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Path tracking control of autonomous agricultural mobile robots*

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Abstract: In a tractor automatic navigation system, path planning plays a significant role in improving operation efficiency. This study aims to create a suboptimal reference course for headland turning of a robot tractor and design a path-tracking controller to guide the robot tractor along the reference course. A time-minimum suboptimal control method was used to generate the reference turning course based on the mechanical parameters of the test tractor. A path-tracking controller consisting of both feedforward and feedback component elements was also proposed. The feedforward component was directly determined by the desired steering angle of the current navigation point on the reference course, whereas the feedback component was derived from the designed optimal controller. Computer simulation and field tests were performed to validate the path-tracking performance. Field test results indicated that the robot tractor followed the reference courses precisely on flat meadow, with average and standard lateral deviations being 0.031 m and 0.086 m, respectively. However, the tracking error increased while operating on sloping meadow due to the employed vehicle kinematic model.

Key words: Headland turning, Path creation, Path-tracking controller, Optimal control, Robot tractor
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INTRODUCTION

Automatic guidance of agricultural vehicles has received the attention of researchers for nearly 80 years (Willrodt, 1924). Especially until the recent 20 years, with the development of computer and sensor technologies, numerous researches on this subject have been reported. Some literature reviews of automatic guidance for agricultural vehicles were presented (Wilson, 2000; Keicher and Seufert, 2000; Reid *et al.*, 2000; Torii, 2000; Lu and Liu, 2002).

In agricultural robotics, path planning can have direct effect on the farming efficiency. To improve the efficiency of field operations, many researchers have proposed their path planning algorithms for field traffic (Al-Hasan and Vachtsevanos, 2002; Palmer *et al.*, 2003; Sørensen *et al.*, 2004; Oksanen *et al.*, 2005). However, most of their researches were focused on planning the whole field traffic, and were seldom related to the detailed course generation in headlands. On the other hand, an optimal headland turn can not only improve the tracking accuracy when a robot tractor transfers from the current working row to the next one, but also minimize the time spent in the headlands so as to increase the efficiency of farming operations.

A dynamic path search algorithm was designed by Zhang and Qiu (2004) to guide an autonomous agricultural tractor to track the desired path and to make turns at the end of the field. The dynamic path search algorithm used posture sensors to determine the current tractor position and a tractor dynamics model to estimate the future tractor position. Their field test showed that the typical lateral deviation at sharp turning points ranged from 0.2 m to 1.5 m with travelling speed of 1.4 m/s to 3.1 m/s. Oksanen and Visala (2004) proposed an optimal control solver to

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create a path for a tractor-trailer combination in specified headland. The vehicle's mechanical, as well as the field's geometrical constraints were considered. In order to use optimal control problem solutions in real-time applications, the solutions in variable headland cases were approximated with Bézier curves. Kise *et al.*(2002) developed a turning algorithm for a robot tractor. Two types of turning paths, namely forward turning and switch-back turning, were created by applying a third-order spline function based on the minimum turning radius and maximum steering speeds. Computer simulation and field tests showed that the maximum tracking error was less than 0.2 m in all headland turns. Torisu *et al.*(1997; 1998) studied the path creating method for a tractor in a headland using optimal control theory. They dealt with the optimal path with respect to travelling in minimum time. A numerical solution was also addressed, and some examples were provided.

This study aimed at creating a suboptimal reference course for headland turning of a robot tractor and designing a path-tracking controller to guide the robot tractor along the reference course.

MATERIALS AND METHODS

Vehicle kinematic model

A kinematic model was applied to describe the tractor motion in Cartesian coordinates. Fig.1 illustrates the simplified tractor bicycle model. For the sake of simplicity, it is assumed that the tractor moves at a low constant speed over a flat surface with no wheel slippage. The tractor's motion equations can be given as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ v\tan\alpha / L \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u , \qquad (1)$$

or

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u),$$

where x and y are the coordinates of the rear axle center; θ is the heading angle; α is the steering angle; u is the steering rate; v is the velocity; L is the wheel base; and $\boldsymbol{x}=[x, y, \theta, \alpha]^{\mathrm{T}}$ is the fourth-order state vector.

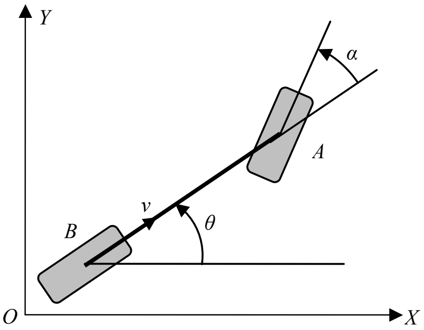


Fig.1 Vehicle bicycle model

Path creation

It is supposed that the work course is composed of straight crop rows and headland turns, as shown in Fig.2. The length of the crop rows is denoted by l, the working breadth by d. An optimal feedforward control theory was used to create the time-minimum turning courses.

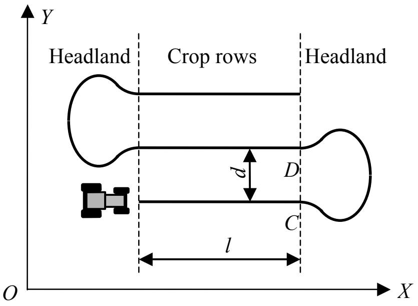


Fig.2 Work course of the robot tractor

The optimal control problem can be formulated as:

Control plant: $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u)$;

Constraint on steering angle:

$$|\alpha| \le \alpha_{\max},\ |u| \le u_{\max}, \qquad (2)$$

where $\alpha_{\max}$ is the maximum steering angle and $u_{\max}$ is the maximum steering rate.

Initial state: $\boldsymbol{x}(t_0)=[0, 0, 0, 0]^{\mathrm{T}}$, $t_0=0$;

Final state: $t=t_{\mathrm{f}}$ is given and $\boldsymbol{x}(t_{\mathrm{f}})$ is free;

Performance index:

$$J = x^2(t_{\mathrm{f}}) + [y(t_{\mathrm{f}}) - d]^2 + [\theta(t_{\mathrm{f}}) - \pi]^2 + \alpha^2(t_{\mathrm{f}}) + \int_0^{t_{\mathrm{f}}} \frac{r}{\alpha_{\max}^2 - \alpha^2}\mathrm{d}t, \qquad (3)$$

where r is the penalty parameter of the interior point methods (Torisu *et al.*, 1996) and $\boldsymbol{x}=[0, d, \pi, 0]^{\mathrm{T}}$ is the target final state.

The above depicted optimal problem, which is to find an optimal course in the fixed time interval t_{f}, can be solved by numerical computation of a second-order algorithm (Imae and Hakomori, 1987a; 1987b). To obtain a time-minimum suboptimal course, the fixed-time problem has to be changed to the free time one. Therefore, a numerical computation of dichotomy was introduced. The steps are listed as follows:

Step 1: The final time t_{f} is replaced with the variable $t_{\mathrm{f}j}$ (j=1, 2, …), so the optimal problem becomes how to find the minimum time t_{fm} and the corresponding optimal trajectory.

Step 2: A final time t_{f1} is initialized to d/v, and the optimal problem is solved by the algorithm mentioned above to obtain the minimum performance index J_1 and the corresponding optimal trajectory. It is estimated that the value J_1 cannot be very small because the final condition in Eq.(3) cannot be satisfied by only rectilinear motion.

Step 3: Another final time t_{f2} is initialized to a certain value which is big enough. As a result, by solving the optimal problem, the minimum performance index J_2 should be less than a predetermined threshold M. If not, another bigger final time should be selected until $J_2<M$.

Step 4: The optimal problem with a final time $t_{\mathrm{f3}}=(t_{\mathrm{f1}}+t_{\mathrm{f2}})/2$ is solved and the minimum performance index J_3 and the corresponding optimal trajectory will be obtained. If $J_3<M$ then let $t_{\mathrm{f2}}=t_{\mathrm{f3}}$, else let $t_{\mathrm{f1}}=t_{\mathrm{f3}}$.

Step 5: If $t_{\mathrm{f2}}-t_{\mathrm{f1}}<0.1$ s then let $t_{\mathrm{fm}}=t_{\mathrm{f2}}$ and stop, else repeat Step 4.

Finally, the suboptimal trajectory is obtained in terms of solution of the optimal problem with the minimum-time t_{fm}, and can be expressed by a series of navigation points. The navigation signal of each point is described by a vector $\boldsymbol{x}_i^*=[x_i^*, y_i^*, \theta_i^*, \alpha_i^*]^{\mathrm{T}}$, where i denotes the number of the navigation points. This trajectory will serve as the reference headland turning course for the robot tractor.

Path-tracking controller

In order to guide the robot tractor along the reference course, a path-tracking controller composed of a feedforward and a feedback component element was designed. So this subsection deals with the development of a path-tracking method and an optimal controller, i.e., the feedback component.

As described in the last subsection, the reference course is composed of a series of navigation points P_i based on the earth fixed coordinate system (XOY), as shown in Fig.3. A moving coordinate system ($X_{\mathrm{r}}O_{\mathrm{r}}Y_{\mathrm{r}}$) is defined where the origin O_{r} is a navigation point on the reference course. It is supposed that O_{r} is the closest navigation point away from the current vehicle position, i.e., the control point B. The X-axis of the moving coordinate system directs at the tangent on the point O_{r}. From the figure, the lateral deviation δy, the heading deviation $\delta\theta$ and the steering error $\delta\alpha$ can be defined as:

$$\delta\alpha=\alpha-\alpha_i^*,\ \delta\theta=\theta-\theta_i^*,$$

$$\delta\dot{y}=v\sin(\delta\theta), \tag{4}$$

$$\delta\dot{\theta}=\dot{\theta}-\dot{\theta}_i^*=\frac{v\tan\alpha}{L}-\frac{v\tan\alpha_i^*}{L}, \tag{5}$$

$$\delta\dot{\alpha}=\delta u. \tag{6}$$

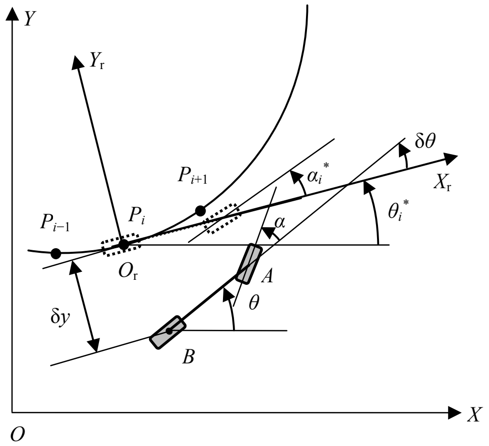


Fig.3 Definition of the navigation signals

Employ first-order Taylor series to approximate Eqs.(4) and (5) in the neighborhood of 0 and α_i^*, respectively, so that

$$\delta\dot{y}=v\delta\theta, \tag{7}$$

$$\delta\dot{\theta}=\frac{v}{L\cos^2\alpha_i^*}\delta\alpha. \tag{8}$$

The linearized state equations can be expressed by

$$\delta\dot{\boldsymbol{y}}=\boldsymbol{A}_i\delta\boldsymbol{y}+\boldsymbol{B}\delta u, \tag{9}$$

where $\delta\boldsymbol{y}=[\delta y\ \delta\theta\ \delta\alpha]^{\mathrm{T}}$,

$$\boldsymbol{A}_i=\begin{bmatrix}0 & v & 0\\ 0 & 0 & \dfrac{v}{L\cos^2\alpha_i^*}\\ 0 & 0 & 0\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}0\\0\\1\end{bmatrix}.$$

The optimal control problem is to find a control law δu for the system stated by Eq.(9) such that the performance index

$$J(\delta u)=\int_0^{\infty}\{\delta\boldsymbol{y}^{\mathrm{T}}\boldsymbol{Q}\delta\boldsymbol{y}+R(\delta u)^2\}\mathrm{d}t \tag{10}$$

is minimized under the constraints stated by Eq.(2), where $\boldsymbol{Q}$ is a 3×3 symmetric and positive semidefinite matrix and R is a positive scalar. In this case,

$$\boldsymbol{Q}=\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix},\quad R=1.$$

For each navigation point, the corresponding $\boldsymbol{A}_i$ is considered as a constant matrix in the short sampling interval; therefore, the optimal feedback gain can be obtained by solving the following algebraic Riccati equation:

$$\boldsymbol{P}_i\boldsymbol{A}_i+\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}_i-\boldsymbol{P}_i\boldsymbol{B}R^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{Q}=\boldsymbol{0}, \tag{11}$$

$$\delta u=-R^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_i\delta\boldsymbol{y}=-\boldsymbol{K}_i\delta\boldsymbol{y}, \tag{12}$$

where $\boldsymbol{K}_i$ is the feedback matrix gain and $\boldsymbol{K}_i=R^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_i\delta\boldsymbol{y}$.

Accordingly the extent of steering angle change $\delta\alpha$ can be calculated by

$$\delta\alpha=\delta u\cdot\delta t, \tag{13}$$

where δt is the sampling interval.

In addition, according to

$$\mathrm{rank}[\boldsymbol{B}\quad \boldsymbol{A}_i\boldsymbol{B}\quad \boldsymbol{A}_i^2\boldsymbol{B}]=3,$$

the control system is controllable.

In the designed path tracking method, the location and orientation of the navigation point were employed to determine the current steering angle for vehicle motion. In terms of this path-tracking method (Fig.4), the steering angle is determined by both feedforward and feedback component elements. The feedforward component, α_i^*, is the desired steering angle of the current navigation point on the reference course. The feedback component $\delta\alpha$ is the amount of steering angle change and is evaluated by the optimal controller. Therefore, the control steering angle α_{c} of the tractor being near the current navigation point is calculated as follows:

$$\alpha_{\mathrm{c}}=\alpha_i^*+\delta\alpha. \tag{14}$$

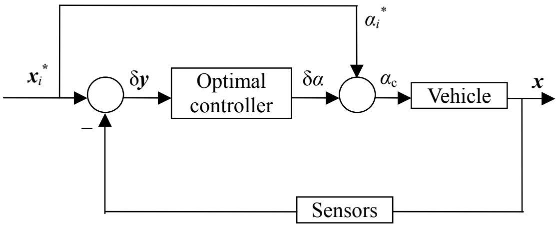


Fig.4 Block diagram of the path-tracking controller

Computer simulation

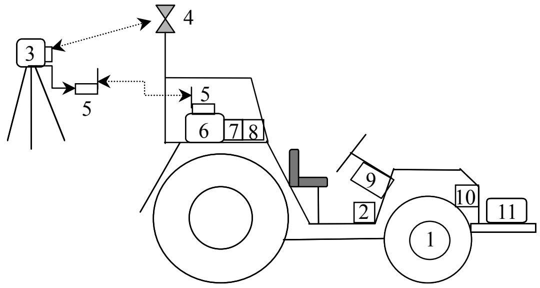
To validate the designed path-tracking controller, computer simulation was done, and the simulated results were compared to the target ones that were created by the optimal control method. During the simulation, the speed of the robot tractor was set constant at 0.5 m/s.

Experiment

An 18 kW four-wheel drive Mitsubishi MT2501D model tractor was modified and served as the test tractor. Total mass of the tractor was 1 125 kg with a wheelbase of 1.595 m and a tread of 1.280 m. Fig.5 shows the prototype test tractor with sensors and equipment. The specifications of instrumentation used in the experiments are listed in Table 1.

Field tests were conducted on both a flat and an average 8° sloping meadow. In the experiment, the TS fixed coordinates (X, Y) should be transformed into inclined plane fixed coordinates (x, y). The relationship between the two coordinate systems is shown in Fig.6. The detailed transformation can be expressed by the equations below:

$$\begin{cases}x_B=X_P+L_G\cos\theta,\\ y_B=-Y_P/\cos\varphi+H_P\tan\varphi+L_G\sin\theta,\end{cases} \tag{15}$$



1: Potentiometer; 2: FOG; 3: Total station; 4: Prism; 5: Wireless modems; 6: PC; 7: AD/DA board; 8: DC motor driver; 9: DC motor; 10: Magnetic sensor; 11: AC generator

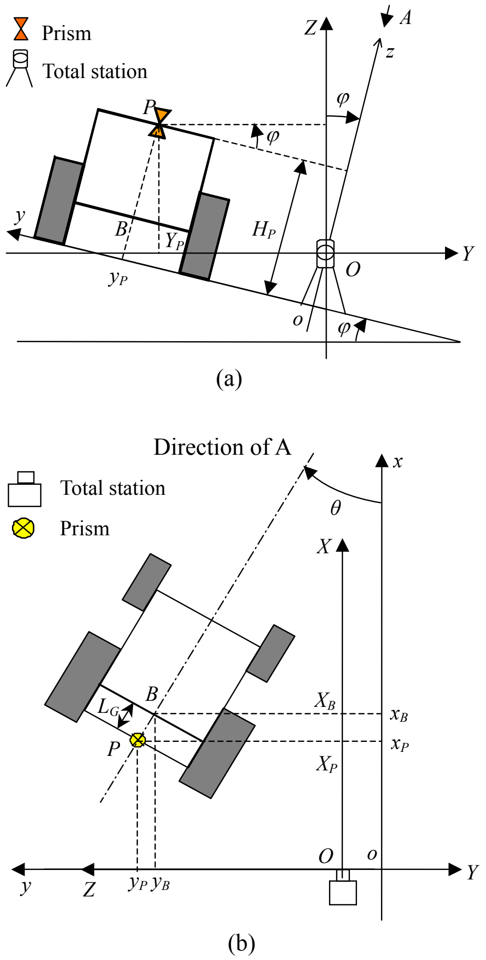
Fig.5 Test tractor and instrument

Table 1 Specifications of instrumentation

Equipment	Function
DC motor	Power is 82 W, used for the steering actuator
1.0 GHz Pentium PC	Mounted on the tractor, used as the central processing unit
Potentiometer	Fixed on the front axle, measures the steering angle
Magnetic sensor	Fixed near the flywheel, measures the engine speed
Fiber optic gyroscope (FOG)	Model is JG-35FD, measures the heading angle with range of ±180°, angular drift is less than ±1.5°/h
Laser auto-tracking range finder	Leica TCA 1105 model, 2 mm positioning accuracy. Composed of a total station (TS) and a prism
Wireless modems	Transmits the signals of the tractor position from the TS to the PC
AD/DA board	Converts the analogue signals to digital (AD) and digital to analogue (DA)

where x_B and y_B are the co-ordinates of the rear axle center in the inclined plane fixed frame; X_P and Y_P are the co-ordinates of the prism in the TS fixed frame; L_G is the distance between the prism and the rear axle center along the vehicle centerline; H_P is the height of the prism from the ground; φ is the slope angle.

For the test on sloping terrain, the 0° heading angle was always set parallel to the contour line. The tractor's speed was set constant at 0.5 m/s for all the tests.



(a)

(b)

Fig.6 Relationship between the total station fixed coordinate system and the inclined plane fixed coordinate system. (a) Front elevation; (b) Platform

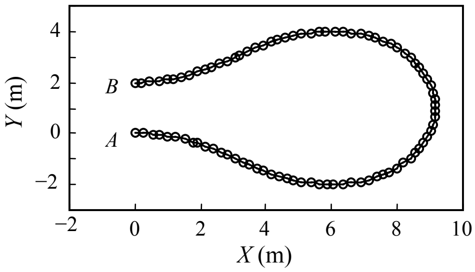


Fig.7 A suboptimal headland turning course

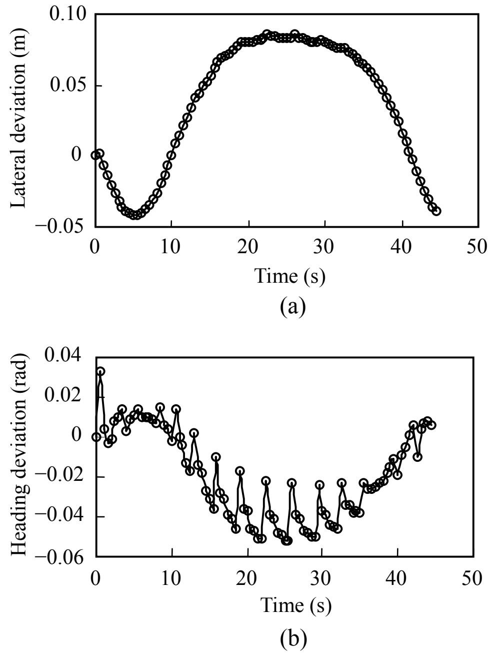
RESULTS AND DISCUSSION

Creation of the reference path

A suboptimal headland turning course (Fig.7) was created by the applied path creation method. In this case, in terms of the mechanical parameters of the robot tractor, the parameters were initialized as follows: $\alpha_{\max}$=0.698 rad; $u_{\max}$=0.890 rad/s; d=2.000 m; v=0.5 m/s; r=0.001; M=0.150; t_{f1}=4 s and t_{f2}=55 s. As a result, it took the robot tractor 44.9 s to travel from the starting point A of the initial state $\boldsymbol{x}(0)=[0, 0, 0, 0]^T$ to the end point B of the final state $\boldsymbol{x}(44.9)$=[0.000, 2.006, 3.152, −0.002]^T. It was found that the parameter $u_{\max}$ played an important role in the time-minimum control problem. If the $u_{\max}$ approached to 2.905 rad/s, the suboptimal control time could be as short as 36.18 s.

Simulation

The simulation results are shown in Fig.8. Figs.8a and 8b illustrate the time histories of the lateral deviation and heading deviation, respectively. It was found that the mean lateral deviation was 0.035 m, with standard deviation of 0.045 m, whereas the mean heading deviation was 0.019 rad, with standard deviation of 0.022 rad. It was evaluated that the designed path-tracking controller could guide the tractor robot to turn along the reference course in the headland.



(a)

(b)

Fig.8 Simulation results
(a) Lateral deviation; (b) Heading deviation

Headland turning tests

Fig.9 shows that the robot tractor made four turns on the flat meadow. The average headland turning time was about 48.9 s. It was noted that this average time was slightly longer than the time (44.9 s) spent in path creation. The reason for which was mainly attributed to the speed calibrated from the engine revolutions per minute, which is usually faster than the actual speed when the vehicle turns.

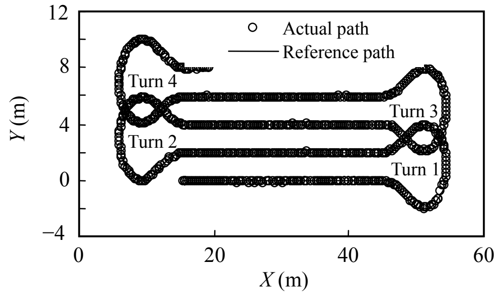
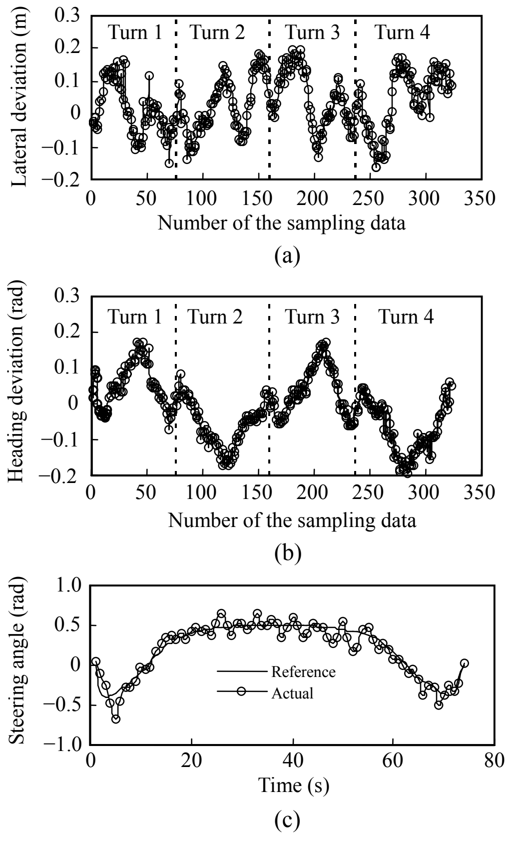


Fig.9 Trajectories of the robot tractor on a flat land

For each turn, the time histories of lateral and heading deviations are shown in Figs.10a and 10b, respectively. The average lateral deviation was 0.031 m, with standard deviation of 0.086 m, whereas the average heading deviation was −0.013 rad with standard deviation of 0.085 rad. It was indicated that the designed path-tracking controller could guide the robot tractor along the suboptimal reference courses precisely on flat land.

The time history of the steering angle of the first headland turn is shown in Fig.10c. It was observed



(a)

(b)

(c)

Fig.10 Field test results on flat land
(a) Lateral deviation; (b) Heading deviation; (c) Steering angle

that the steering wheel was controlled by the feedback controller around the reference steering angle. This phenomenon is in accord with the path-tracking method composed of feedforward and feedback component elements.

While operating on sloping terrain, the control accuracy decreased and the error increased, as shown in Fig.11. For each turn, the time histories of lateral and heading deviations are shown in Figs.11a and 11b, respectively. The average and standard deviations of the lateral error were 0.099 m and 0.105 m, respectively, and those of the heading error were 0.039 rad and 0.088 rad, respectively. However, the robot tractor could still track the reference course, even when the slope angle reached to around 11°. On the other hand, the testing demonstrated that the kinematic model-based path creation and path-tracking method were not well suited for vehicle automatic navigation on sloping terrain.

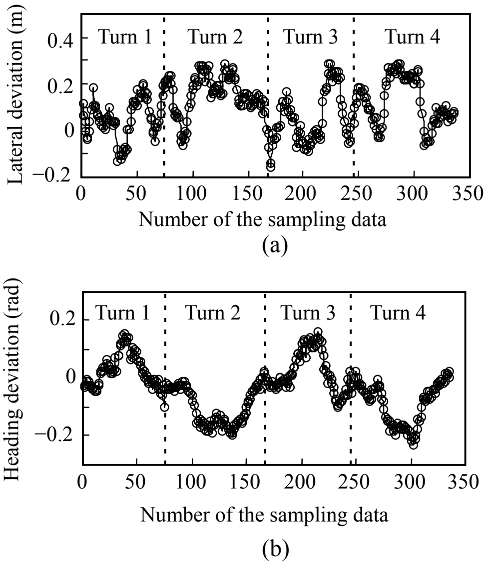


Fig.11 Field test results on sloping terrain
(a) Lateral deviation; (b) Heading deviation

CONCLUSION

A suboptimal control algorithm for generating reference turning courses for a robot tractor was presented. This algorithm could find a time-minimum headland turn based on the mechanical parameters of the tractor. A path-tracking controller consisting of both feedforward and feedback component elements was also proposed to guide the robot tractor along the reference turning courses. Computer simulation and field test results showed that the path tracking controller was applicable to sharp headland turns and could precisely guide the tractor along the reference courses on flat meadow with the average lateral deviation of 0.031 m and standard deviation of 0.086 m. However, the tracking error increased while operating on sloping meadow due to vehicle kinematic model employed. Therefore, in future work, attention must be paid to providing an accurate vehicle model incorporating variations in ground surface profiles, tire-ground friction forces and terrain slope information.

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