Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

A generalization of Kempe's linkages*

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Received Dec. 19, 2006; revision accepted Jan. 25, 2007

Abstract: A new, general type of planar linkages is presented, which extends the classical linkages developed by Kempe consisting of two single-looped kinematic chains of linkages, interconnected by revolute hinges. Together with a locking device, these new linkages have only one degree of freedom (DOF), which makes them ideal for serving as deployable structures for different purposes. Here, we start with a fresh matrix method of analysis for double-loop planar linkages, using 2D transformation matrices and a new symbolic notation. Further inspection for one case of Kempe's linkages is provided. Basing on the inspection, by means of some novel algebraic and geometric techniques, one particularly fascinating solution was found. Physical models were built to show that the derivation in this paper is valid and the new mechanisms are correct.

Key words: Double-loop, Kempe's linkages, Transformation matrices, Deployable structures

doi:10.1631/jzus.2007.A1084 **Document code:** A **CLC number:** TU2; TU3

INTRODUCTION

More than a century ago, Kempe (1878) synthesized six classes of over-constrained planar linkages which became a classic in kinematics later on. For the first time Kempe listed conditions under which a plane linkage represented in Fig.1, consisting of two planar four-bar linkages pivoted together became a movable system. Such combinations are called double-loop planar linkages, whose internal degree of freedom (DOF) is one. Kempe based his investigation on the assumption that the two quadrilaterals respectively formed by attachment points of the two four-bar remained the same while the system was deforming. Darboux (1879), Fontené (1904), and Baker and Yu (1983), impressed by Kempe's results, took up the task to re-examine Kempe's findings. They isolated all solutions of the type sought by Kempe and gave some new linkages. However, their linkages were all limited to two four-bar. In the early 1990s, Hoberman (1990; 1991) proposed a special planar linkage of angulated beams, which break the limit of number of bars. The concept was extended by You and Pellegrino (1997) which gave birth to a more general family of double-loop linkages. Later, Wohlhart (2000) presented another type of double-loop.

Together with a locking device, double-loop linkages can serve as deployable structures for different purposes. Therefore, the exploitation of the new type double-loop linkages has theoretical importance. We intend, in this paper, to find a generalization of Kempe's linkages, in order to determine the conditions for its mobility. Meanwhile, the paper will explicitly state how to construct the generalization of Kempe's linkages. Besides the chief purpose of the paper, we are going to employ special notation which can be used to analyze in double-loop planar linkages.

The layout of the paper is as follows. Section 2 introduces a standard matrix method of analysis for a mechanism proposed by Denavit and Hartenberg (1955) and derives its 2D form. Section 3 firstly re-examines a case of Kempe's linkages and lists two

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* Project supported by the National Natural Science Foundation of China (No. 50638050), and the Research Foundation for the Doctoral Program of Higher Education of China (No. 20050335097)

geometric characteristics. According to these conditions, describe Kempe's linkages in a set of transform matrices by the matrix method. A fascinating generalization of Kempe's linkages is found. Section 4 concludes this paper.

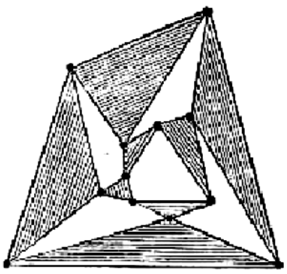

Fig.1 Kempe's linkage

MATRIX METHODS OF ANALYSIS FOR PLANAR LINKAGES

Transformation matrices

In a classical paper, Denavit and Hartenberg (1955) set forth a standard approach to the analysis of linkages, where the geometric conditions are taken into account. Beggs (1966) used similar methods for analysis of a set of mechanisms. According to coordinate transformations and transformation matrices, these scholars were the first to demonstrate what closed-chain mechanisms form a movable system. However, they discussed mechanisms in 3D coordinate system, which brings unnecessary complexity for analysis of planar linkages. For simplicity, we shall deduce transformation matrices in 2D coordinate system.

Assume that we have found two coordinate systems, $X_1O_1Y_1$ and $X_2O_2Y_2$, shown in Fig.2. X_2 axis locates at the extension of O_1O_2, and the coordinates of point O_2 in system $X_1O_1Y_1$ is (x_0, y_0). Now let us show how to get a transform matrix $[\boldsymbol{T}_{12}]$ that transforms the coordinates of a point P in system $X_1O_1Y_1$ to its coordinates in system $X_2O_2Y_2$. Thus, the coordinates of P is (x_1, y_1) in system $X_1O_1Y_1$ and (x_2, y_2) in system $X_2O_2Y_2$. The relationship of the above coordinates can be written as

$$x_1=x_2\cos\theta_{12}-y_2\sin\theta_{12}+x_0,\ y_1=x_2\sin\theta_{12}+y_2\cos\theta_{12}+y_0, \quad (1)$$

where θ_{12} is the angle of rotation from X_1 to X_2 positively about anti-clockwise. Here, it is specified that θ_{12} equals $-\theta_{21}$. Then, the distance between O_1 and O_2 is denoted by a_{12}, so Eq.(1) can be rewritten in matrix form as

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & a_{12}\cos\theta_{12} \\ \sin\theta_{12} & \cos\theta_{12} & a_{12}\sin\theta_{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}. \quad (2)$$

Hence, the transformation matrix $[\boldsymbol{T}_{12}]$ in planar case can be denoted by

$$[\boldsymbol{T}_{12}] = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & a_{12}\cos\theta_{12} \\ \sin\theta_{12} & \cos\theta_{12} & a_{12}\sin\theta_{12} \\ 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

The inverse of $[\boldsymbol{T}_{12}]$ is given as

$$[\boldsymbol{T}_{21}] = \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & -a_{12} \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

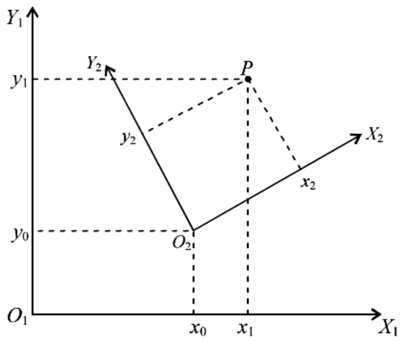

Fig.2 Two coordinate systems

Mobility condition

Denavit and Hartenberg (1955) pointed out that, for a closed loop in a linkage, the necessary and sufficient mobility condition is that the product of the transform matrices equals the unit matrix, i.e.,

$$[\boldsymbol{T}_{12}][\boldsymbol{T}_{23}][\boldsymbol{T}_{34}]...[\boldsymbol{T}_{n1}]=[\boldsymbol{I}]. \quad (5)$$

Using mathematical induction, we can have

$$[\boldsymbol{T}_{12}][\boldsymbol{T}_{23}]\ldots[\boldsymbol{T}_{n1}]= \begin{bmatrix} \cos(\theta_{12}+\theta_{23}+...+\theta_{n1}) & -\sin(\theta_{12}+\theta_{23}+...+\theta_{n1}) & K_1 \\ \sin(\theta_{12}+\theta_{23}+...+\theta_{n1}) & \cos(\theta_{12}+\theta_{23}+...+\theta_{n1}) & K_2 \\ 0 & 0 & 1 \end{bmatrix}, \quad (6)$$

where,

$$K_1=a_{12}\cos\theta_{12}+a_{23}\cos(\theta_{12}+\theta_{23})+\ldots+a_{n1}\cos(\theta_{12}+\theta_{23}+\ldots+\theta_{n1}),$$
$$K_2=a_{12}\sin\theta_{12}+a_{23}\sin(\theta_{12}+\theta_{23})+\ldots+a_{n1}\sin(\theta_{12}+\theta_{23}+\ldots+\theta_{n1}). \quad (7)$$

According to Eq.(5), the following conditions must be met:

$$\theta_{12}+\theta_{23}+\ldots+\theta_{n1}=2k\pi\ (k\in\mathbb{N}),\ \ K_1=K_2=0. \quad (8)$$

The advantage of applying this matrix method is that we can describe a double-loop into a set of transform matrices. According to the theory that the product of these transform matrices equals the unit matrix, we can code a short piece of software enabling rapid determination of suitability of the double-loop.

CREATION OF A GENERALIZATION OF KEMPE'S LINKAGES

Re-examine Case 5 of Kempe's linkages

Among six cases of Kempe's linkages, we have found that Case 5, shown in Fig.3, has greater potential to be exploited than other cases. The geometrical characteristics of Case 5, given by Kempe, are as follows. First, the root polygon (the dashed quadrilaterals in Fig.3) of two deploying four-bar always remain similar; secondly, in Fig.3, the triangles ABM, BCN, CDP, DAQ are similar and the triangles $A'B'M'$, $B'C'N'$, $C'D'P'$, $D'A'Q'$ are also similar. Hence, the following conditions can be obtained.

$$\angle QAM=\angle Q'A'M'=\angle DAB,$$
$$\angle MBN=\angle M'B'N'=\angle ABC,$$
$$\angle NCP=\angle N'C'P'=\angle BCD,$$
$$\angle PDQ=\angle P'D'Q'=\angle CDA,$$
$$AM/A'M'=AQ/A'Q'=CN/C'N'=CP/C'P',$$
$$BM/B'M'=BN/B'N'=DP/D'P'=DQ/D'Q'. \quad (9)$$

Case 5 is a typical class of over-constrained mechanism because Kutzbach formula gives for the degree of mobility: $F=-3$. However, in fact if one relative angle of two adjacent links, for example $\angle AMB$ in Fig.3, is known, the quadrilateral $AMBM'$ is determined and the positions of all the other links are also uniquely determined, i.e., it can move with only 1 DOF. Accordingly, we dare to make a guess that a class of double-loop planar linkages matches the following conditions: (1) The root polygon stays similar to its initial shape throughout the motion; (2) The triangles built on each edge of the root polygon are cyclically similar.

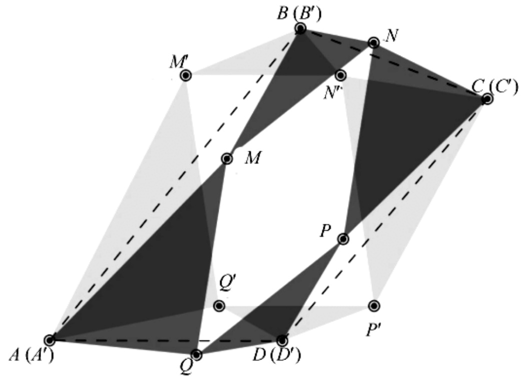

Fig.3 Case 5 of Kempe's linkages

It is possible to extend the concept to a series of combinations of a pair of six-bar, eight-bar, even $2n$-bar where n is a positive integer greater than 2. For brevity, we shall henceforth refer to this kinematics chain as type SQ which has a 1 DOF motion. In the subsequent discussion we shall prove our guess, by the use of the matrix method mentioned in Section 2.

Description of Kempe's linkages

Before using the matrix method, it is necessary to give the symbolic notation in double-loop planar linkages for applying the transformation. Denavit and Hartenberg (1955) gave the rules of notation in a closed single chain, which cannot be directly used in double-loop planar linkages for the geminate joints. Therefore, we rewrote the rules of notation as follows: (1) Number the revolute hinges in sequence around one loop, then the other loop; (2) The axis from $i-1$ to i is X_i. Suppose the total number of hinges is n, the axis from n to 1 is X_1; (3) The distance between X_{i-1} and X_i is $a_{i-1,i}$; (4) The angle between X_{i-1} and X_i is $\theta_{i-1,i}$, which is measured positively about anti-clockwise.

This notation will now be used to describe Case 5 of Kempe's linkages in Fig.4. Additionally for simplicity, in the subsequent analysis, we shall use a set of angulated beams instead of the plates in Fig.1.

The connections are located at exactly the same position as the original plates. The symbol ⊙ indicates the positions of hinges. Note that 2 and 10 are used to describe the same hinge, the same with 4 and 12, 6 and 14, 8 and 16.

From Eq.(9), the relationship of all angles and lengths of bars in Fig.4 can be shown in Fig.5. Note that we respectively denoted the supplementary angles of $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle CDA$ as α, β, γ and δ. Geometrically we have

$$\alpha+\beta+\gamma+\delta=2\pi. \tag{10}$$

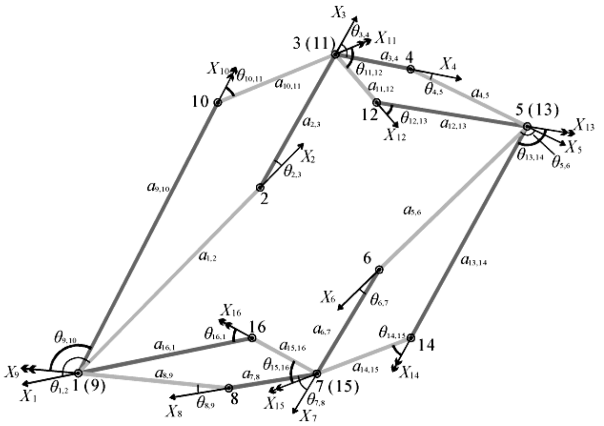

Fig.4 Symbolic notations in double-loop planar linkages

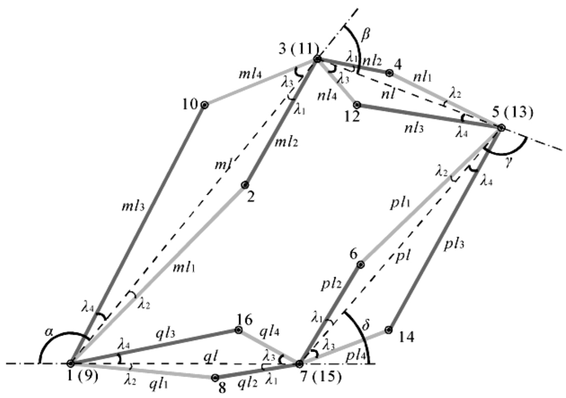

Fig.5 The value of each $a_{i-1,i}$ and $\theta_{i-1,i}$ in Case 5 of Kempe's linkages

Therefore, the values of each $a_{i-1,i}$ and $\theta_{i-1,i}$ in Case 5 of Kempe's linkages can be shown in Table 1.

Substituting the values of every $a_{i-1,i}$ and $\theta_{i-1,i}$ into Eq.(8) yields

$$\theta_{12}+\theta_{23}+\ldots+\theta_{n1}=-2(\alpha+\beta+\gamma+\delta)=-4\pi.$$

Table 1 The value of $a_{i-1,i}$ and $\theta_{i-1,i}$ (i=17, $a_{i-1,i}$=$a_{16,1}$, $\theta_{i-1,i}$=$\theta_{16,1}$)

$a_{i-1,i}$	Value	$\theta_{i-1,i}$	Value
$a_{1,2}$	ml_1	$\theta_{1,2}$	$-(\alpha+\lambda_2+\lambda_4)$
$a_{2,3}$	ml_2	$\theta_{2,3}$	$\lambda_1+\lambda_2$
$a_{3,4}$	nl_2	$\theta_{3,4}$	$-\beta$
$a_{4,5}$	nl_1	$\theta_{4,5}$	$-(\lambda_1+\lambda_2)$
$a_{5,6}$	pl_1	$\theta_{5,6}$	$-\gamma$
$a_{6,7}$	pl_2	$\theta_{6,7}$	$\lambda_1+\lambda_2$
$a_{7,8}$	ql_2	$\theta_{7,8}$	$-\delta$
$a_{8,9}$	ql_1	$\theta_{8,9}$	$-(\lambda_1+\lambda_2)$
$a_{9,10}$	ml_3	$\theta_{9,10}$	$-(\alpha-\lambda_2-\lambda_4)$
$a_{10,11}$	ml_4	$\theta_{10,11}$	$-(\lambda_3+\lambda_4)$
$a_{11,12}$	nl_4	$\theta_{11,12}$	$-\beta$
$a_{12,13}$	nl_3	$\theta_{12,13}$	$\lambda_3+\lambda_4$
$a_{13,14}$	pl_3	$\theta_{13,14}$	$-\gamma$
$a_{14,15}$	pl_4	$\theta_{14,15}$	$-(\lambda_3+\lambda_4)$
$a_{15,16}$	ql_4	$\theta_{15,16}$	$-\delta$
$a_{16,1}$	ql_3	$\theta_{16,1}$	$\lambda_3+\lambda_4$

So the first condition in Eq.(8) has been met. Moreover, we have

$$\begin{aligned}
K_1=&l_1[m\cos(\alpha+\lambda_2+\lambda_4)+n\cos(\alpha+\beta+\lambda_2+\lambda_4)\\
&+p\cos(\alpha+\beta+\gamma+\lambda_2+\lambda_4)+q\cos(\alpha+\beta+\gamma+\delta+\lambda_2+\lambda_4)]\\
&+l_2[m\cos(\alpha-\lambda_1+\lambda_4)+n\cos(\alpha+\beta-\lambda_1+\lambda_4)\\
&+p\cos(\alpha+\beta+\gamma-\lambda_1+\lambda_4)+q\cos(\alpha+\beta+\gamma+\delta-\lambda_1+\lambda_4)]\\
&+l_3[m\cos\alpha+n\cos(\alpha+\beta)+p\cos(\alpha+\beta+\gamma)\\
&+q\cos(\alpha+\beta+\gamma+\delta)]+l_4[m\cos(\alpha+\lambda_3+\lambda_4)\\
&+n\cos(\alpha+\beta+\lambda_3+\lambda_4)+p\cos(\alpha+\beta+\gamma+\lambda_3+\lambda_4)\\
&+q\cos(\alpha+\beta+\gamma+\delta+\lambda_3+\lambda_4)]=0,\\
K_2=&-l_1[m\sin(\alpha+\lambda_2+\lambda_4)+n\sin(\alpha+\beta+\lambda_2+\lambda_4)\\
&+p\sin(\alpha+\beta+\gamma+\lambda_2+\lambda_4)+q\sin(\alpha+\beta+\gamma+\delta+\lambda_2+\lambda_4)]\\
&-l_2[m\sin(\alpha-\lambda_1+\lambda_4)+n\sin(\alpha+\beta-\lambda_1+\lambda_4)\\
&+p\sin(\alpha+\beta+\gamma-\lambda_1+\lambda_4)+q\sin(\alpha+\beta+\gamma+\delta-\lambda_1+\lambda_4)]\\
&-l_3[m\sin\alpha+n\sin(\alpha+\beta)+p\sin(\alpha+\beta+\gamma)+q\sin(\alpha+\beta+\gamma+\delta)]\\
&-l_4[m\sin(\alpha+\lambda_3+\lambda_4)+n\sin(\alpha+\beta+\lambda_3+\lambda_4)\\
&+p\sin(\alpha+\beta+\gamma+\lambda_3+\lambda_4)+q\sin(\alpha+\beta+\gamma+\delta+\lambda_3+\lambda_4)]=0.
\end{aligned} \tag{11}$$

It must be noted that λ_1, λ_2, λ_3, λ_4 are a group of linearly dependent variables which demonstrates type SQ has a DOF motion. It is easy to know the sufficient condition of Eq.(11) is the following condition:

$$\begin{aligned}
&ml\cos(\alpha+\varepsilon)+nl\cos(\alpha+\beta+\varepsilon)+pl\cos(\alpha+\beta+\gamma+\varepsilon)\\
&\qquad+ql\cos(\alpha+\beta+\gamma+\delta+\varepsilon)=0,\\
&ml\sin(\alpha+\varepsilon)+nl\sin(\alpha+\beta+\varepsilon)+pl\sin(\alpha+\beta+\gamma+\varepsilon)\\
&\qquad+ql\sin(\alpha+\beta+\gamma+\delta+\varepsilon)=0.
\end{aligned} \tag{12}$$

where ε is variable angle. Evidently, Eq.(12) is true if we explain it in vector space.

Assume that we have found four vectors $\boldsymbol{AB}$, $\boldsymbol{BC}$, $\boldsymbol{CD}$ and $\boldsymbol{DA}$, which are shown in Fig.6 and expressed in Eq.(13):

$$\begin{aligned}&\boldsymbol{AB}=ml(\mathrm{i}\cos(\alpha+\varepsilon)+\mathrm{j}\sin(\alpha+\varepsilon)),\\&\boldsymbol{BC}=nl(\mathrm{i}\cos(\alpha+\beta+\varepsilon)+\mathrm{j}\sin(\alpha+\beta+\varepsilon)),\\&\boldsymbol{CD}=pl(\mathrm{i}\cos(\alpha+\beta+\gamma+\varepsilon)+\mathrm{j}\sin(\alpha+\beta+\gamma+\varepsilon)),\\&\boldsymbol{DA}=ql(\mathrm{i}\cos(\alpha+\beta+\gamma+\delta+\varepsilon)+\mathrm{j}\sin(\alpha+\beta+\gamma+\delta+\varepsilon)),\end{aligned} \quad (13)$$

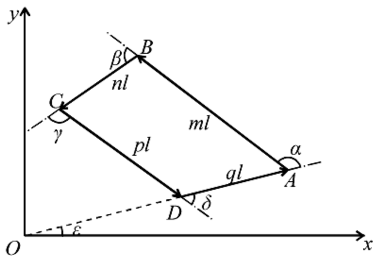

Fig.6 Four closed vectors

From Fig.6, we know that *ABCD* is a closed quadrilateral, which leads to

$$\boldsymbol{AB}+\boldsymbol{BC}+\boldsymbol{CD}+\boldsymbol{DA}=0. \quad (14)$$

Substituting Eq.(13) into Eq.(14), Eq.(12) can be obtained, which indicates we have $K_1=K_2=0$ in Case 5 of Kempe's linkages. That means, the second condition in Eq.(8) has been met, too. Hence, according to Denavit and Hartenberg's standard approach to the analysis of linkages, we can prove all double-loop planar linkages have one DOF.

A generalization of Kempe's linkages

From Section 3.2, for a group of closed vectors, the conditions similar to Eq.(12) can be always obtained:

$$\begin{aligned}&m_1\cos(\alpha_1+\varepsilon)+m_2\cos(\alpha_1+\alpha_2+\varepsilon)+\ldots\\&\qquad+m_i\cos(\alpha_1+\alpha_2+\ldots+\alpha_i+\varepsilon)=0,\\&m_1\sin(\alpha_1+\varepsilon)+m_2\sin(\alpha_1+\alpha_2+\varepsilon)+\ldots\\&\qquad+m_i\sin(\alpha_1+\alpha_2+\ldots+\alpha_i+\varepsilon)=0.\end{aligned} \quad (15)$$

Consequently, a generalization of Kempe's linkages can be derived from the root polygon. Now let us show how to construct a general type SQ.

For n even and greater than 2: (1) draw an arbitrary n-sided root polygon, for instance a closed vector hexagon shown in Fig.7a which satisfies Eq.(15), (2) choose four suitable angles, λ_1, λ_2, λ_3, λ_4, and (3) construct similar triangles on the sides of the root polygon in the pattern of Fig.7b. In this case, the pattern using λ_1, λ_2, closes up with triangles only on one side of each edge of the root polygon, and the pattern using λ_3, λ_4, fills in the other triangles.

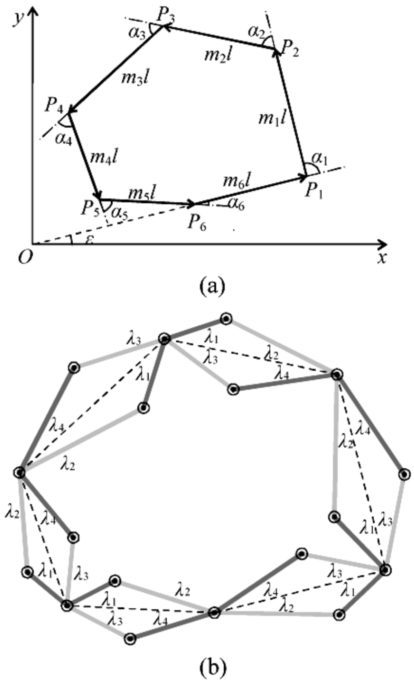

Fig.7 A generalization of Kempe's linkage in the even case. (a) An arbitrary root polygon; (b) Constructional pattern

For n odd and greater than 1: similarly, (1) draw an arbitrary n-sided root polygon, for instance a closed vector pentagon shown in Fig.8a which satisfies Eq.(15), (2) choose two suitable angles, λ_1, λ_2, and (3) construct similar triangles on the sides of the root polygon alternating in the pattern of Fig.8b. In the odd case, the pattern, by sequence as shown in Fig.8b, using just λ_1, λ_2, fills in all inner and outer triangles before closing up. It must be pointed out that all quadrilaterals are similar parallelograms which were found by You and Pellegrino (1997).

The most symmetric linkage, Hoberman linkage as shown in Fig.9a, also belongs to the generalization of Kempe's linkages, whose triangles are all equal. It is the most special case in type SQ for $\lambda_1=\lambda_2=\lambda_3=\lambda_4$. As an extension of this concept, the Elliptic-like linkage shown in Fig.9b can be built which is also highly symmetric.

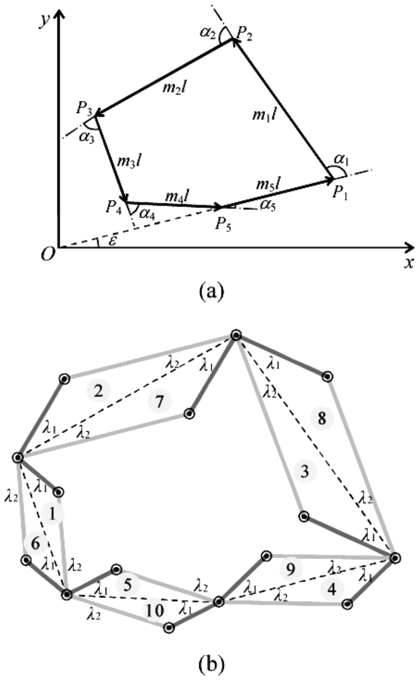

Fig.8 A generalization of Kempe's linkage in the odd case. (a) An arbitrary root polygon; (b) Constructional pattern

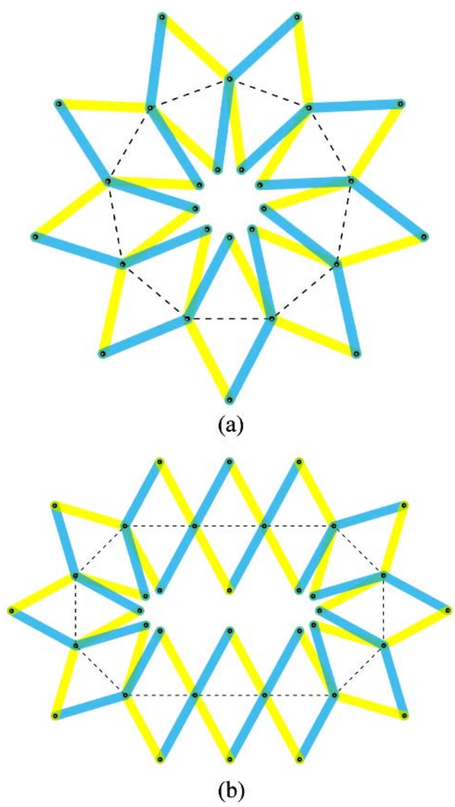

Fig.9 Highly symmetrical linkages. (a) Hoberman linkage; (b) Elliptic-like linkage

By now, the guess in Section 3.1 has been totally confirmed. Physical models built to demonstrate all of the extended concepts have shown that the above derivation is valid and the new mechanisms are correct, which is shown in Fig.10. Following the geometrical conditions of type SQ, the model consists of double-loop six-bar linkages made of red and blue Perspex. With a single degree of freedom, these models can move perfectly well as expected, which make them ideal for use as foldable structures.

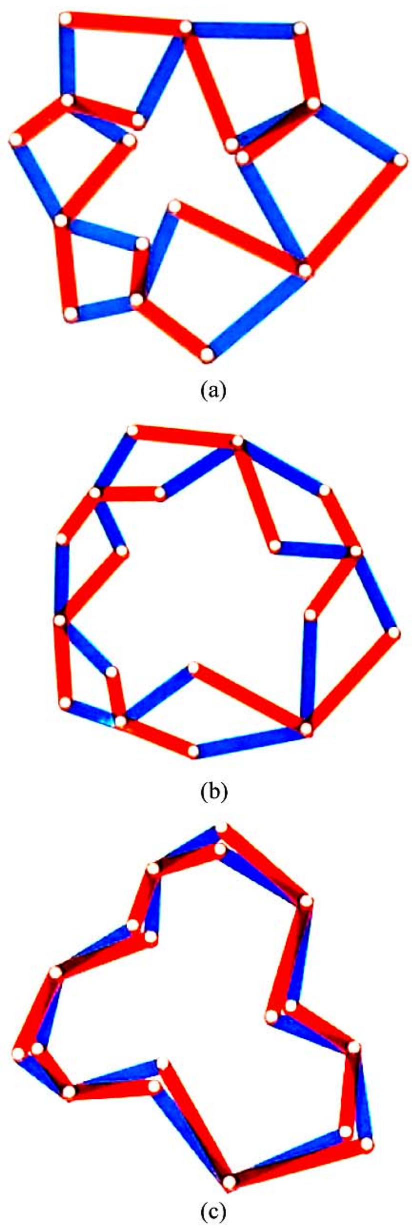

Fig.10 (a)~(c) Motion sequence of a model consisting of double-loop six-bar linkages

CONCLUSION

This paper has extended Case 5 of Kempe's linkages to a generalization of Kempe's linkages. Using transformation matrices rewritten by us in 2D coordinate system, we have introduced a matrix method into the analysis of planar linkages. A new symbolic notation for the double-loop planar linkages has been introduced and thus the designers can use this symbolic notation to transform double-loop planar linkages into a set of transformation matrices.

It was found that, adopting the matrix methods of analysis extended by us, Case 5 has been re-explained, mathematically. More significantly, basing on the two geometric characteristics of Case 5 described in Section 3.1, we have given birth to type SQ mechanisms, including the even case and the odd case. Physical models built have shown that the derivation in this paper is valid and the new mechanisms are correct.

The new linkages provide more choices to designers of foldable structures for applications such as retractable roofs. However, due to the lower manufacturing costs, highly symmetrical linkages are preferred in applications, as far as possible. Based on the analysis in the paper, we have coded a short piece of software enabling rapid determination of suitability of double-loop.

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