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Initial pre-stress finding procedure and structural performance research for Levy cable dome based on linear adjustment theory*

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Abstract: The cable-strut structural system is statically and kinematically indeterminate. The initial pre-stress is a key factor for determining the shape and load carrying capacity. A new numerical algorithm is presented herein for the initial pre-stress finding procedure of complete cable-strut assembly. This method is based on the linear adjustment theory and does not take into account the material behavior. By using this method, the initial pre-stress of the multi self-stress modes can be found easily and the calculation process is simplified and efficient also. Finally, the initial pre-stress and structural performances of a particular Levy cable dome are analyzed comprehensively. The algorithm has proven to be efficient and correct, and the numerical results are valuable for practical design of Levy cable dome.

Key words: Linear adjustment theory, Cable-strut structure, Initial pre-stress, Levy cable dome, Structural performances analysis
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INTRODUCTION

The cable-strut structure is a special light weight structural system suitable for large span space structures. Cable dome is a particular cable-strut system, such as Geiger cable dome and Levy cable dome. Tensile cables and compression struts make up this system with the specific initial pre-stress in a self-equilibrium state. In other words, a special initial pre-stress will determine a unique cable-strut geometry. The initial pre-stress is a key factor for analyzing the shape and carrying capacity of the cable-strut system. On the contrary, for form-finding of cable nets and membrane structures (Otter, 1964; Argyris and Scharpf, 1972; Schek, 1974; Linkwitz and Grutndig, 1987; Gomez Estrada *et al.*, 2006), the essential analysis is to calculate the specific initial pre-stress with specified geometry and topology is usually referred to the architect and engineer's idea. This procedure may be referred to as force finding. In the past years, some methods have been presented to resolve this question, including the method based on infinitesimal mechanisms (Calladine, 1991; Pellegrino, 1990; 1993), the nonlinear force method (Luo and Dong, 2000) and the overall feasible pre-stress method (Yuan, 2003; Yuan and Dong, 2005), the method based on infinitesimal mechanisms assumes the cable-strut system as a statically and kinematically indeterminate system. That can only be used to calculate the initial pre-stress of first-order infinitesimal mechanisms and judge the potential stability. The nonlinear force method can avoid the stiffness matrix singularity from the displacement method by using singular value decomposition of equilibrium matrix. For self-stress modes of more than one, this method can get many sets of initial pre-stress, but the numerical procedure is inefficient. The overall feasible pre-stress method provides the lowest pre-stress level for the initial pre-stress of the cable-strut system. The lowest pre-stress level can keep the stability of the system, but this may not be the optimal initial

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pre-stress of practical cable-strut system.

This paper presents a new method for calculation of initial pre-stress of cable-strut assembly. This method is based on the linear adjustment theory without considering the material behavior (i.e. no specification of constitutive relationship). A schematic iterative procedure is employed for the specified initial pre-stress to find the equilibrated force. Using this method, the initial pre-stress of the multi self-stress modes can be found easily and efficiently. As an example, the initial pre-stress is calculated for a particular Levy cable dome where the load capacity and structural performances are investigated thoroughly.

LINEAR ADJUSTMENT THEORY

Theoretical background

Fig.1 shows a link of cable-strut structural assembly. The node coordinate vector, the node force vector and the residual vector for the force or for the coordinate are written as

$$\boldsymbol{X}^{\mathrm{T}}=[\boldsymbol{X}_j^{\mathrm{T}} \quad \boldsymbol{X}_k^{\mathrm{T}}]=[x_j \quad y_j \quad z_j \quad x_k \quad y_k \quad z_k], \tag{1}$$

$$\boldsymbol{r}^{\mathrm{T}}=[r_{xj} \quad r_{yj} \quad r_{zj} \quad r_{xk} \quad r_{yk} \quad r_{zk}], \tag{2}$$

$$\boldsymbol{v}^{\mathrm{T}}=[v_{xj} \quad v_{yj} \quad v_{zj} \quad v_{xk} \quad v_{yk} \quad v_{zk}]. \tag{3}$$

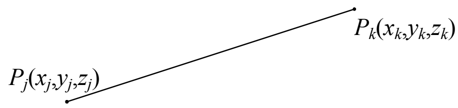

Fig.1 A link of cable-strut assembly

The branch node matrix for a link can be written as

$$\boldsymbol{c}=[\boldsymbol{E},-\boldsymbol{E}]=\begin{bmatrix}1 & 0 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & -1\end{bmatrix}, \tag{4}$$

where $\boldsymbol{E}$ is the 3×3 identity matrix.

The differences vector of the coordinates yields

$$\boldsymbol{u}=[u \quad v \quad w]^{\mathrm{T}}=[x_j-x_k \quad y_j-y_k \quad z_j-z_k]^{\mathrm{T}}=\boldsymbol{cX}. \tag{5}$$

The length of the link can be calculated from

$$l^2=(x_j-x_k)^2+(y_j-y_k)^2+(z_j-z_k)^2=\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}=\boldsymbol{X}^{\mathrm{T}}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{cX}. \tag{6}$$

In the case of the force density, q is the quotient of the force s divided by the stressed length l, so

$$q=s/l. \tag{7}$$

Based on the equilibrium equation of the force method, the equilibrium equation of the link can be written as follows:

$$\boldsymbol{r}+\boldsymbol{v}=\boldsymbol{c}^{\mathrm{T}}\boldsymbol{uq}=\boldsymbol{c}^{\mathrm{T}}\boldsymbol{Qu}=\boldsymbol{c}^{\mathrm{T}}\boldsymbol{QcX}. \tag{8}$$

For a cable-strut system, the equilibrium equation can be obtained as

$$\boldsymbol{r}+\boldsymbol{v}=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Uq}=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Qu}=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{QCX}. \tag{9}$$

Linear adjustment theory

Supposing that the node force vector is $\boldsymbol{b}_{n\times1}$ and that the individual weights in the diagonal matrix $\boldsymbol{P}_{n\times n}$, together with the residual vector $\boldsymbol{v}_{n\times1}$ known, and based on the force density method subjected to additional conditions (Singer, 1995), the residual equation is defined as

$$\boldsymbol{b}+\boldsymbol{v}=f(\boldsymbol{x})=\boldsymbol{Ax}, \tag{10}$$

where $\boldsymbol{A}_{n\times h}=\partial f(\boldsymbol{x})/\partial \boldsymbol{x}$ is a Jacobian matrix, $\boldsymbol{x}_{h\times1}$ is the h unknown nodes force vector. If the number of the known quantity is greater than the number of the unknown quantity, the matrix $\boldsymbol{A}$ is not square and there are many feasible combinations for determining the unknowns. Using the linear adjustment theory aforementioned, the equation can yield the optimal solution. The least square theory can be employed to find

$$\phi(\boldsymbol{x})=\boldsymbol{v}^{\mathrm{T}}\boldsymbol{Pv}\rightarrow \min, \tag{11}$$

where $\phi(\boldsymbol{x})$ is a function of the unknowns $\boldsymbol{x}$. The residual force vector will be minimum when $\phi(\boldsymbol{x})$ reaches minimum. Then

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Pv}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}(f(\boldsymbol{x})-\boldsymbol{b}). \tag{12}$$

The unknowns $\boldsymbol{x}$ can be calculated by linear iteration as

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Pb}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{PAx}. \tag{13}$$

This is the basic outline of the linear adjustment theory.

FORCE-FINDING USING LINEAR ADJUSTMENT THEORY

Force density equilibrium equation

A cable-strut assembly is assumed to have n_s nodes and m links, with the node coordinate vector $\boldsymbol{X}_{3n_s\times 1}$ being unknown. Based on Eq.(11), a least square formula for node coordinate can be written as:

$$\boldsymbol{\phi}(\boldsymbol{X})=\boldsymbol{v}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{v}=(f(\boldsymbol{X})-\boldsymbol{b})^{\mathrm{T}}\boldsymbol{P}(f(\boldsymbol{X})-\boldsymbol{b})\rightarrow\min. \quad (14)$$

An additional condition is that the distances of two adjacent nodes are zero

$$\boldsymbol{0}_i+\boldsymbol{v}_i=\sqrt{(x_j-x_k)^2+(y_j-y_k)^2+(z_j-z_k)^2}. \quad (15)$$

So, Eq.(14) can be written as

$$\boldsymbol{\phi}(\boldsymbol{X})=\sum_{i=1}^{m}q_i l_i^2=\sum_{i=1}^{m}q_i(u_i^2+v_i^2+w_i^2)\rightarrow\min. \quad (16)$$

The nonlinear residual Eq.(15) can be split into three linear residual equations

$$\begin{aligned}\boldsymbol{0}+\boldsymbol{v}_{xi}&=x_j-x_k=\boldsymbol{u},\\ \boldsymbol{0}+\boldsymbol{v}_{yi}&=y_j-y_k=\boldsymbol{v},\\ \boldsymbol{0}+\boldsymbol{v}_{zi}&=z_j-z_k=\boldsymbol{w}.\end{aligned} \quad (17)$$

With use of the node coordinate vector, Eq.(16) can be written as

$$\boldsymbol{\phi}(\boldsymbol{X})=\boldsymbol{u}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{u}=\boldsymbol{X}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_{\mathrm{s}}\boldsymbol{X}\rightarrow\min. \quad (18)$$

Then the equilibrium equation can be developed by linear adjustment theory

$$\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}\right)^{\mathrm{T}}\boldsymbol{Q}\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}\right)\boldsymbol{X}=\boldsymbol{C}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_{\mathrm{s}}\boldsymbol{X}=0. \quad (19)$$

The unknown node coordinates can thus be calculated by linear iteration.

Initial pre-stress determination of the cable-strut system

From Eq.(9): $\boldsymbol{r}+\boldsymbol{v}=\boldsymbol{C}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{U}\boldsymbol{q}$, without any additional condition, the Jacobian matrix is $\boldsymbol{A}=\boldsymbol{C}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{U}$. Given the weight identity matrix $\boldsymbol{P}=\boldsymbol{E}$, the node force vector $\boldsymbol{r}$ and the node residual force vector $\boldsymbol{v}$, the equilibrium equation can be formulated as

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}\boldsymbol{q}=\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{r}, \quad (20)$$

which can be rewritten as

$$\boldsymbol{U}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{s}}\boldsymbol{C}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{U}\boldsymbol{q}=\boldsymbol{U}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{s}}\boldsymbol{r}. \quad (21)$$

Given the force density of any link $\boldsymbol{q}_i$, the initial force density $\boldsymbol{q}_0$ of all links can be obtained from Eq.(21). The residual vector $\boldsymbol{v}$ can be obtained through $\boldsymbol{q}_0$, then substituted into Eq.(21) and re-calculated, until the iteration threshold is reached. Finally,

$$\boldsymbol{q}=\boldsymbol{q}_0+\Delta\boldsymbol{q}. \quad (22)$$

The force density vector $\boldsymbol{q}$ is the optimal solution and the initial pre-stress of the cable-strut structure can be calculated from Eq.(7).

The number of the known force density is determined by the self-stress modes of the cable-strut system. If there are s self-stress modes, the quantity of the given force density cannot exceed $s-1$. If there is only one self-stress mode, any one link force density can yield the solution. To choose the links of given force, one should consider the geometry symmetry and make sure that the link in the symmetrical location has the same force density. Based on the initial pre-stress, Hooke's Law and Conjugate gradient iteration method (Strobel, 2004), nonlinear structural analysis (Shan *et al.*, 1993; Kmet and Kokorudová, 2006) can be done for various load case.

ANALYSIS OF THE LEVY CABLE DOME

Analysis of the initial pre-stress

The Levy cable dome, invented by American engineer Levy, is a particular cable-strut system (Chen, 2005). Though the first Levy cable dome, Georgic Dome was successfully used for the Olympic Stadium in 1996, many scientific and technical questions still need thorough researching. In contrast to the

Geiger cable dome, Levy cable dome adopts triangle cable gird to form local negative Gauss curvature membrane which increases the complexity of the design details, but also greatly improves the geometric stability (Zhang, 2004). In this paper, a Levy cable dome with center hoop will be analyzed comprehensively regarding the initial pre-stress and various load cases. The diameter of the cable dome is 120 m, with a rise of 17.58 m, a center hoop diameter of 6 m, and with 450 link elements and 72 struts. Fig.2 presents the perspective and section views of Levy cable dome.

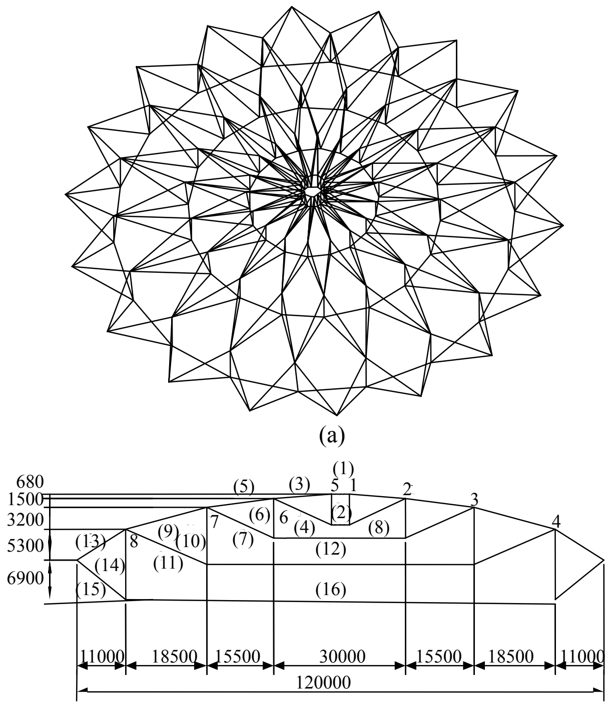

Fig.2 Levy cable dome (unit: mm)
(a) Isometric view; (b) Section view

From matrix analysis of the equilibrium matrix, the number of the self-stress modes $n-r$ is 19 and the number of the mechanisms $m-r$ is 1. Therefore the cable dome is found to be statically and kinematically indeterminate. Given that the initial pre-stress of all of the outer 18 hoop cables is 2600 kN, the dome can be calculated according to the linear adjustment theory, and the results are listed in Table 1.

The struts all adopt Q235 circular hollow section (CHS) whose elastic modulus is 2.06×10^{11} N/m^2. All cables are helical steel wire strand (ASTM A586-91) whose effective elastic modulus is 1.7×10^{11} N/m^2.

Table 1 Link type and initial pre-stress

No.	Link type	Initial pre-stress (kN)
1	∅325×12	1191.3 (184)
2	∅325×12	−21.2
3	∅42 (1×91)	212
4	∅54 (1×91)	34.2
5	∅42 (1×91)	261.6
6	∅325×12	−51.7
7	∅54 (1×91)	85.8
8	2∅39.7 (1×61)	442
9	∅36.5 (1×61)	380.5
10	∅325×12	−125
11	∅63.5 (1×127)	215.5
12	2∅39.7 (1×61)	1070
13	∅54 (1×91)	818.4
14	∅400×12	−553
15	∅63.5 (1×127)	684.3
16	2∅50.8 (1×91)	2600

Note: 1191.3 (184) represents the element force of the top and bottom chord of the inner hoop

For the load case, the additional membrane self-weight is 0.0125 kN/m^2, and the reference snow pressure and wind pressures are 0.3 kN/m^2 and 0.45 kN/m^2 respectively, then the design value of load is 0.66 kN/m^2 according to LRFD code. To evaluate the structural performance of the cable dome, a fully loaded and a half loaded case are the two necessary states analyzed. Due to large geometrical nonlinearity, the load is applied in 5 incremental steps.

Nodal deflection analysis

1. Fully loaded case

The nodal deflections of the nodes 1~4 are the same as those of nodes 5~8 due to symmetry, so the nodal deflections 1~4 show the dome deflections. As shown in Fig.3, the deflections of the center hoop are the largest with the deflections being nearly proportional to that of the load incremental step. The largest deflection to span ratio is about 1/270. The dome exhibits fairly high overall stiffness when the pressure loading is that of a symmetric load case.

2. Half partially loaded case

In this load case, loads are only applied to the nodes 5~8. As shown in Fig.4, node 1 descends, and nodes 2~4 actually ascend. The dome deflects downward in the loaded areas and exhibits an overall asymmetrically deformed shape. The maximum deflection appears at node 6, and is about 75% greater than the larger deflection in the fully loaded case. This indicates the dome has rather low stiffness under asymmetrical loading, which is to be expected since the inherent mechanism in the structure is being excited.

Internal force analysis

1. Fully loaded case

As the dome applied fully symmetrical load, the axial forces of the representative members are presented as shown in Fig.5. The tension of hoop and diagonal cables increases, but the tension of ridge cables decreases with increasing downward load. The tension changes of the cables are little. The tension change of the ridge cables is greater than that of hoop and diagonal cables; but they are still under 30% of the initial pre-stress. This indicates that initial pre-stress forms a great part of the permanent force and plays a key role on load capacity and deformation.

2. Half partially loaded case

For the dome with half load applied, the axial forces of the cables and struts are shown in Fig.6, where the α figures show the axial force of links in

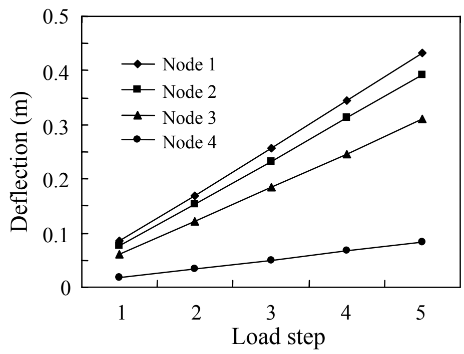

Fig.3 Nodal deflection of upper chord

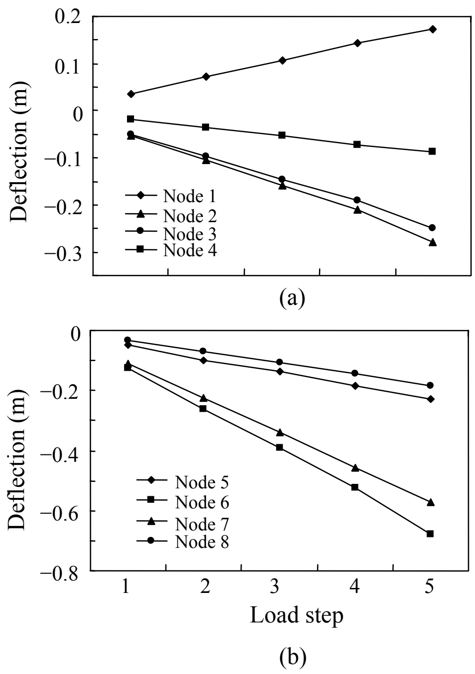

Fig.4 Nodal deflections of upper chord
(a) Deflection of nodes 1 to 4; (b) Deflection of nodes 5 to 8

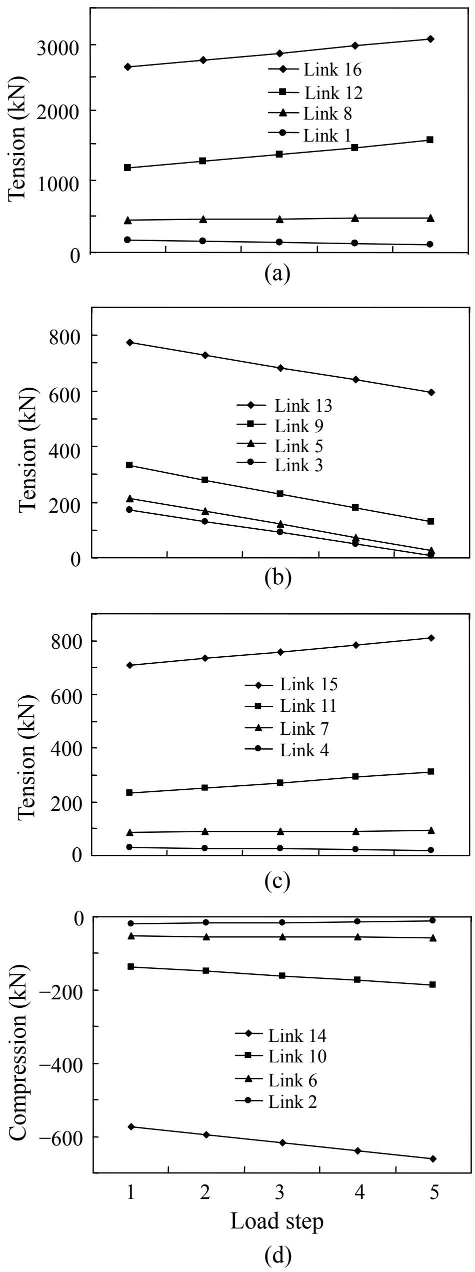

Fig.5 (a) Hoop cable tension; (b) Ridge cable tension; (c) Diagonal cable tension; (d) Strut compression

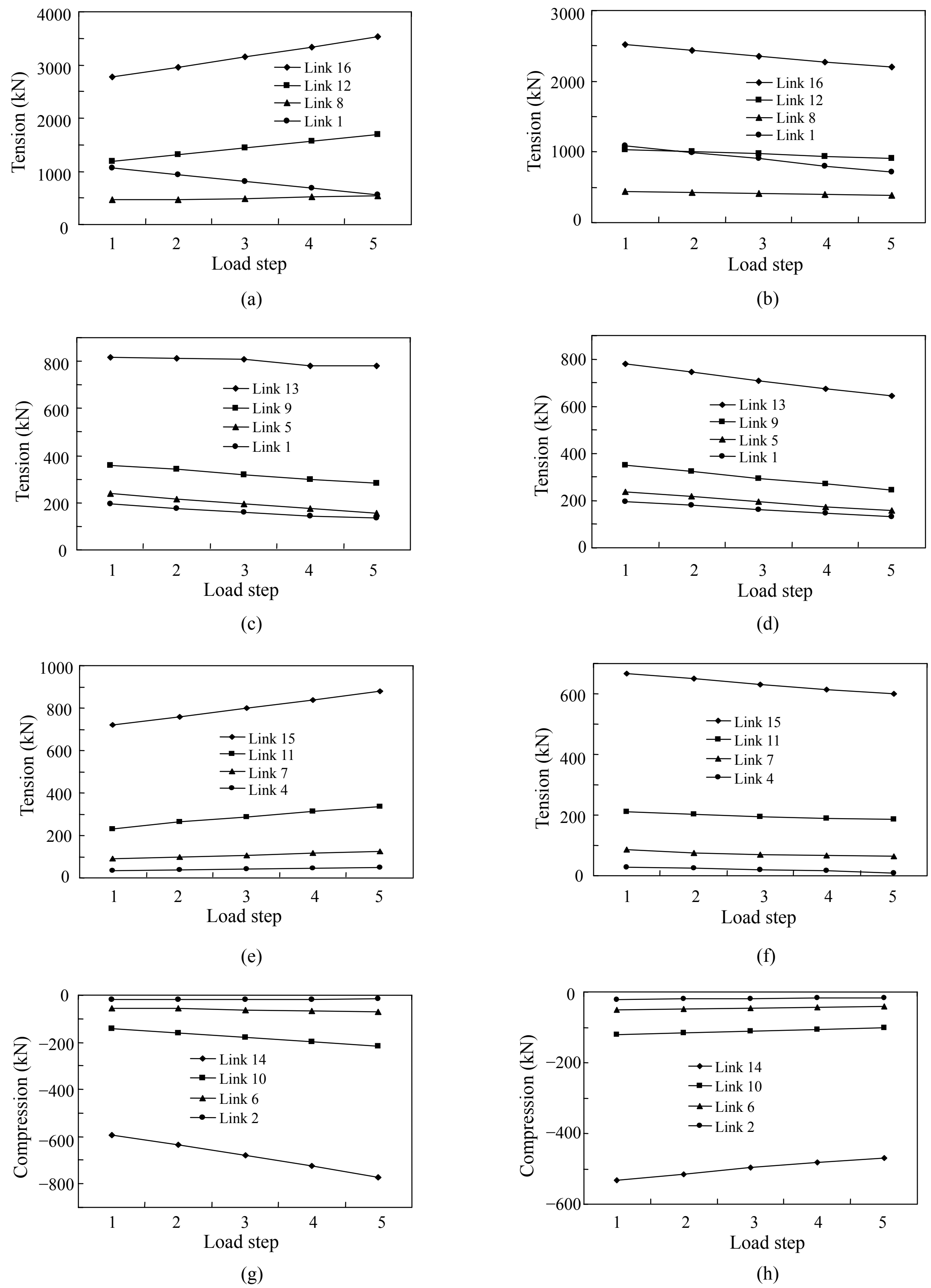

Fig.6 (a) Hoop cable tension (α); (b) Hoop cable tension (β); (c) Ridge cable tension (α); (d) Ridge cable tension (β); (e) Diagonal cable tension (α); (f) Diagonal cable tension (β); (g) Strut compression (α); (h) Strut compression (β)

the unloaded part and the β figures represent the axial force of links in loaded area.

In the loaded part, the tension of hoop cables, ridge cables and diagonal cables decreases little with increasing load. On the contrary, the counter part links in the unloaded regions changes in multiple ways. The force of the diagonal cables and the compression struts increases with the increasing load. However, the force of the ridge cables decreases with increasing load. The force of the inner hoop cables decreases while the force of the outer hoop cables increases with the load step.

CONCLUSION

In this paper, a new numerical algorithm is presented for force-finding of the cable-strut system. This algorithm is based on the linear adjustment theory and does not consider the material behavior. By using this method, the initial pre-stress of the multi self-stress modes can be found easily and the calculation process is simplified and efficient. Based on the initial pre-stress, Hooke's Law with convenient solving method of conjugate gradient iteration method can be used to analyze the loaded structure. Finally, using this algorithm, the initial pre-stress of a particular Levy cable dome can be found. Nonlinear structural analysis of the Levy cable dome was done for two load cases. The results indicate that the algorithm is efficient and correct and that the numerical results are valuable in practical design of Levy cable dome.

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