear modeling and adaptive fuzzy control of MCFC stack. *Journal of Process Control*, **12**(8):831-839. [doi:10.1016/S0959-1524(02)00013-6]

Suykens, J.A.K., Vandewalle, J., 2000. Recurrent least squares support machines. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, **47**(7):1109-1114. [doi:10.1109/81.855471]

Vong, C.M., Wong, P.K., Li, Y.P., 2006. Prediction of automotive engine power and torque using least squares support machines and Bayesian inference. *Engineering Applications of Artificial Intelligence*, **19**(3):277-287. [doi:10.1016/j.engappai.2005.09.001]

Wang, S.W., Yu, D.L., Gomm, J.B., Page, G.F., Douglas, S.S., 2006. Adaptive neural network model based predictive control for air-fuel ratio of SI engines. *Engineering Applications of Artificial Intelligence*, **19**(2):189-200. [doi:10.1016/j.engappai.2005.08.005]

Yoshida, F., Izaki, Y., Watanabe, T., 2004. Wide range load controllable MCFC cycle with pressure swing operation. *Journal of Power Sources*, **137**(2):196-205. [doi:10.1016/j.jpowsour.2004.06.001]

Zhu, J., 2002. Intelligent Predictive Control Technology and Application. Zhejiang University Press, Hangzhou, China, p.138-140 (in Chinese).