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On exponential stability for systems with state delays*

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Abstract: This paper considers the issue of delay-dependent exponential stability for time-delay systems. Both nominal and uncertain systems are investigated. New sufficient conditions in terms of linear matrix inequalities (LMIs) are obtained. These criteria are simple owing to the use of an integral inequality. The model transformation approaches, bounding techniques for cross terms and slack matrices are all avoided in the derivation. Rigorous proof and numerical examples showed that the proposed criteria and those based on introducing slack matrices are equivalent.

Key words: Exponential stability, Uncertain time-delay systems, Integral inequality, Slack matrix, Linear matrix inequality (LMI)
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INTRODUCTION

Time delays are frequently encountered in various practical systems, such as manufacturing systems, neural networks, population dynamics models, and so on (Gu *et al.*, 2003). The existing stability results for time-delay systems can be classified into two categories: delay-dependent criteria and delay-independent criteria. Delay-dependent conditions, which take the delay into account, are generally less conservative than delay-independent ones. A lot of delay-dependent stability results have been reported during the past decades, see e.g. (Chen, 1995; Cao *et al.*, 1998; Cao and Xue, 2005; Fridman, 2001; Fridman and Shaked, 2003; Gu, 1997; Gu and Niculescu, 2000; Gu *et al.*, 2003; Han, 2005; Ivanescu *et al.*, 2003; Kolmanovskii and Richard, 1999; Moon *et al.*, 2001; Park, 1999; Wu *et al.*, 2004a; 2004b; Xu and Lam, 2005).

All of the above-mentioned stability conditions for time-delay systems are concerned with asymptotic stability instead of exponential ones. But it is very important to estimate the decay rates (i.e. exponential stability degrees) of time-delay systems in many dynamical systems. The issue of exponential stability for delay systems has received considerable attention in recent years. For example, based on the concept of matrix measure, decay rate estimates were investigated in (Lehman and Shujaee, 1994; Niculescu *et al.*, 1998; Sun and Hsieh, 1998), but these conditions are difficult to test. Liu (2003) provided a delay-dependent exponential stability condition for systems without uncertainties in terms of LMI, which is easy to be verified. Both exponential stability conditions and exponential estimates for retarded and neutral systems were addressed in (Mondié and Kharitonov, 2005) and (Kharitonov *et al.*, 2005), respectively. Most recently, based on a parameterized neutral-type transformation, Kwon and Park (2006) discussed the exponential stability of uncertain delayed systems. The descriptor-type transformation and some free-weighting matrices were introduced to obtain exponential stability conditions for generalized state-space systems with parametric uncertainties (Yu and Lien,

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2006). Some exponential stability criteria based on introducing some slack matrices were presented in (Xu *et al.*, 2006).

In this paper, we focus on the problem of delay-dependent robustly exponential stability for a class of linear time-delay systems with norm-bounded uncertainties. We avoid applying model transformation and bounding techniques for cross terms. An integral inequality is used to obtain sufficient conditions in terms of LMIs, which can be efficiently solved by LMI Toolbox in Matlab. The proposed results are simpler than those of (Xu *et al.*, 2006) because no slack matrices are involved. The effectiveness of our approach and its equivalence to the method proposed in (Xu *et al.*, 2006) are illustrated by numerical examples as well as mathematical proof.

Throughout this paper, the notations are fairly standard. The symmetric term in a symmetric matrix is denoted as *, i.e.

$$\begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ * & \boldsymbol{P}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ \boldsymbol{P}_2^{\mathrm{T}} & \boldsymbol{P}_3 \end{bmatrix}.$$

PROBLEM FORMULATION

Consider the following nominal time-delay system:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{A}_1\boldsymbol{x}(t-h), \\ \boldsymbol{x}(\theta) = \boldsymbol{\varphi}(\theta),\ \forall\theta \in [-h, 0], \end{cases} \quad (1)$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$ is the state vector, $\boldsymbol{A}(t)$, $\boldsymbol{A}_1(t)\in\mathbb{R}^{n\times n}$ are known real constant matrices, and $\boldsymbol{\varphi}(\cdot)\in\mathbb{R}^n$ is the initial condition function assumed to be continuously differentiable on $[-h, 0]$. Here, $h>0$ is a constant scalar indicating the delay time.

Xu *et al.*(2006) provided a stability criterion stated as follows:

Proposition 1 [Theorem 1, (Xu *et al.*, 2006)] For given scalars $\lambda>0$ and $h>0$, the nominal time-delay system (1) is exponentially stable at decay rate λ, if there exist symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$, and matrices $\boldsymbol{P}_2$, $\boldsymbol{Y}$, $\boldsymbol{W}$, $\boldsymbol{S}\in\mathbb{R}^{n\times n}$ such that

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} & \boldsymbol{\Gamma}_{13} & h\boldsymbol{Y} & \boldsymbol{\Gamma}_{15} \\ * & \boldsymbol{\Gamma}_{22} & \boldsymbol{\Gamma}_{23} & h\boldsymbol{W} & \boldsymbol{\Gamma}_{25} \\ * & * & -h\boldsymbol{Z}_2 & h^2\boldsymbol{S} & \boldsymbol{0} \\ * & * & * & -h\boldsymbol{Z}_1 & \boldsymbol{0} \\ * & * & * & * & -h\boldsymbol{Z}_1 \end{bmatrix} < \boldsymbol{0}, \quad (2a)$$

$$\begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ * & \boldsymbol{P}_3 \end{bmatrix} > \boldsymbol{0}, \quad (2b)$$

where

$$\begin{cases} \boldsymbol{\Gamma}_{11} = \boldsymbol{P}_1(\boldsymbol{A}+\lambda\boldsymbol{I}) + (\boldsymbol{A}+\lambda\boldsymbol{I})^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}} \\ \qquad + \boldsymbol{Q} + h\boldsymbol{Z}_2 - \boldsymbol{Y} - \boldsymbol{Y}^{\mathrm{T}}, \\ \boldsymbol{\Gamma}_{12} = \mathrm{e}^{\lambda h}\boldsymbol{P}_1\boldsymbol{A}_1 - \boldsymbol{P}_2 + \boldsymbol{Y} - \boldsymbol{W}^{\mathrm{T}}, \\ \boldsymbol{\Gamma}_{13} = h(\boldsymbol{A}+\lambda\boldsymbol{I})^{\mathrm{T}}\boldsymbol{P}_2 + h\boldsymbol{P}_3 - h\boldsymbol{S}^{\mathrm{T}}, \\ \boldsymbol{\Gamma}_{15} = h(\boldsymbol{A}+\lambda\boldsymbol{I})^{\mathrm{T}}\boldsymbol{Z}_1,\ \boldsymbol{\Gamma}_{22} = -\boldsymbol{Q} + \boldsymbol{W} + \boldsymbol{W}^{\mathrm{T}}, \\ \boldsymbol{\Gamma}_{23} = h\mathrm{e}^{\lambda h}\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_2 - h\boldsymbol{P}_3 + h\boldsymbol{S}^{\mathrm{T}},\ \boldsymbol{\Gamma}_{25} = h\mathrm{e}^{\lambda h}\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{Z}_1. \end{cases} \quad (3)$$

Then, they considered the following time-delay system with norm-bounded uncertainties:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = [\boldsymbol{A} + \Delta\boldsymbol{A}(t)]\boldsymbol{x}(t) + [\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1(t)]\boldsymbol{x}(t-h), \\ \boldsymbol{x}(\theta) = \boldsymbol{\varphi}(\theta), \forall\theta \in [-h, 0], \end{cases} \quad (4)$$

where $\Delta\boldsymbol{A}(t)$, $\Delta\boldsymbol{A}_1(t)\in\mathbb{R}^{n\times n}$ are time-varying parameter uncertainties, described by

$$[\Delta\boldsymbol{A}(t), \Delta\boldsymbol{A}_1(t)] = \boldsymbol{D}\boldsymbol{F}(t)[\boldsymbol{E}, \boldsymbol{E}_1], \quad (5)$$

where $\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{E}_1$ are constant matrices with appropriate dimensions, and $\boldsymbol{F}(t)$ is an unknown time-varying matrix function satisfying $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\le\boldsymbol{I}$.

Proposition 2 [Theorem 2, (Xu *et al.*, 2006)] For given scalars $\lambda>0$ and $h>0$, the uncertain time-delay system (4) is robustly exponentially stable at decay rate λ, if there exist a scalar $\varepsilon>0$, symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$ and matrices $\boldsymbol{P}_2$, $\boldsymbol{Y}$, $\boldsymbol{W}$, $\boldsymbol{S}\in\mathbb{R}^{n\times n}$ such that the LMIs (6) hold,

$$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\Theta}_{11} & \boldsymbol{\Theta}_{12} & \boldsymbol{\Gamma}_{13} & h\boldsymbol{Y} & \boldsymbol{\Gamma}_{15} & \boldsymbol{P}_1\boldsymbol{D} \\ * & \boldsymbol{\Theta}_{22} & \boldsymbol{\Gamma}_{23} & h\boldsymbol{W} & \boldsymbol{\Gamma}_{25} & \boldsymbol{0} \\ * & * & -h\boldsymbol{Z}_2 & h^2\boldsymbol{S} & \boldsymbol{0} & h\boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{D} \\ * & * & * & -h\boldsymbol{Z}_1 & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -h\boldsymbol{Z}_1 & h\boldsymbol{Z}_1\boldsymbol{D} \\ * & * & * & * & * & -\varepsilon\boldsymbol{I} \end{bmatrix} < \boldsymbol{0}, \quad (6a)$$

$$\begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ * & \boldsymbol{P}_3 \end{bmatrix} > \mathbf{0}, \tag{6b}$$

where

$$\begin{cases} \boldsymbol{\Theta}_{11} = \boldsymbol{\Gamma}_{11} + \varepsilon \boldsymbol{E}^{\mathrm{T}} \boldsymbol{E}, \\ \boldsymbol{\Theta}_{12} = \boldsymbol{\Gamma}_{12} + \varepsilon \mathrm{e}^{\lambda h} \boldsymbol{E}^{\mathrm{T}} \boldsymbol{E}_1, \\ \boldsymbol{\Theta}_{22} = \boldsymbol{\Gamma}_{22} + \varepsilon \mathrm{e}^{2\lambda h} \boldsymbol{E}_1^{\mathrm{T}} \boldsymbol{E}_1. \end{cases} \tag{7}$$

It should be noted that the coefficients $\mathrm{e}^{\lambda h}$ and $\mathrm{e}^{2\lambda h}$ are missing in Theorem 2 of (Xu *et al.*, 2006).

By the use of double integrals, Propositions 1 and 2 have introduced more slack matrices than the results of (Xu and Lam, 2005). The matrix variable $\boldsymbol{S}$ did not appear there because only single integrals were used there. It is well known that the introduction of some additional slack matrix variables will make it more flexible in solving LMIs, and reduce the conservatism (Cao and Xue, 2005; Wu *et al.*, 2004a; 2004b; Xu and Lam, 2005). However, too many slack matrices will make the stability criteria more complicated and the computation more time-consuming, which motivate us to find simpler stability criteria without increasing the conservatism compared with (Xu *et al.*, 2006).

MAIN RESULTS

In this section, we present new delay-dependent stability criteria for systems (1) and (4) by using strict LMI optimization approaches.

We first present two lemmas that will be useful for deriving our main results.

Lemma 1 (Boyd *et al.*, 1994) Let $\boldsymbol{\Phi}$ be a given symmetric matrix, $\boldsymbol{H}$ and $\boldsymbol{G}$ be matrices with approximate dimensions. Then for all $\boldsymbol{F}(t)$ satisfying $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leq\boldsymbol{I}$, the following inequality

$$\boldsymbol{\Phi}+\boldsymbol{HF}(t)\boldsymbol{G}+\boldsymbol{G}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{H}^{\mathrm{T}}<\mathbf{0}$$

holds if and only if there exists a scalar $\varepsilon>0$ such that

$$\boldsymbol{\Phi}+\varepsilon\boldsymbol{HH}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}<\mathbf{0}.$$

Lemma 2 (Han, 2005) For any constant symmetric matrix $\boldsymbol{R}>\mathbf{0}$, scalar $h>0$, and vector function $\boldsymbol{x}(\cdot)$: $[-h, 0]\rightarrow\mathbb{R}^n$ such that the following integral is well defined, then

$$-h\int_{t-h}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s \leq \boldsymbol{\eta}^{\mathrm{T}}(t)\begin{bmatrix} -\boldsymbol{R} & \boldsymbol{R} \\ \boldsymbol{R} & -\boldsymbol{R} \end{bmatrix}\eta(t), \tag{8}$$

where $\boldsymbol{\eta}^{\mathrm{T}}(t)=[\boldsymbol{x}^{\mathrm{T}}(t), \boldsymbol{x}^{\mathrm{T}}(t-h)]$.

Let us introduce the following transformation (Liu, 2003; Kwon and Park, 2006; Xu *et al.*, 2006):

$$\boldsymbol{z}(t)= \mathrm{e}^{\lambda t}\boldsymbol{x}(t), \tag{9}$$

where the positive scalar λ is the decay rate (i.e. exponential stability degree), then the nominal system (1) is transformed into:

$$\dot{\boldsymbol{z}}(t) = \bar{\boldsymbol{A}}\boldsymbol{z}(t) + \bar{\boldsymbol{A}}_1\boldsymbol{z}(t-h), \tag{10}$$

where $\bar{\boldsymbol{A}} = \boldsymbol{A} + \lambda\boldsymbol{I}$ and $\bar{\boldsymbol{A}}_1 = \mathrm{e}^{\lambda h}\boldsymbol{A}_1$.

Accordingly, the uncertain system (4) can be rewritten as

$$\dot{\boldsymbol{z}}(t) = \tilde{\boldsymbol{A}}\boldsymbol{z}(t) + \tilde{\boldsymbol{A}}_1\boldsymbol{z}(t-h), \tag{11}$$

where $\tilde{\boldsymbol{A}}=\boldsymbol{A}+\lambda\boldsymbol{I}+\Delta\boldsymbol{A}(t)$ and $\tilde{\boldsymbol{A}}_1=\mathrm{e}^{\lambda h}[\boldsymbol{A}_1+\Delta\boldsymbol{A}_1(t)]$. The correspondingly exponential stability criteria are given by the following theorems.

Theorem 1 Considering the system (1), for given scalars $\lambda>0$ and $h>0$, if there exist positive-definite matrices symmetric $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{P}_2\in\mathbb{R}^{n\times n}$ satisfying the following LMIs:

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{\Omega}_{13} & \boldsymbol{\Gamma}_{15} \\ * & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{23} & \boldsymbol{\Gamma}_{25} \\ * & * & -h\boldsymbol{Z}_2 & \mathbf{0} \\ * & * & * & -h\boldsymbol{Z}_1 \end{bmatrix} < \mathbf{0}, \tag{12a}$$

$$\begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ * & \boldsymbol{P}_3 \end{bmatrix} > \mathbf{0}, \tag{12b}$$

where

$$\begin{cases} \boldsymbol{\Omega}_{11} = \boldsymbol{P}_1(\boldsymbol{A}+\lambda\boldsymbol{I}) + (\boldsymbol{A}+\lambda\boldsymbol{I})^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}} \\ \qquad +\boldsymbol{Q} - \boldsymbol{Z}_1/h + h\boldsymbol{Z}_2, \\ \boldsymbol{\Omega}_{12} = \mathrm{e}^{\lambda h}\boldsymbol{P}_1\boldsymbol{A}_1 - \boldsymbol{P}_2 + \boldsymbol{Z}_1/h,\ \boldsymbol{\Omega}_{22} = -\boldsymbol{Q} - \boldsymbol{Z}_1/h, \\ \boldsymbol{\Omega}_{13} = h\mathrm{e}^{\lambda h}(\boldsymbol{A}+\lambda\boldsymbol{I})^{\mathrm{T}}\boldsymbol{P}_2 + h\boldsymbol{P}_3, \\ \boldsymbol{\Omega}_{23} = h\mathrm{e}^{\lambda h}\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_2 - h\boldsymbol{P}_3, \end{cases} \tag{13}$$

then the system (1) is exponentially stable at decay rate λ.

Proof Choose the following legitimate Lyapunov-Krasovskii functional candidate (Xu *et al*., 2006)

$$V(t,z(t)) = V_1 + V_2 + V_3, \tag{14}$$

where

$$V_1 = \varsigma^{\mathrm{T}}(t)\begin{bmatrix} P_1 & P_2 \\ P_2^{\mathrm{T}} & P_3 \end{bmatrix}\varsigma(t),\quad V_2 = \int_{t-h}^{t} z^{\mathrm{T}}(s)Qz(s)\mathrm{d}s,$$

$$V_3 = \int_{-h}^{0}\int_{t+\beta}^{t} \dot{z}^{\mathrm{T}}(s)Z_1\dot{z}(s)\mathrm{d}s\mathrm{d}\beta + \int_{-h}^{0}\int_{t+\beta}^{t} z^{\mathrm{T}}(s)Z_2 z(s)\mathrm{d}s\mathrm{d}\beta,$$

$$\varsigma^{\mathrm{T}}(t) = \begin{bmatrix} z^{\mathrm{T}}(t) & \left(\int_{t-h}^{t} z(s)\mathrm{d}s\right)^{\mathrm{T}} \end{bmatrix}.$$

Then the time-derivative of $V(t, z(t))$ with respect to t along the system (10) is

$$\dot{V}(t,z(t)) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3,$$

where

$$\begin{aligned}\dot{V}_1 &= 2z^{\mathrm{T}}(t)P_1\left[\bar{A}z(t) + \bar{A}_1 z(t-h)\right] \\ &\quad + 2\left[\bar{A}z(t) + \bar{A}_1 z(t-h)\right]^{\mathrm{T}} P_2\int_{t-h}^{t} z(s)\mathrm{d}s \\ &\quad + 2z^{\mathrm{T}}(t)P_2[z(t) - z(t-h)] \\ &\quad + 2[z(t) - z(t-h)]^{\mathrm{T}} P_3\int_{t-h}^{t} z(s)\mathrm{d}s,\end{aligned}$$

$$\dot{V}_2 = z^{\mathrm{T}}(t)Qz(t) - z^{\mathrm{T}}(t-h)Qz(t-h),$$

and by means of Lemma 2

$$\begin{aligned}\dot{V}_3 &= h\dot{z}^{\mathrm{T}}(t)Z_1\dot{z}(t) - \int_{t-h}^{t}\dot{z}^{\mathrm{T}}(s)Z_1\dot{z}(s)\mathrm{d}s \\ &\quad + hz^{\mathrm{T}}(t)Z_2 z(t) - \int_{t-h}^{t} z^{\mathrm{T}}(s)Z_2 z(s)\mathrm{d}s \\ &\le h\dot{z}^{\mathrm{T}}(t)Z_1\dot{z}(t) - \frac{1}{h}\eta^{\mathrm{T}}(t)\begin{bmatrix} -Z_1 & Z_1 \\ Z_1 & -Z_1 \end{bmatrix}\eta(t) \\ &\quad + hz^{\mathrm{T}}(t)Z_2 z(t) - \int_{t-h}^{t} z^{\mathrm{T}}(s)Z_2 z(s)\mathrm{d}s,\end{aligned}$$

it is easy to get

$$\dot{V}(t,z(t)) \le \frac{1}{h}\int_{t-h}^{t}\mu^{\mathrm{T}}(t,s)\Xi\mu(t,s)\mathrm{d}s, \tag{15}$$

where

$$\mu^{\mathrm{T}}(t,s) = \begin{bmatrix} z^{\mathrm{T}}(t) & z^{\mathrm{T}}(t-h) & z^{\mathrm{T}}(s) \end{bmatrix},$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & h\bar{A}^{\mathrm{T}}P_2 + hP_3 \\ * & \Xi_{22} & h\bar{A}_1^{\mathrm{T}}P_2 - hP_3 \\ * & * & -hZ_2 \end{bmatrix},$$

with

$$\begin{cases}\Xi_{11} = P_1\bar{A} + \bar{A}^{\mathrm{T}}P_1 + P_2 + P_2^{\mathrm{T}} + Q \\ \qquad - Z_1/h + hZ_2 + h\bar{A}^{\mathrm{T}}Z_1\bar{A}, \\ \Xi_{12} = P_1\bar{A}_1 - P_2 + Z_1/h + h\bar{A}^{\mathrm{T}}Z_1\bar{A}_1, \\ \Xi_{22} = -Q - Z_1/h + h\bar{A}_1^{\mathrm{T}}Z_1\bar{A}_1.\end{cases}$$

If $\Omega<0$, then $\Xi<0$ by applying Schur complement lemma (Gu *et al*., 2003), which implies $\dot{V}(t,z(t)) < 0$.

So, as a consequence of Lyapunov-Krasovskii stability theorem (Gu *et al*., 2003), system (10) is asymptotically stable.

It is well known that the system (1) is exponentially stable at decay rate λ, provided that the system (10) is asymptotically stable (Liu, 2003). This completes the proof.

Theorem 1 can be extended to the uncertain delayed system (4) as shown below.

Theorem 2 For given scalars $\lambda>0$ and $h>0$, the uncertain time-delay system (4) is robustly exponentially stable at decay rate λ, if there exist a positive scalar $\varepsilon>0$, symmetric positive-definite matrices P_1, P_3, Q, Z_1, $Z_2\in\mathbb{R}^{n\times n}$, and a matrix $P_2\in\mathbb{R}^{n\times n}$ satisfying the following LMIs:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Omega_{13} & \Gamma_{15} & P_1D \\ * & \Lambda_{22} & \Omega_{23} & \Gamma_{25} & 0 \\ * & * & -hZ_2 & 0 & hP_2^{\mathrm{T}}D \\ * & * & * & -hZ_1 & hZ_1D \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{16a}$$

$$\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0, \tag{16b}$$

where

$$\begin{cases}\Lambda_{11} = \Omega_{11} + \varepsilon E^{\mathrm{T}}E, \\ \Lambda_{12} = \Omega_{12} + \varepsilon\mathrm{e}^{\lambda h}E^{\mathrm{T}}E_1, \\ \Lambda_{22} = \Omega_{22} + \varepsilon\mathrm{e}^{2\lambda h}E_1^{\mathrm{T}}E_1.\end{cases} \tag{17}$$

Proof Taking the Lyapunov-Krasovskii functional candidate as Eq.(14), then we can obtain the following inequality by a similar derivation as in the proof of Theorem 1:

$$\dot{V}(t,z(t)) \le \frac{1}{h}\int_{t-h}^{t}\mu^{\mathrm{T}}(t,s)\Sigma\mu(t,s)\mathrm{d}s, \tag{18}$$

where

$$\boldsymbol{\Sigma}=\begin{bmatrix}\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} & h\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_2+h\boldsymbol{P}_3\\ * & \boldsymbol{\Sigma}_{22} & h\tilde{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{P}_2-h\boldsymbol{P}_3\\ * & * & -h\boldsymbol{Z}_2\end{bmatrix}, \quad (19)$$

with

$$\begin{cases}\boldsymbol{\Sigma}_{11}=\boldsymbol{P}_1\tilde{\boldsymbol{A}}+\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_1+\boldsymbol{P}_2+\boldsymbol{P}_2^{\mathrm{T}}+\boldsymbol{Q}\\ \qquad -\boldsymbol{Z}_1/h+h\boldsymbol{Z}_2+h\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Z}_1\tilde{\boldsymbol{A}},\\ \boldsymbol{\Sigma}_{12}=\boldsymbol{P}_1\tilde{\boldsymbol{A}}_1-\boldsymbol{P}_2+\boldsymbol{Z}_1/h+h\tilde{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{Z}_1\tilde{\boldsymbol{A}}_1,\\ \boldsymbol{\Sigma}_{22}=-\boldsymbol{Q}-\boldsymbol{Z}_1/h+h\tilde{\boldsymbol{A}}_1^{\mathrm{T}}\boldsymbol{Z}_1\tilde{\boldsymbol{A}}_1.\end{cases}$$

By Schur complement lemma, we have

$$\boldsymbol{\Sigma}=\boldsymbol{\Omega}+\begin{bmatrix}\boldsymbol{P}_1\boldsymbol{D}\\ \boldsymbol{0}\\ h\boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{D}\\ h\boldsymbol{Z}_1\boldsymbol{D}\end{bmatrix}\boldsymbol{F}(t)\begin{bmatrix}\boldsymbol{E} & \mathrm{e}^{\lambda h}\boldsymbol{E}_1 & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}$$
$$+\begin{bmatrix}\boldsymbol{E}^{\mathrm{T}}\\ \mathrm{e}^{\lambda h}\boldsymbol{E}_1^{\mathrm{T}}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{D}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{P}_1 & \boldsymbol{0} & h\boldsymbol{P}_2 & h\boldsymbol{Z}_1\end{bmatrix}<\boldsymbol{0}. \quad (20)$$

According to Lemma 1, $\boldsymbol{\Sigma}<\boldsymbol{0}$ if and only if there exists a positive scalar ε such that:

$$\boldsymbol{\Sigma}=\boldsymbol{\Omega}+\varepsilon^{-1}\begin{bmatrix}\boldsymbol{P}_1\boldsymbol{D}\\ \boldsymbol{0}\\ h\boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{D}\\ h\mathrm{Z}_1\boldsymbol{D}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}_1\boldsymbol{D}\\ \boldsymbol{0}\\ h\boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{D}\\ h\mathrm{Z}_1\boldsymbol{D}\end{bmatrix}^{\mathrm{T}}+\varepsilon\begin{bmatrix}\boldsymbol{E}^{\mathrm{T}}\\ \mathrm{e}^{\lambda h}\boldsymbol{E}_1^{\mathrm{T}}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{E}^{\mathrm{T}}\\ \mathrm{e}^{\lambda h}\boldsymbol{E}_1^{\mathrm{T}}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}<\boldsymbol{0}. \quad (21)$$

Then, it follows from Schur complement directly that $\boldsymbol{\Sigma}<\boldsymbol{0}$ is equivalent to $\boldsymbol{\Lambda}<\boldsymbol{0}$.

Thus, if $\boldsymbol{\Lambda}<\boldsymbol{0}$ then $\dot{V}(t,\boldsymbol{z}(t))<\boldsymbol{0}$, and the system (11) is robustly asymptotically stable, which guarantees that the system (4) is robustly exponentially stable at decay rate λ.

Remark 1 Theorems 1 and 2 are new delay-dependent exponential stability criteria derived by an integral inequality, without resorting to model transformation and cross terms bounding techniques. The corresponding stability conditions are expected to be less conservative.

Remark 2 The forms of Theorems 1 and 2 are simpler than Propositions 1 and 2, respectively, since fewer matrix variables and lower dimensions of the LMIs are involved, due to the fact that no additional slack matrices (no matrix variables $\boldsymbol{Y}$, $\boldsymbol{W}$ and $\boldsymbol{S}$) have been introduced. Precisely, the dimensions of the matrix inequalities (12a) and (16a) are $4n\times4n$ and $5n\times5n$, respectively. And the dimensions of Eq.(2a) and Eq.(6a) are $5n\times5n$ and $6n\times6n$, respectively.

Remark 3 By solving the LMIs (12) and (16) iteratively with respect to h, one can obtain the maximal allowable delays $\bar{h}$ guaranteeing the nominal system (1) and uncertain system (4) to be exponentially stable and robustly exponentially stable at given decay rates λ, and vice versa.

By setting λ=0 in Theorems 1 and 2, we can obtain the asymptotic stability criteria of systems (1) and (4), respectively, as follows:

Corollary 1 If the following LMIs

$$\boldsymbol{\Phi}=\begin{bmatrix}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} & \boldsymbol{\Phi}_{13} & h\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Z}_1\\ * & \boldsymbol{\Phi}_{22} & \boldsymbol{\Phi}_{23} & h\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{Z}_1\\ * & * & -h\boldsymbol{Z}_2 & \boldsymbol{0}\\ * & * & * & -h\boldsymbol{Z}_1\end{bmatrix}<\boldsymbol{0}, \quad (22a)$$

$$\begin{bmatrix}\boldsymbol{P}_1 & \boldsymbol{P}_2\\ * & \boldsymbol{P}_3\end{bmatrix}>\boldsymbol{0}, \quad (22b)$$

where

$$\begin{cases}\boldsymbol{\Phi}_{11}=\boldsymbol{P}_1\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_1+\boldsymbol{P}_2+\boldsymbol{P}_2^{\mathrm{T}}+\boldsymbol{Q}-\boldsymbol{Z}_1/h+h\boldsymbol{Z}_2,\\ \boldsymbol{\Phi}_{12}=\boldsymbol{P}_1\boldsymbol{A}_1-\boldsymbol{P}_2+\boldsymbol{Z}_1/h,\ \boldsymbol{\Phi}_{22}=-\boldsymbol{Q}-\boldsymbol{Z}_1/h,\\ \boldsymbol{\Phi}_{13}=h\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_2+h\boldsymbol{P}_3,\ \boldsymbol{\Phi}_{23}=h\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{P}_2-h\boldsymbol{P}_3,\end{cases} \quad (23)$$

have a matrix symmetric and positive-definite solutions $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$, then the time-delay system (1) is asymptotically stable.

Corollary 2 The uncertain time-delay system (4) is asymptotically stable, if there exist a scalar ε>0, a matrix $\boldsymbol{P}_2\in\mathbb{R}^{n\times n}$ and symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$ such that the following LMIs are satisfied:

$$\boldsymbol{\Psi}=\begin{bmatrix}\boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{12} & \boldsymbol{\Phi}_{13} & h\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Z}_1 & \boldsymbol{P}_1\boldsymbol{D}\\ * & \boldsymbol{\Psi}_{22} & \boldsymbol{\Phi}_{23} & h\boldsymbol{A}_1^{\mathrm{T}}\boldsymbol{Z}_1 & \boldsymbol{0}\\ * & * & -h\boldsymbol{Z}_2 & \boldsymbol{0} & h\boldsymbol{P}_2^{\mathrm{T}}\boldsymbol{D}\\ * & * & * & -h\boldsymbol{Z}_1 & h\boldsymbol{Z}_1\boldsymbol{D}\\ * & * & * & * & -h\boldsymbol{Z}_1\end{bmatrix}<\boldsymbol{0}, \quad (24a)$$

$$\begin{bmatrix}\boldsymbol{P}_1 & \boldsymbol{P}_2\\ * & \boldsymbol{P}_3\end{bmatrix}>\boldsymbol{0}, \quad (24b)$$

where

$$\begin{cases} \boldsymbol{\Psi}_{11} = \boldsymbol{\Phi}_{11} + \varepsilon \boldsymbol{E}^{\mathrm{T}} \boldsymbol{E}, \\ \boldsymbol{\Psi}_{12} = \boldsymbol{\Phi}_{12} + \varepsilon \boldsymbol{E}^{\mathrm{T}} \boldsymbol{E}_1, \\ \boldsymbol{\Psi}_{22} = \boldsymbol{\Phi}_{22} + \varepsilon \boldsymbol{E}_1^{\mathrm{T}} \boldsymbol{E}_1. \end{cases} \tag{25}$$

Similar to Remark 2, Corollaries 1 and 2 are simpler than the counterparts of (Xu *et al.*, 2006).

ILLUSTRATIVE EXAMPLES

In this section, we provide three numerical examples to demonstrate the effectiveness of the proposed approaches in the previous section.

Example 1 Consider the nominal linear time-delay system (1) with:

$$\boldsymbol{A} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \boldsymbol{A}_1 = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix}. \tag{26}$$

This system was considered in (Liu, 2003) and (Xu *et al.*, 2006). For given h, the maximal allowable decay rates $\bar{\lambda}$ computed by Theorem 1 and some existing approaches are shown in Table 1.

Table 1 Comparison of the maximal allowable decay rates $\bar{\lambda}$ for Example 1

h	$\bar{\lambda}$			
	(Liu, 2003)	(Mondié and Kharitonov, 2005)	Proposition 1	Theorem 1
0.8	0.9366	0.7344	0.9366	0.9366
1.0	0.5903	0.6715	0.9192	0.9192
1.2	0.3400	0.6145	0.8990	0.8990
1.4	0.1813	0.5642	0.8115	0.8115
1.6	0.0752	0.5202	0.6990	0.6990
1.8	0.0014	0.4818	0.6148	0.6148
2.0	0	0.4481	0.5494	0.5494

Example 2 Consider the following uncertain time-delay system (Xu *et al.*, 2006):

$$\begin{cases} \boldsymbol{A} = \begin{bmatrix} -4 & 1 \\ 0 & -4 \end{bmatrix}, \boldsymbol{A}_1 = \begin{bmatrix} 0.1 & 0 \\ 4 & 0.1 \end{bmatrix}, \\ \boldsymbol{D} = 0.2\boldsymbol{I}, \ \boldsymbol{E} = \boldsymbol{E}_1 = \boldsymbol{I}. \end{cases} \tag{27}$$

Table 2 lists the maxima of decay rate $\bar{\lambda}$ obtained by Theorem 2 and two other methods for different h.

Table 2 Comparison of the maximal allowable decay rates $\bar{\lambda}$ for Example 2

h	$\bar{\lambda}$		
	(Mondié and Kharitonov, 2005)	Proposition 2	Theorem 1
0.3	0.6255	0.9531	0.9531
0.5	0.4760	0.7687	0.7687
0.7	0.3825	0.6408	0.6408
0.9	0.3191	0.5480	0.5480
1.1	0.2735	0.4781	0.4781
1.3	0.2392	0.4236	0.4236
1.5	0.2125	0.3802	0.3802

It indicates that, from the above two examples, Propositions 1, 2 and Theorems 1, 2 are both less conservative than some other methods in the literature. On the other hand, the conditions based on these two methods can achieve the same upper bounds of $\bar{\lambda}$ for both nominal system (1) and uncertain system (4). In other words, the simpler Theorems 1 and 2 do not introduce additional conservatism compared with Propositions 1 and 2.

Example 3 Consider the asymptotic stability of the following time-delay system in (Xu and Lam, 2005) with norm-bounded parameter uncertainties:

$$\begin{cases} \boldsymbol{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \boldsymbol{A}_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \\ \boldsymbol{D} = \gamma \boldsymbol{I}, \ \boldsymbol{E} = \boldsymbol{E}_1 = \boldsymbol{I}. \end{cases} \tag{28}$$

Table 3 shows the comparison of the maximum allowable delays $\bar{h}$ for various uncertainty degrees γ by different methods. It is clear that $\bar{h}$ by Corollary 1 of this paper and (Han, 2005) are the equivalent to those by (Xu *et al.*, 2006) and (Xu and Lam, 2005), respectively.

Table 3 Comparison of the maximal allowable delays $\bar{h}$ for Example 3

γ	$\bar{h}$			
	(Xu and Lam, 2005)	(Han, 2005)	(Xu *et al.*, 2006)	Corollaries 1 and 2
0	4.4721	4.4721	4.4721	4.4721
0.1	3.2172	3.2172	3.2415	3.2415
0.2	2.3970	2.3970	2.4317	2.4317
0.3	1.8607	1.8607	1.8961	1.8961
0.4	1.4950	1.4950	1.5185	1.5185
0.5	1.2310	1.2310	1.2374	1.2374
0.6	1.0181	1.0181	1.0181	1.0181

EQUIVALENCE OF THE TWO METHODS

From the numerical examples in the previous section, we present the following theorems.

Theorem 3 For given scalars $\lambda>0$ and $h>0$, there exist symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$ and matrices $\boldsymbol{P}_2$, $\boldsymbol{Y}$, $\boldsymbol{W}$, $\boldsymbol{S}\in\mathbb{R}^{n\times n}$ such that Eqs.(2a) and (2b) hold if and only if there exist symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{P}_2\in\mathbb{R}^{n\times n}$ such that Eqs.(12a) and (12b) hold.

Proof (1) $\boldsymbol{\Gamma}<\boldsymbol{0}\Rightarrow\boldsymbol{\Omega}<\boldsymbol{0}$: To obtain Eq.(12a), it suffices to pre-multiply and post-multiply $\boldsymbol{\Gamma}<\boldsymbol{0}$ with $\boldsymbol{T}(h)$ and $\boldsymbol{T}^{\mathrm{T}}(h)$, respectively, where

$$\boldsymbol{T}(h)=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}/h & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{I}/h & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}. \tag{29}$$

Thus, $\boldsymbol{\Gamma}<\boldsymbol{0}$ implies $\boldsymbol{\Omega}=\boldsymbol{T}(h)\boldsymbol{\Gamma}\boldsymbol{T}^{T}(h)<\boldsymbol{0}$.

(2) $\boldsymbol{\Omega}<\boldsymbol{0}\Rightarrow\boldsymbol{\Gamma}<\boldsymbol{0}$: By solving the LMIs (12), we can obtain a matrix $\boldsymbol{P}_2$ and symmetric positive- definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$ such that $\boldsymbol{\Omega}<\boldsymbol{0}$ is satisfied.

For a given symmetric and positive-definite matrix $\boldsymbol{Z}_1$, setting $\boldsymbol{W}=\boldsymbol{W}^{\mathrm{T}}=-\boldsymbol{Z}_1/h$, and $\boldsymbol{Y}=\boldsymbol{Y}^{\mathrm{T}}=\boldsymbol{Z}_1/h$, then we have

$$\begin{cases}\boldsymbol{\Delta}_{11}=\boldsymbol{Z}_1/h-\boldsymbol{Y}-\boldsymbol{Y}^{\mathrm{T}}+h\boldsymbol{Y}\boldsymbol{Z}_1^{-1}\boldsymbol{Y}^{\mathrm{T}}=\boldsymbol{0},\\ \boldsymbol{\Delta}_{12}=-\boldsymbol{Z}_1/h+\boldsymbol{Y}-\boldsymbol{W}^{\mathrm{T}}+h\boldsymbol{Y}\boldsymbol{Z}_1^{-1}\boldsymbol{W}^{\mathrm{T}}=\boldsymbol{0},\\ \boldsymbol{\Delta}_{13}=-h\boldsymbol{S}^{\mathrm{T}}+h^2\boldsymbol{Y}\boldsymbol{Z}_1^{-1}\boldsymbol{S}^{\mathrm{T}}=\boldsymbol{0},\\ \boldsymbol{\Delta}_{22}=\boldsymbol{Z}_1/h+\boldsymbol{W}+\boldsymbol{W}^{\mathrm{T}}+h\boldsymbol{W}\boldsymbol{Z}_1^{-1}\boldsymbol{W}^{\mathrm{T}}=\boldsymbol{0},\\ \boldsymbol{\Delta}_{23}=h\boldsymbol{S}^{\mathrm{T}}+h^2\boldsymbol{W}\boldsymbol{Z}_1^{-1}\boldsymbol{S}^{\mathrm{T}}=\boldsymbol{0}.\end{cases} \tag{30}$$

And for any matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_2$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$ given by Eq.(12), the following inequality is true:

$$\boldsymbol{\Omega}+\begin{bmatrix}\boldsymbol{\Delta}_{11} & \boldsymbol{\Delta}_{12} & \boldsymbol{\Delta}_{13} & \boldsymbol{0}\\ * & \boldsymbol{\Delta}_{22} & \boldsymbol{\Delta}_{23} & \boldsymbol{0}\\ * & * & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}<\boldsymbol{0}. \tag{31}$$

It implies that there exists a scalar $\delta>0$ such that

$$\boldsymbol{\Omega}+\begin{bmatrix}\boldsymbol{\Delta}_{11} & \boldsymbol{\Delta}_{12} & \boldsymbol{\Delta}_{13} & \boldsymbol{0}\\ * & \boldsymbol{\Delta}_{22} & \boldsymbol{\Delta}_{23} & \boldsymbol{0}\\ * & * & \delta\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}<\boldsymbol{0}. \tag{32}$$

On the other hand, for a given positive-definite symmetric matrix $\boldsymbol{Z}_1$, we can always find a matrix $\boldsymbol{S}\in\mathbb{R}^{n\times n}$ satisfying $h^3\boldsymbol{S}\boldsymbol{Z}_1^{-1}\boldsymbol{S}^{\mathrm{T}}=\delta\boldsymbol{I}$. That is

$$\boldsymbol{\Omega}+\begin{bmatrix}\boldsymbol{\Delta}_{11} & \boldsymbol{\Delta}_{12} & \boldsymbol{\Delta}_{13} & \boldsymbol{0}\\ * & \boldsymbol{\Delta}_{22} & \boldsymbol{\Delta}_{23} & \boldsymbol{0}\\ * & * & h^3\boldsymbol{S}\boldsymbol{Z}_1^{-1}\boldsymbol{S}^{\mathrm{T}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}<\boldsymbol{0}. \tag{33}$$

With the definition of $\boldsymbol{\Omega}$ in Theorem 1 and Schur complement, inequality (33) becomes $\boldsymbol{\Gamma}<\boldsymbol{0}$, which means that $\boldsymbol{\Gamma}<\boldsymbol{0}$ is implied by $\boldsymbol{\Omega}<\boldsymbol{0}$.

Theorem 4 For given scalars $\lambda>0$ and $h>0$, there exist a scalar $\varepsilon>0$, symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$ and matrices $\boldsymbol{P}_2$, $\boldsymbol{Y}$, $\boldsymbol{W}$, $\boldsymbol{S}\in\mathbb{R}^{n\times n}$ such that Eqs.(6a) and (6b) hold if and only if there exist symmetric positive-definite matrices $\boldsymbol{P}_1$, $\boldsymbol{P}_3$, $\boldsymbol{Q}$, $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2\in\mathbb{R}^{n\times n}$, a matrix $\boldsymbol{P}_2\in\mathbb{R}^{n\times n}$ and a scalar $\varepsilon>0$ such that Eqs.(16a) and (16b) hold.

Proof We can obtain $\boldsymbol{\Lambda}=\boldsymbol{U}(h)\boldsymbol{\Theta}\boldsymbol{U}^{\mathrm{T}}(h)<\boldsymbol{0}$, where

$$\boldsymbol{U}(h)=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}/h & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{I}/h & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}. \tag{34}$$

This means $\boldsymbol{\Lambda}<\boldsymbol{0}$ provided $\boldsymbol{\Theta}<\boldsymbol{0}$.

Following similar lines of the second part of the proof in Theorem 3, we can show that $\boldsymbol{\Theta}<\boldsymbol{0}$ as long as $\boldsymbol{\Lambda}<\boldsymbol{0}$.

If we choose the following Lyapunov-Krasovskii functional

$$V(t,\boldsymbol{x}(t))=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)+\int_{t-h}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\mathrm{d}s+\int_{-h}^{0}\int_{t+\beta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\beta, \tag{35}$$

we can conclude that the asymptotic stability conditions of (Xu and Lam, 2005) and those of (Han, 2005) are equivalent.

The simulation data in Table 3 have shown the equivalence of the two methods.

Remark 4 It follows from Theorems 3 and 4 that the method based on integral inequality (8) can simplify the exponential stability conditions in (Xu *et al.*, 2006). These simpler results will reduce the computation amount.

Remark 5 The improvements of the results of this paper and (Xu *et al.*, 2006) over those of (Xu and Lam, 2005) and (Han, 2005) given by Table 3 result from the different types of Lyapunov-Krasovskii functionals as Eq.(14) compared with Eq.(35). This gives us a hint that the possible directions for further improvements for time-delay systems are to choose different types of Lyapunov-Krasovskii functionals.

CONCLUSION

New delay-dependent exponential stability conditions based on an integral inequality have been obtained. Our criteria are simpler than those of (Xu *et al.*, 2006) without introducing any slack matrices, which lead to simpler forms of LMIs. We have proved that the proposed conditions are equivalent to those of (Xu *et al.*, 2006). Three illustrative examples have been provided to demonstrate the effectiveness.

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