



## A novel MMSE based codebook construction for MIMO precoded spatial multiplexing with limited feedback<sup>\*</sup>

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**Abstract:** This paper deals with the design and performance analysis of the transmission precoder optimization for multiple-input multiple-output (MIMO) systems with limited feedback of channel state information (CSI). We assume that the receiver can get perfect channel knowledge by channel estimation while the transmitter only has partial channel knowledge from limited feedback. We present a minimum mean square error (MMSE) criterion based codebook construction algorithm for MIMO precoded spatial multiplexing systems under a specific average power constraint. The optimal transmitter structure is employed in this paper. Simulation results show that the MMSE criteria based codebook construction algorithm with hybrid design of power allocation and precoding can achieve better performance than that of equal power allocation based codebook of previous research.

**Key words:** Multiple-input multiple-output (MIMO), Precoding, Channel state information (CSI), Feedback

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### INTRODUCTION

Multiple-input multiple-output (MIMO) system is well motivated for wireless communications through fading channels because of the potential improvements in transmit rate or diversity gain (Jafar, 2003). The achievable performance of multi-antenna systems depends on the availability of channel state information (CSI) at the transmitter and the receiver. When the channel state information is unavailable at the transmitter, the diversity can be obtained by applying spacing-time coding (Jafar and Goldsmith, 2004). When the transmitter could get perfect CSI, the system can obtain diversity as well as additional array gain by transmitting beamforming. However, in practical wireless systems, the transmitter cannot obtain perfect CSI because of inaccurate estimation, feedback delay, feedback errors, and various other reasons. Fortunately, it has been proved that for multi-antenna systems, even partial channel knowl-

edge is valuable in enhancing system performance (Narula *et al.*, 1998; Jafar and Goldsmith, 2004). Thus, exploiting channel partial CSI at the transmitter in a MIMO wireless system has attracted great interest in communication society (Rey and Lamarca, 2005; Xia and Giannakis, 2006).

Transmitter CSI (CSIT) can enhance MIMO system performance by using precoder. The precoder is an optimal transmit processing block based on CSIT, and CSIT can be obtained at the transmitter by feedback from the receiver in frequency division duplexing systems or by measuring the reverse channel in time division duplexing systems. Generally, it is assumed that perfect CSI is available at the receiver while the transmitter can only get partial CSI. There are two ways to exploit partial CSI (Xia and Giannakis, 2006). One is to use the statistical characteristic of CSI (Mukkavilli *et al.*, 2003; Jafar and Goldsmith, 2004). Another is using the limited feedback bits which are the index of the quantized transmit schemes. A set of quantized transmit schemes is called codebook which can be designed off-line (Xia and Giannakis, 2006). In this paper, we

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study the optimal transmit scheme for MIMO systems with limited feedback. The previous researches on limited feedback CSI based transmitter beamforming and precoding are based on various criteria (Love and Heath, 2003; 2005a). However, in performance sense, previous works are all based on equal power allocation. In other words, the transmitter structure in previous works is not the optimal transmitter structure in (Palomar and Cioffi, 2003).

This paper considers power allocation for MIMO systems with limited feedback. We investigate the MMSE based codebook construction algorithm and propose a practical solution of power allocation in codebook construction.

### SYSTEM MODEL

We consider that the spatial multiplexing MIMO wireless communication system is with  $N_t$  transmit antennas and  $N_r$  receive antennas.

Assume  $N_s$  symbols be transmitted in one time slot. Then we have  $N_s \leq \min\{N_r, N_t\}$ . The precoder is applied to allocating power across  $N_t$  antennas. Thus, the precoder  $F$  is an  $N_t \times N_s$  matrix.

In this context, we consider a MIMO block-fading channel model where the channel state remains quasi-static within a fading block, but behaves independently across different fading blocks. If the channel keeps quasi-static, the  $N_s \times 1$  received symbol vector is

$$\mathbf{r} = \mathbf{G}\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{n}, \quad (1)$$

where the receiver  $\mathbf{G}$  is an  $N_s \times N_r$  matrix, and  $\mathbf{n}$  is an  $N_r \times 1$  noise vector, which has the Gaussian statistics.

Let  $\mathbf{x}$  and  $\mathbf{r}$  be transmitted symbol vector and received symbol vector, respectively. The output-input relation can be written as a set of equations:

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}, \quad (2)$$

$$\mathbf{r} = \mathbf{G}\mathbf{y} = \mathbf{G}\mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{n}. \quad (3)$$

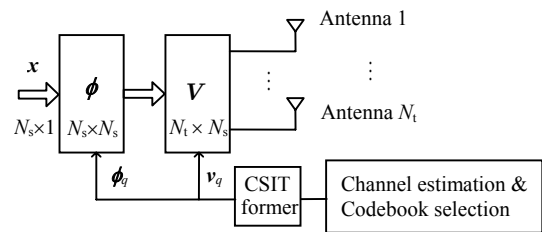
The optimal structure of  $F$  is  $F=V\phi$ , where the  $N_t \times N_s$  matrix  $V$  denotes the precoding matrix and  $\phi$  is  $N_s \times N_s$  diagonal matrix with the entries obtained by water-filling based power allocation (Scaglione et al., 2002; Palomar and Cioffi, 2003; Vu and Paulraj,

2006), which means that the transmit power is allocated across  $N_t$  antennas.

In this paper, we combine precoding and power allocation together (Fig.1). Thus Eq.(2) can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{v}\sqrt{\phi}\mathbf{x} + \mathbf{n}, \quad (4)$$

where the  $N_t \times N_s$  matrix  $\mathbf{v}$  denotes the precoding matrix and  $\phi$  is the  $N_s \times N_s$  matrix with the entries denoting the allocated power along each beamforming mode.



**Fig.1 The optimal transmit structure**  
 $\phi_q$ : power loading matrix;  $\mathbf{v}_q$ : precoding matrix

### Feedback model

In this paper, we consider frequency division duplexing systems. First CSI is estimated at the receiver. Then the receiver will select the optimal transmit schemes from the codebook and feed back the index of the optimal codeword, i.e., the quantized transmit schemes in the codebook, to the transmitter. By each received codeword index, the transmitter chooses the corresponding transmit schemes from the codebook.

The number of feedback bits  $B$  denotes the index of each non-overlapping region  $\{R_1, \dots, R_{N_B}\}$  in codebook, where  $N_B=2^B$ . The limited feedback design is a kind of quantization problem based on some criteria. The quantizer divides the space of channel matrix into  $N_B$  non-overlapping regions. For each of these regions, the transmitter selects the optimal transmit scheme that reaches the expected design object. The transmission determined by power loading matrix  $\phi_q$  and the precoding matrices  $\mathbf{v}_q$ , where  $q \in [1, N_B]$ , is designed off-line and stored at both the transmitter and the receiver. Each feedback index  $q \in [1, N_B]$  is corresponding to a stored transmit scheme in the codebook. The power loading matrices

are diagonal with nonnegative entries. The column vectors of the precoding matrix are unitary, i.e.,  $\|\mathbf{v}_l(q)\|^2=1$  for  $q \in [1, N_B]$ , where  $\mathbf{v}_l(q)$  denotes the  $l$ th column of the  $q$ th precoding matrix.

Within a fading signal block, the fading state is assumed to be quasi-statistic. Thus the effect of finite feedback delay would be irrelevant and the effect of feedback delay would be asymptotically negligible (Lau and Liu, 2004).

### MMSE CODEBOOK CONSTRUCTION

In this section, we will first briefly outline the transmitter optimization, and then consider Lloyd algorithm for the codebook construction.

#### Transmitter optimization

Given an independent identical distribution (i.i.d.) block-fading MIMO channel with the transmit CSI of  $B$  bits. The average minimum mean square error (MMSE)  $P_e(\mathbf{V}, \boldsymbol{\phi})$  of the MIMO system can be computed from the  $B$ -bit index codeword. Our goal is to find the MMSE based transmit scheme for controlled  $P_0$ .

This optimal performance problem can be formulated as

$$\min P_e(\mathbf{v}, \boldsymbol{\phi}), \quad \text{s.t.} \quad E \left( \sum_{l=1}^{N_s} \text{tr}(\tilde{\mathbf{x}}(l)\tilde{\mathbf{x}}^H(l)) \right) \leq P_0, \quad (5)$$

where  $\tilde{\mathbf{x}}(l) = \mathbf{v}(l)\sqrt{\boldsymbol{\phi}(l)}\mathbf{x}(l)$ , and  $\boldsymbol{\phi}(l)$  denotes the power allocated on the corresponding symbol.

Although previous research has proposed many codebook construction criteria (Love and Heath, 2003), there are no MMSE based codebook construction criteria with optimal transmitter structure. Sampath *et al.*(2001) and Scaglione *et al.*(2002) deduced the optimal MSE expression of linear transmitter and receiver, but it is based on perfect CSI on both sides. In this section we manage to directly use the exact MSE as the codebook design criterion. Let  $\mathcal{W}$  be codebook designed off-line and known to both the transmitter and the receiver, the proposed MSE based selection rule is as follows:

$$\mathbf{F}_{\text{opt}} = \arg \min_{\mathbf{F} \in \mathcal{W}} \text{MSE}(\mathbf{H}, \mathbf{F}). \quad (6)$$

The MSE expression can be computed by a simple expression (Palomar and Cioffi, 2003). Let  $P_e(\boldsymbol{\phi})$  denote the MSE expression by allocated power  $\boldsymbol{\phi}$ , we can write it as

$$P_e(\boldsymbol{\phi}) = \sum_{l=1}^{N_s} \frac{1}{1 + \phi_l \lambda_l}, \quad (7)$$

where the constants  $\lambda_l$  is the eigenvalue of  $\mathbf{H}^H \mathbf{R}_{nn}^{-1} \mathbf{H}$  and  $\phi_l$  is the power allocated on the  $l$ th subchannel.

#### MMSE based codebook construction algorithm

To design the MMSE based codebook, the joint optimization of the channel vector region and the transmit modes is required. Two key works involved are:

(1) Given the channel vector region, to find an optimal design of the transmit modes.

(2) Given a set of transmit modes, to find an optimal design of channel vector region.

It is noticed that the optimization of finding the region  $R = \{R_1, \dots, R_{N_B}\}$  and transmit scheme  $\{\mathbf{v}_q, \boldsymbol{\phi}_q\}_{q=1}^{N_B}$  is equivalent to designing a vector quantizer with a modified distortion measure.

Let  $A_q$  denote the probability that the channel matrix  $\mathbf{H}$  lies in the region  $R_q$ , i.e.,  $P(\mathbf{H} \in R_q) = A_q$ . By the statement and experiment in (Love and Heath, 2005b), the probabilities of selection for each possible codebook precoding matrix are the same. Thus, the uniform distribution across the codebook would mean that each codeword is selected with a probability of  $1/N_B$ .  $P_0$  stands for the average transmit power. By jointly designing  $\{R_q\}_{q=1}^{N_B}$  and transmitter strategy  $\{\mathbf{v}_q, \boldsymbol{\phi}_q\}_{q=1}^{N_B}$ , our ultimate goal is to

$$\min J = \sum_{q=1}^{N_B} A_q P_e(q), \quad \text{s.t.} \quad \sum_{q=1}^{N_B} \left( A_q \sum_{l=1}^{N_s} \phi_l(q) \right) \leq P_0. \quad (8)$$

Using Lagrange multiplier  $\mu$ , the distortion measure for optimizing the MSE performance is given by

$$D = \sum_{q=1}^{N_B} A_q \frac{1}{N_s} \sum_{l=1}^{N_s} \frac{1}{1 + \phi_l(q) \lambda_l(q)} + \mu \left( \sum_{q=1}^{N_B} \left( A_q \sum_{l=1}^{N_s} \phi_l(q) \right) - P_0 \right). \quad (9)$$

The distortion measure is a function of  $\mathbf{H}$ ,  $\mathbf{v}_q$  and  $\phi_q$ . The partition index  $q$  is sent to the transmitter, and the transmitter selects the optimal schemes by  $q$ .

Hence, the optimization problem could be solved by Lloyd's algorithm, which can be outlined in the following steps (Gersho and Gray, 1992):

Step 0: To generate a training set with  $N_{tr}$  samples  $\{T_n\}_{n=1}^{N_{tr}}$ . Start with an initial transmit schemes (obtained via random computer search in  $N_{tr}$  samples). In the first iteration, let power be equally allocated on each data substreams. Then, carry out the following two steps iteratively.

Step 1: Given a certain channel condition region of  $\{R_1, \dots, R_{N_B}\}$ , to find the optimal transmit scheme  $\{T_1, \dots, T_{N_B}\}$ , where the transmitter scheme  $T_q := \{\mathbf{v}_q, \phi_q\}$  denotes the multimode beamforming matrix and the power allocation along different modes. The optimal transmit scheme  $T_q$  is given by the generalized region centroid condition:

$$T_q = \left( \arg \min_{\mathbf{H} \in R_q} E[D] \right) A_q. \quad (10)$$

Step 2: Given a transmit scheme  $T_q$ , to find the optimal channel regions  $R_q$  ( $1 \leq q \leq N_B$ ). The optimal region is given by the nearest neighbor rule:

$$R_q = \{ \mathbf{H} : D(\mathbf{H}, T_q) \leq D(\mathbf{H}, T_j), \forall j, q \in [1, \dots, N_B] \}. \quad (11)$$

During the iteration, we examine the tentative codebook, and record the minimum distance of Eq.(9) if it is larger than the currently best. We do not stop this process until no further improvement on the minimum distance is observed.

The followings are the close-form solutions of the above Step 1 and Step 2.

### Solution of Step 1

For a given region  $R_q$  ( $1 \leq q \leq N_B$ ), find the optimal  $T_q$ .

Given a certain region  $R_q$ , the probability  $A_q$  and the channel covariance matrix can be calculated for every  $q$ th region. Let the eigenvalue decomposition of  $\mathbf{H}^H \mathbf{R}_{nm}^{-1} \mathbf{H} = \mathbf{V} \mathbf{A}_q \mathbf{V}^H$ , where  $\mathbf{A}_q = \text{diag}(\lambda_1(q), \dots, \lambda_{N_t}(q))$  is an  $N_t \times N_t$  diagonal matrix and  $\lambda_1(q) \geq \lambda_2(q)$

$\geq \dots \geq \lambda_{N_t}(q)$ , and  $\mathbf{V}_q$  is an  $N_t \times N_t$  unitary matrix. The optimal unquantized precoder for each channel condition should be  $\mathbf{V}_{opt} = \bar{\mathbf{V}}_q$ , where  $\bar{\mathbf{V}}_q$  is the matrix constructed from the first  $N_s$  columns of  $\mathbf{V}_q$  (Love and Heath, 2005b; Xia and Giannakis, 2006).  $\mathbf{R}_{nm}$  is the covariance of noise. Based on those decomposition matrices, we can select the optimal precoding matrix belonging to the region  $R_q$  consequently,

$$\mathbf{v}_q^{opt} = \arg \min_{\{\mathbf{v} | \mathbf{H} \in R_q\}} D(\mathbf{H}, \mathbf{v}, \phi). \quad (12)$$

**Theorem 1** With optimal eigen-beamforming combined with power allocation, we get the optimal transmitter power allocation schemes by water-filling principle:

$$\phi_l(q) = \left( \frac{P_{tot} + \sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} \lambda_l^{-1}(q)}{\sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} \lambda_l^{-1/2}(q)} \lambda_l^{-1/2}(q) - \lambda_l^{-1}(q) \right)^+, \quad (13)$$

where  $(x)^+ = \max(0, x)$ ,  $q=1, \dots, N_B$ , and  $\phi_l(q)$  denotes the  $l$ th entry of the diagonal power allocation matrix of the  $q$ th region. Note that  $\bar{N}_s \leq N_s$ . The reason is that under some conditions, the worst eigenmode would be dropped off.

**Proof** The cost function can be expressed as follows:

$$D_{\mathbf{H}, T_q} = \sum_{q=1}^{N_B} A_q \frac{1}{\bar{N}_s} \sum_{l=1}^{\bar{N}_s} \frac{1}{1 + \phi_l(q) \lambda_l(q)} + \mu \left( \sum_{q=1}^{N_B} A_q \sum_{l=1}^{\bar{N}_s} \phi_l(q) - P_0 \right). \quad (14)$$

Gradient provides us the following conditions:

$$\frac{\partial D_{\mathbf{H}, T_q}}{\partial \phi_l(q)} = \frac{-\lambda_l(q)}{\bar{N}_s [1 + \phi_l(q) \lambda_l(q)]^2} + \mu = 0, \quad (15)$$

$$\frac{\partial D_{\mathbf{H}, T_q}}{\partial \mu} = \sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} A_q \phi_l(q) - P_0 = 0. \quad (16)$$

From Eq.(15), we get

$$\frac{\partial D_{\mathbf{H}, T_q}}{\partial \phi_l(q)} = \frac{[\lambda_l(q) / (\bar{N}_s \mu) - 1]^{1/2}}{\lambda_l(q)}. \quad (17)$$

From Eq.(16), we get

$$\sqrt{\frac{1}{\bar{N}_s \mu}} = \frac{P_0 + \sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} A_q \lambda_l^{-1}(q)}{\sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} A_q \lambda_l^{-1/2}(q)}. \quad (18)$$

Substituting Eq.(18) into Eq.(17), the result will be

$$\phi_l(q) = \left( \frac{P_0 + \sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} A_q \lambda_l^{-1}(q)}{\sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} A_q \lambda_l^{-1/2}(q)} \lambda_l^{-1/2}(q) - \lambda_l^{-1}(q) \right)^+. \quad (19)$$

Because the channel  $\mathbf{H}$  is i.i.d,  $A_q$  can be dealt with constant (Love and Heath, 2005b). Let  $P_{\text{tot}} = P_0/A_q$ , Eq.(19) can be written as

$$\phi_l(q) = \left( \frac{P_{\text{tot}} + \sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} \lambda_l^{-1}(q)}{\sum_{q=1}^{N_B} \sum_{l=1}^{\bar{N}_s} \lambda_l^{-1/2}(q)} \lambda_l^{-1/2}(q) - \lambda_l^{-1}(q) \right)^+. \quad (20)$$

**Solution of Step 2**

For a given transmit scheme, determine the optimal region  $\{R_q\}_{q=1}^{N_B}$ .

The optimal partition region is given by the nearest neighbor rule.

In our case, we will use the distortion measure Eq.(9) to get the new regions as follows:

$$R_q = \{\mathbf{H} : D(\mathbf{H}, T_q) \leq D(\mathbf{H}, T_j), \forall j \neq q \in [1, \dots, N_B]\}. \quad (21)$$

The above two necessary conditions of optimality proved to be essential for the codebook construction (Xia, 2004). Firstly, for each region, the optimal codeword can be chosen to minimize the distortion. In our case, multimode beamforming together with power allocation is employed in centroid condition. Secondly, the nearest neighbor rule is used to find the optimal region around each codeword.

**Solution of codewords selection**

Here, we present the optimal codeword selection algorithm for MIMO system. Let  $T_q = \{\mathbf{v}_q, \phi_q\}$  denote

the  $q$ th codeword in codebook  $\{T_q\}_{q=1}^{N_B}$ , then the description of the codewords selection is:

(1) Get CSI of  $\mathbf{H}$  by channel estimation at the receiver.

(2) Introduce the following eigenvalue decomposition (EVD):  $\mathbf{v}^H \mathbf{H}^H \mathbf{R}_{mm}^{-1} \mathbf{H} \mathbf{v} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^H$ , where  $\mathbf{\Lambda}$  is an  $N_s \times N_s$  diagonal matrix with diagonal entries  $\lambda_{l=1}^{N_s}$  and matrix  $\mathbf{V}$  denotes the eigen matrix of  $\mathbf{v}^H \mathbf{H}^H \mathbf{R}_{mm}^{-1} \mathbf{H} \mathbf{v}$ .

(3) Let  $D = \sum_{l=1}^{N_s} \frac{1}{1 + \lambda_l \phi}$ , where  $N_s$  denotes the

data stream on each sub-channel. The selected optimal codeword is

$$T_{\text{opt}} = \arg \min D(T_q), T_q \in T_{q=1}^{N_B}. \quad (22)$$

**SIMULATION AND DISCUSSION**

In this section, we provide some numerical examples to illustrate the performance of the optimal limited feedback design. We present Monte Carlo simulation for the  $N_t=4, N_r=2$ , MIMO system with the constellation used being QPSK.

We assumed the discrete-time channel impulse response be generated according to the Hiperlan2 Channel Model C in (Medbo and Schramm, 1998). The channels between different transmit and receiver antennas were assumed independent. The channel was fixed for a frame and randomly varied between frames. The transmitter power was allocated by water-filling principle. The receiver used linear decoder with perfect channel knowledge. It was assumed that the feedback CSI had no delay and no transmit error.

**Experiment 1** The first experiment compares the performance of one system with ideal CSI and the other with partial CSI at the transmitter. When the transmitter has perfect CSI, the best performance can be achieved. While in partial CSI scenario, we compare the performance of MMSE based codebook and the existing codebooks in (Zhou, 2006) and at <http://dynamo.ecn.purdue.edu/~djlove/grass.html>. We use actual  $Q$ -function as codebook selection criteria for the existing codebook. Fig.2 shows that MMSE based codebook can get a moderate performance gain over the existing codebook, and the codebook in (Zhou,

2006) is a little better than that at <http://dynamo.ecn.purdue.edu/~djlove/grass.html>. We also use Chernoff-bound approximation as codeword selection method. The result shows that Chernoff-bound approximation selection is as good as the actual  $Q$ -function selection.

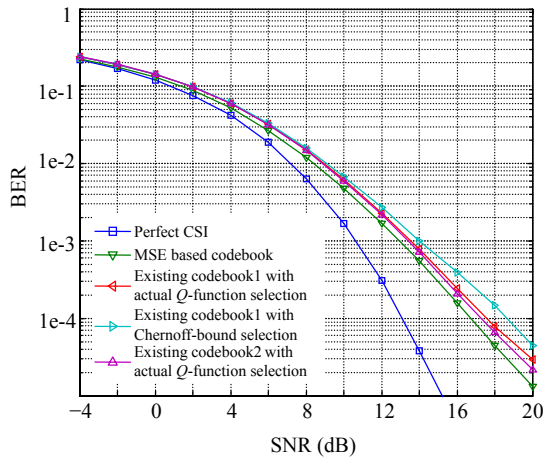


Fig.2 BER performance comparison of optimal power allocation and codebook based limited feedback

**Experiment 2** The codebook construction algorithm couples two steps which are finding the optimal transmit schemes  $T_q$  for a certain channel condition regions and finding the optimal channel regions  $R_q$  for a transmit scheme  $T_q$ , where  $q \in [1, N_B]$ . These two steps are worked repeatedly to enable the final design of the transmit modes and the fading regions. In this experiment, the codebook size is  $N_B=64$ , and the data stream is  $N_s=2$ . Fig.3 shows that after three to four iterations, the distortion can be converged. The iterative algorithm is guaranteed to converge in a few iterations.

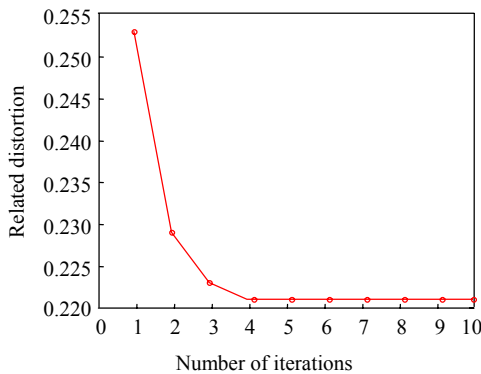


Fig.3 Convergence of codebook construction algorithm

**Experiment 3** The influence of codebook size with MMSE receiver is addressed in Fig.4. The  $B=0$  case corresponds to a MIMO precoding system without CSI in the transmitter. In this scenario, the transmitter has to take equal power allocation transmit scheme. The  $B=4$  case corresponds to the codebook size 16, and  $B=6, 7, 8$  correspond to the codebook size 64, 128, 256, respectively. When  $B=\text{inf}$ , it means that the transmitter can get perfect CSI. We observe that:

(1) The feedback link can improve the system performance;

(2) When the feedback bits increase, the system can have performance gain with  $B=\text{inf}$  as a performance benchmark;

(3) The gap between  $B=6, B=7$  and  $B=8$  is small, so the codebook size in a practical system is not necessary to be large.

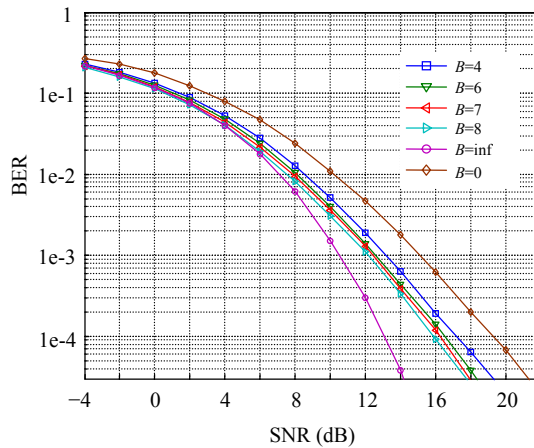


Fig.4 BER performance comparison among codebooks of different sizes

CONCLUSION

In this paper, a precoded spatial multiplexing MIMO system with a limited feedback of CSI is considered and a new MMSE based codebook construction algorithm is proposed. The performance of MMSE based codebook outperforms that of the existing codebooks. The essential component of this paper is the codebook design by using optimal structure of the transmitter and receiver, which employs the optimal transmit structure combining precoding and power allocation. Although the presented codebook design algorithm is complex, it should not be an issue because the codebook is designed off-line.

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