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Equalization for continuous phase modulation over frequency-selective fading channels

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Abstract: This paper presents an equalization algorithm for continuous phase modulation (CPM) over frequency-selective channels. A specific training sequence is first embedded in each data packet. By recursive least-squares (RLS) estimation, the channel information parameters can be acquired, and a fractionally spaced equalizer performs joint decoding and equalization. Simulation results show that the proposed algorithm can acquire the channel information parameters rapidly and accurately, and that the fractionally spaced equalizer can eliminate the intersymbol interference (ISI) effectively, and is not sensitive to timing inaccuracy, so this algorithm can be exploited for demodulation system in burst mode.

Key words: *M*-ary continuous phase modulation (CPM), Channel-estimation, Joint decoding and equalization, Fractionally spaced

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INTRODUCTION

Continuous phase modulation (CPM), a class of constant-envelope, bandwidth-efficient modulation schemes (Aulin and Sundberg, 1981; Aulin *et al*., 1981; Anderson *et al*., 1986), has long been popular because its constant envelope allows use of saturated non-linear amplifiers without spectral distortion. When transmitted over frequency-selective fading channels, the CPM signals may be seriously corrupted with intersymbol interference (ISI). Effective channel information estimation and equalization must be required to improve the performance. The optimum time-domain equalization technique for CPM in the maximum-likelihood (ML) sense is ML sequence estimation (MLSE) (Qureshi, 1985; Proakis, 2001). Recently, maximum likelihood sequence estimations of both PSK and CPM signals over Rayleigh flat-fading channels are presented (Lodge and Moher, 1990; Vitetta and Taylor, 1995a; 1995b; 1996). These techniques provide good error rate performance and do not have irreducible error rate, but these receivers usually have high implementation complexity, especially for channels with long impulse responses. It may be even hard to be implemented in some case due to its high complexity. The basic methodology for removing ISI with frequency-domain equalization (FDE) has been applied (Tan and Stuber, 2005; Pancaldi and Vitetta, 2005; 2006). As mentioned in Tan and Stuber (2005), we may first exploit the linear decomposition of CPM signals as proposed by Laurent (1986) and extended by Mengali and Morelli (1995). But except for the binary case, it has found limited application to reduced complexity designs (Mengali and Morelli, 1995), so FDE may be unsuitable for *M*-ary CPM. Reduced-complexity sequence-estimation techniques such as delayed decision-feedback sequence estimation (DDFSE) (Duel-Hallen and Heegard, 1989) can be applied at the expense of some performance degradation. However, the symbol-rate sampling used at the equalizer causes loss of valuable signal information.

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In this paper, a fractionally spaced DDFSE equalizer is proposed to perform the decoding and the equalization. Simulation results show that the fractionally spaced equalizer can eliminate the ISI effectively, and is not sensitive to timing inaccuracy. The complexity of the algorithm is controlled by a parameter. At its maximum value, it is equivalent to the Viterbi algorithm (Forney, 1973); and at its minimal value, it reduces to decision-feedback detection. So it is a direct tradeoff between complexity and performance.

The remainder of this paper is organized as follows. Section 2 describes the system and the channel model. A fractionally spaced equalizer is proposed in Section 3. Section 4 presents the simulation results of the proposed algorithm, followed by the conclusions in Section 5.

SYSTEM AND CHANNEL MODEL

First let the M-ary symbol sequence be $\alpha=\ldots, a_{-1}, a_0, a_1, \ldots$, where $a_n\in\{\pm1, \pm3, \ldots, \pm(M-1)\}$. For the time interval $nT\leq t<(n+1)T$, the equivalent complex low-pass CPM signal may be expressed by (Proakis, 2001)

$$\begin{cases} s(t,\alpha)=\sqrt{2E_\mathrm{s}/T}\exp(\mathrm{j}\phi(t,\alpha)), \\ \phi(t,\alpha)=2\pi h\sum_{k=-\infty}^{+\infty}a_k q(t-kT), \end{cases} \quad (1)$$

where E_s denotes the symbol energy, T denotes the symbol period, and $h=m/p$ is the modulation index. The phase response function is

$$q(t)=\int_{-\infty}^{t}g(\tau)\mathrm{d}\tau, \quad (2)$$

where $g(t)$ is a frequency pulse function of duration LT.

For practical purpose, we can assume that the CPM signal is strictly band-limited to $|f|\leq W/2$, for some W. A tapped delay-line (TDL) model is usually used for a time-varying frequency selective fading channel (Yiin and Stuber, 1997), namely

$$c(t,\tau)=\sum_{n=0}^{M_D-1}c_n(t)\delta(\tau-nT_\mathrm{c}), \quad (3)$$

where T_c is the channel resolution which satisfies $T_\mathrm{c}<1/W$; for convenience, T_c is chosen such that $T=N_\mathrm{c}T_\mathrm{c}$ with N_c being an integer. M_D is the length of the channel ISI in terms of T_c, $c_n(t)$ is the tap weight coefficient at the time nT_c. The received signal $r(t)$ is then given by

$$r(t)=y(t)+z(t)=\sum_{j=0}^{M_D-1}c_j(t)s(t-jT_\mathrm{c},\alpha)+z(t), \quad (4)$$

where $z(t)$ is the additive white Gaussian noise (AWGN).

At the receiver, the signal is sampled at $t=kT_\mathrm{c}$ to provide a set of sufficient statistics for estimation and equalization. The kth sample of $r(t)$ is

$$\begin{aligned} r_k\equiv r(kT_\mathrm{c}) &= \sum_{j=0}^{M_D-1}c_j(kT_\mathrm{c})s(kT_\mathrm{c}-jT_\mathrm{c},\alpha)+z(kT_\mathrm{c}) \\ &= \sum_{j=0}^{M_D-1}c_{j,k}s_{k-j}+z_k. \end{aligned} \quad (5)$$

COMBINED DECODING AND EQUALIZATION

The carrier phase of a CPM signal with a fixed modulation index of h can be expressed as

$$\begin{aligned} \phi(t,\alpha) &= 2\pi h\sum_{k=-\infty}^{+\infty}a_k q(t-kT) \\ &= \pi h\sum_{k=-\infty}^{n-L}a_k+2\pi h\sum_{k=n-L+1}^{n}a_k q(t-kT) \\ &= \theta_n+\theta(t,\alpha), \quad (k\in\mathbb{Z},\ nT\leq t\leq(n+1)T), \end{aligned} \quad (6)$$

where we have assumed that $q(t)=0$ for $t<0$, and $q(t)=1/2$ for $t\geq LT$.

Let $L_\mathrm{c}=\lfloor M_D/N_\mathrm{c}\rfloor+1$ be the length of the channel ISI in terms of the number of symbols. At $t=nT$, the state of an ISI-corrupted CPM signal can be defined by

$$S_n=\left\{\theta_n, a_{n-1}, a_{n-2}, \ldots, a_{n-L+1}, \ldots, a_{n-L_\mathrm{c}-L+1}\right\}. \quad (7)$$

The optimum receiver for the CPM signal consists of a correlator followed by a maximum likelihood sequence detector that searches the paths through the state trellis for the minimum Euclidean distance path. The Viterbi algorithm is an efficient method for performing this research (Proakis, 2001). Suppose that the channel tap gains have been estimated, then the trellis decoding and channel equalization can be performed jointly with a single MLSE detector. It is easy to show that for a particular sequence of transmitted symbols $\alpha=\ldots, a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots$, the metric can be computed as follows:

$$CM_n(\alpha)=CM_{n-1}(\alpha)+v_n(\alpha). \tag{8}$$

The term $CM_{n-1}(\alpha)$ represents the metrics for the surviving sequences up to time nT, and the term

$$v_n(\alpha)=-\sum_{m=0}^{N_s-1}\left|r_k-\sum_{j=0}^{M_D-1}\hat{c}_{j,k}\hat{s}_{k-j}\right|^2 \tag{9}$$

represents the additional increments to the metrics contributed by the signal in the time interval $nT\le t\le(n+1)T$, where N_s is the sample rate which equals N_c, $\hat{c}_{j,k}$ is the estimation of the channel tap gain.

Note that the complexity of the MLSE detector is proportional to M^{L_c+L-1}, as mentioned in Yiin and Stuber (1997), the complexity is too high especially when the length of the channel ISI L_c and the alphabet size M increase and it may be hard to be implemented in some case. So we propose a reduced-complexity sequence-estimation technique which is suitable for CPM signal named "delayed decision-feedback sequence estimation" (DDFSE) (Duel-Hallen and Heegard, 1989).

DDFSE is a method to reduce the number of states, where each state provides only partial information about the actual state of the channel. The required residual information is provided by an estimation associated with each state of the trellis. At $t=nT$, the state of the CPM signal can be given by

$$S_n=\{\theta_n,\ a_{n-1},\ a_{n-2},\ \ldots,\ a_{n-L'+1}\}, \tag{10}$$

where $L\le L'\le L_c+L$.

The additional increments to the metrics contributed by the signal in the time interval $nT\le t\le(n+1)T$ can be modified as

$$v_n(\alpha)=-\sum_{m=0}^{N_s-1}|r_k-C(S_{n+1};S_n)|^2,$$

where

$$C(S_{n+1};S_n)=\sum_{j=0}^{L'N_s-1}\hat{c}_{j,k}\hat{s}_{k-j}+\sum_{j=L'N_s}^{M_D-1}\hat{c}_{j,k}\tilde{s}_{k-j}. \tag{11}$$

The second term of the right side of Eq.(11) represents the estimation of the ISI caused by the time-delay spread longer than $L'T$. The complexity of the DDFSE is proportional to $M^{L'-1}$. When $L'=L_c+L$, DDFSE is equivalent to the Viterbi algorithm, and at its minimal value, the algorithm reduces to decision-feedback detection.

Until now, we all assumed that the channel tap gains were known. However, we must get channel information parameters rapidly and accurately in practical conditions, and it is important for the performance of a receiver. So we propose recursive least-squares (RLS) algorithm to estimate the channel information parameters.

The RLS estimation of channel information parameters may be formulated as follows (Qureshi, 1985; Proakis, 2001). Suppose the train words sequence is $\boldsymbol{y}_N(n)$ ($n=0, 1, \ldots, t$), we have observed the sequence $\boldsymbol{I}(n)$, and we wish to determine the coefficient vector $\boldsymbol{c}_N(t)$ of the equalizer that minimizes the time-average weighted squared error.

$$\varepsilon_N^{LS}=\sum_{n=0}^{t}w^{t-n}\left|\boldsymbol{e}_N(n,t)\right|^2, \tag{12}$$

where the error is defined as

$$\boldsymbol{e}_N(n,t)=\boldsymbol{I}(n)-\boldsymbol{c}'_N(t)\boldsymbol{y}_N(n), \tag{13}$$

and w represents a weighting factor, $0<w<1$.

Minimization of ε_N^{LS} with respect to the coefficient vector yields the sets of linear equations

$$\boldsymbol{R}_N(t)\boldsymbol{c}_N(t)=\boldsymbol{D}_N(t), \tag{14}$$

where $\boldsymbol{R}_N(t)$ is the signal correlation matrix defined as

$$\boldsymbol{R}_N(t)=\sum_{n=0}^{t}w^{t-n}\boldsymbol{y}_N^*(n)\boldsymbol{y}'_N(n), \tag{15}$$

and $\boldsymbol{D}_N(t)$ is the cross-correlation vector:

$$\boldsymbol{D}_N(t)=\sum_{n=0}^{t} w^{t-n}\boldsymbol{I}(n)\boldsymbol{y}_N^*(n). \quad (16)$$

The solution is

$$\boldsymbol{c}_N(t)=\boldsymbol{R}_N^{-1}(t)\boldsymbol{D}_N(t). \quad (17)$$

In practical implementations, the value of $\boldsymbol{R}_N^{-1}(t)$ can be stored in a look-up table for fast processing.

SIMULATION AND RESULTS

In this section, the performance of the fractionally spaced equalizer will be evaluated for several schemes. The CPM schemes belong to the length L raised cosine (LRC) pulse family. For convenience, we take 2-ary 1RC with h=0.5, 4-ary 1RC with h=0.25 and 8-ary 1RC with h=0.2 for example. The sample rate at the receiver N_s=8, and the channel impulse responses are

Channel 1: $h_1(t)=\delta(t)+0.3\delta(t-T)$,
Channel 2: $h_2(t)=\delta(t)+0.4\delta(t-T)+0.2\delta(t-2T)$,

where T is the symbol period.

Fig.1 shows the comparison of the performances of 2, 4, 8-ary CPM signals over an ideal channel with those without equalization over Channel 1 and Channel 2.

It is clear that when the time-delay spread is longer than the symbol period, the information transmission will be seriously adversely influenced, especially as the alphabet size M increases. However, the complexity of the MLSE detector increases quickly as M increases. So in high data rate transmission condition, equalization is necessary and the study of reduced-complexity equalization technique also makes sense.

Fig.2a shows the simulation results of the fractionally spaced equalizer with L'=1 for 4-ary 1RC CPM signals over Channel 2. First we suppose that the channel tap gains have been obtained. The simulation result is given in Fig.2a corresponding to "DDFSE c_n known". We can see that the performance is very close to that over an ideal channel. Then we use the RLS algorithm to estimate the channel tap gains. The parameter w=0.999, and the length of train words is 20 or 10. We also give the performance of 4-ary CPM signals over an ideal channel and that without equalization over Channel 2 for comparison.

The results showed that if the channel tap gains are known, the receiver complexity is greatly reduced while the performance degradation is less than 0.5 dB compared to the MLSE detector. The results also showed that for RLS algorithm, the bit error rate performance improves as the length of train words increases.

Fig.2b gives the simulation results when the timing is not accurate. Suppose that the length of train words for channel estimation is 20, and the timing error $\Delta\tau$ is T/8, T/4 or T/2. It is clear that a fractionally spaced equalizer is not sensitive to timing error. When $\Delta\tau=T/2$, the performance degradation is less than 1 dB.

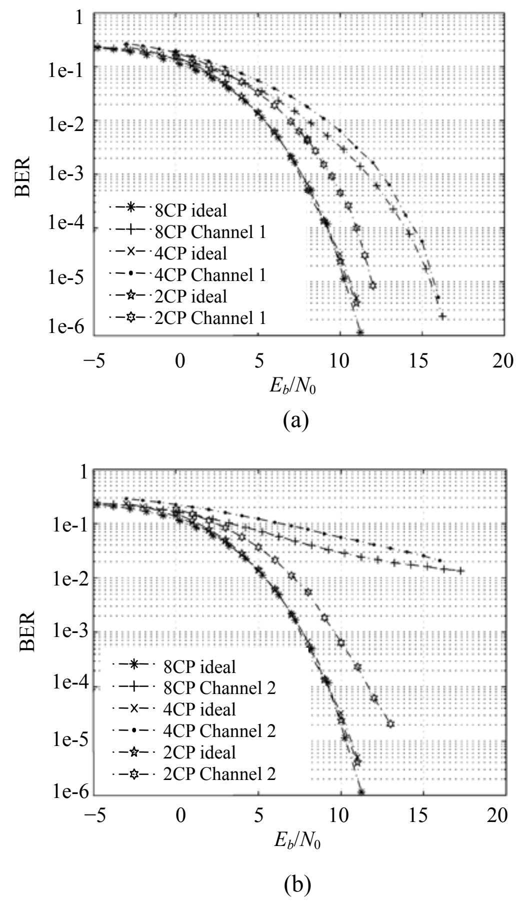

Fig.1 The comparison of the performances of 2, 4, 8-ary CPM signals on an ideal channel with those without equalization on (a) Channel 1 and (b) Channel 2

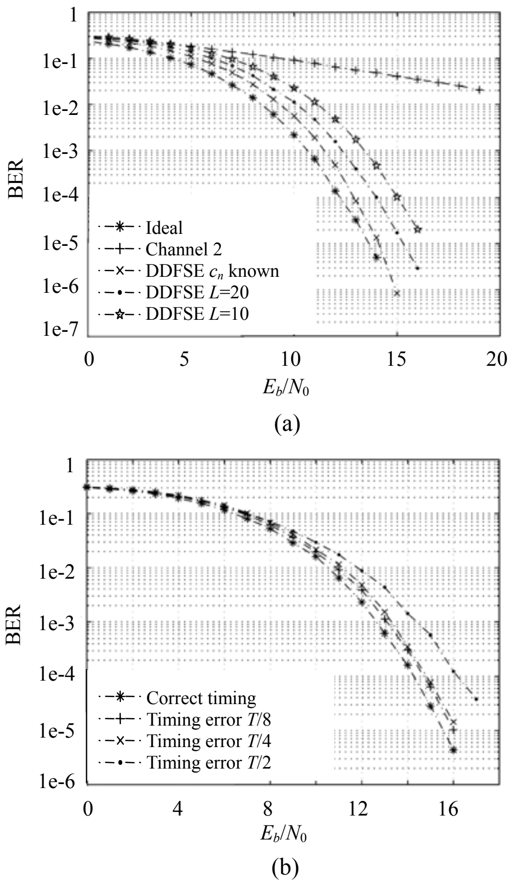

Fig.2 Simulation results of the fractionally spaced equalizer for 4-ary 1RC CPM signals on channel 2. (a) The timing is accurate; (b) The timing has assumed error

CONCLUSION

A joint decoding and equalization algorithm has been proposed for CPM signals over frequency-selective fading channels. Simulation results show that the proposed algorithm can acquire the channel information parameters rapidly and accurately, that the fractionally spaced equalizer can eliminate the ISI effectively, and that it does not exhibit any sensitivity to timing inaccuracy. The algorithm combines structures of the reduced-state Viterbi algorithm and a decision-feedback detector, and provides a tradeoff between complexity and performance. So this algorithm can be exploited for demodulation system in burst mode.

References

Anderson, J.B., Aulin, T., Sundberg, C.E., 1986. Digital Phase Modulation. Plenum Press, New York.

Aulin, T., Sundberg, C.E.W., 1981. Continuous phase modulation—Part I: full response signaling. *IEEE Trans. on Commun.*, **29**(3):196-209. [doi:10.1109/TCOM.1981.1095001]

Aulin, T., Rydbeck, N., Sundberg, C.E.W., 1981. Continuous phase modulation—Part II: partial response signaling. *IEEE Trans. on Commun.*, **29**(3):210-225. [doi:10.1109/TCOM.1981.1094985]

Duel-Hallen, A., Heegard, C., 1989. Delayed decision-feedback sequence estimation. *IEEE Trans. on Commun.*, **37**(5):428-436. [doi:10.1109/26.24594]

Forney, G.D., 1973. The Viterbi algorithm. *Proc. IEEE*, **61**(3):268-278.

Laurent, P., 1986. Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP). *IEEE Trans. on Commun.*, **34**(2):150-160. [doi:10.1109/TCOM.1986.1096504]

Lodge, J.H., Moher, M.L., 1990. Maximum likelihood sequence estimation of CPM signals transmitted over Rayleigh flat-fading channels. *IEEE Trans. on Commun.*, **38**(6):787-794. [doi:10.1109/26.57471]

Mengali, U., Morelli, M., 1995. Decomposition of M-ary CPM signals into PAM waveforms. *IEEE Trans. on Inf. Theory*, **41**(5):1265-1275. [doi:10.1109/18.412675]

Pancaldi, F., Vitetta, G.M., 2005. Block channel equalization in the frequency domain. *IEEE Trans. on Commun.*, **53**(3):463-471. [doi:10.1109/TCOMM.2005.843412]

Pancaldi, F., Vitetta, G.M., 2006. Equalization algorithms in the frequency domain for continuous phase modulations. *IEEE Trans. on Commun.*, **54**(4):648-658. [doi:10.1109/TCOMM.2006.873073]

Proakis, J.G., 2001. Digital Communications (4th Ed.). McGraw-Hill, New York.

Qureshi, S.U.H., 1985. Adaptive equalization. *Proc. IEEE*, **73**(9):1349-1387.

Tan, J., Stuber, G.L., 2005. Frequency-domain equalization for continuous phase modulation. *IEEE Trans. on Wirel. Commun.*, **4**(5):2479-2490. [doi:10.1109/TWC.2005.853901]

Vitetta, G.M., Taylor, D.P., 1995a. Viterbi decoding of differentially encoded PSK signals transmitted over Rayleigh flat-fading channels. *IEEE Trans. on Commun.*, **43**(2-4):1256-1259. [doi:10.1109/26.380163]

Vitetta, G.M., Taylor, D.P., 1995b. Maximum likelihood decoding of uncoded and coded PSK signal sequences transmitted over Rayleigh flat-fading channels. *IEEE Trans. on Commun.*, **43**(11):2750-2758. [doi:10.1109/26.481226]

Vitetta, G.M., Taylor, D.P., 1996. Multisampling receiver for uncoded and coded PSK signal sequences transmitted over Rayleigh flat-fading channels. *IEEE Trans. on Commun.*, **44**(2):130-133. [doi:10.1109/26.486600]

Yiin, L., Stuber, G.L., 1997. MLSE and soft-output equalization for trellis-coded continuous phase modulation. *IEEE Trans. on Commun.*, **45**(6):651-659. [doi:10.1109/26.592602]