



## Modelling of ultrasonic motor with dead-zone based on Hammerstein model structure\*

Xin-liang ZHANG<sup>1</sup>, Yong-hong TAN<sup>2</sup>

(<sup>1</sup>Department of Automation, School of Electronic, Information and Electrical Engineering,  
 Shanghai Jiao Tong University, Shanghai 200030, China)

(<sup>2</sup>Laboratory of Intelligent Systems and Control Engineering, Guilin University of Electronic Technology, Guilin 541004, China)

E-mail: xinliangzhang@sjtu.edu.cn; tany@gliet.edu.cn

Received Mar. 21, 2007; revision accepted June 3, 2007; published online Nov. 25, 2007

**Abstract:** The ultrasonic motor (USM) possesses heavy nonlinearities which vary with driving conditions and load-dependent characteristics such as the dead-zone. In this paper, an identification method for the rotary travelling-wave type ultrasonic motor (RTWUSM) with dead-zone is proposed based on a modified Hammerstein model structure. The driving voltage contributing effect on the nonlinearities of the RTWUSM was transformed to the change of dynamic parameters against the driving voltage. The dead-zone of the RTWUSM is identified based upon the above transformation. Experiment results showed good agreement between the output of the proposed model and actual measured output.

**Key words:** Ultrasonic motor (USM), Hammerstein model, Dead-zone, Nonlinearity, Identification

**doi:**10.1631/jzus.A071146

**Document code:** A

**CLC number:** TM32

### INTRODUCTION

The ultrasonic motor (USM) is a new type motor, which is driven by the ultrasonic vibration force of piezoelectric elements. It has excellent performance and many useful features, e.g., high torque density at low speed, no electron-magic interference, high holding torque without any applied electric power, and so on (Sashida, 1993; Ueha and Tomikawa, 1993). The typical rotary travelling-wave ultrasonic motor (RTWUSM) is composed of a rotor, a stator made of elastic body and piezoelectric elements, and friction materials between stator and rotor. When two standing waves feeding voltages with proper amplitudes and phase difference between them are applied to the piezoelectric layer, two orthogonal vibration modes are excited to their single eigen-frequency. A travelling bending wave is generated with an elliptic locus

motion on the stator surface. The rotor is driven by frictional forces generated by the contact between the rotor and stator due to the preload force (Wallaschek, 1998; Hagood IV and McFarland, 1995; Yang and Que, 2001).

The dynamics of the USM have many complicated and nonlinear characteristics due to load torque, driving frequency, applied voltages, etc. It is difficult to derive a mathematical model allowing for all the USM dynamics. Many theories and methods have been reported for the modelling of the USM in recent years, such as the equivalent circuit model (Aoyagi *et al.*, 1996; Tomikawa *et al.*, 1991), the finite element model (Wang, 2004; Frangi *et al.*, 2005; Kagawa *et al.*, 1996), and the analysis model (Jeong *et al.*, 1997; Zhu, 2004; Schmidt *et al.*, 1996). These models are generally used to analyze the static characteristics with many assumptions or motor design. They are mostly too complex or inexact to be used directly in the design of the controllers. From the control point of view, the identification method to model the USM can

\*Project supported by the National Natural Science Foundation of China (No. 60572055) and the Natural Science Foundation of Guangxi Province (No. 0339068), China

overcome the problem based on the above models in the design of the controller. The experimental results showed that the identification models like Hammerstein model by analogy with the operation of the USM can work well with good agreement with the practical dynamics of the USM (Bigdeli and Haeri, 2004; Senjyu et al., 2001). Senjyu et al.(2006) introduced a second order transfer function to identify the dynamics of the USM to obtain the mathematical model for speed control, and achieved satisfactory simulation results. Experimental results showed that the control based on the identification model can yield satisfactory performance for the prototype USM (Senjyu et al., 2001).

The nonlinearities of the USM dynamics due to load-dependent dead-zone and driving conditions are mostly important limiting nonlinearities in controller design approaches. In this paper, the typical rotary travelling-wave ultrasonic motor (RTWUSM) is studied. The effect of the driving voltage is focused on and can be expressed by the change of the dynamic parameters in the proposed model. The relation between the dynamic parameters and driving voltage is approximated by polynomial equations against driving voltage based on experimental measurements. The whole system is taken as a nonlinear ARX model based Hammerstein structure with its nonlinear part being expressed by a polynomial of a known order of driving voltage. And then the dead-zone is identified based on the above analysis.

The paper is organized as follows. Section 2 describes the problem formulation related to the RTWUSM based on Hammerstein model structure. Section 3 derives the iterative Least Square Algorithm for the identification. Section 4 gives the identification for the RTWUSM dead-zone. Section 5 presents the simulation results to show the effectiveness of the developed model and finally Section 5 gives concluding remarks and plans for further development.

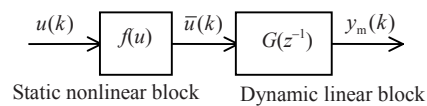
### PROBLEM FORMULATION

The focused Hammerstein model structure is shown in Fig.1, which consists of a formally linear dynamic element in series with a nonlinear static element (Ding and Chen, 2005), where  $u(k)$  denotes the input,  $y_m(k)$  the measured output of the system,

and  $\bar{u}(k)$  the inner immeasurable variable, namely the output of the nonlinear element. The structure in Fig.1 is used to identify the subsystem for RTWUSM excluding the dead-zone, and  $u(k)$  is defined as the contributing input voltages after the dead-zone detailed in Section 5. The structure of the nonlinear static block can vary from exponential functions to complex neural networks. In this paper, for a SISO process, we model the nonlinear static portion of the system by a power series which is composed of nonlinear polynomial equations in ascending integer powers from one onward described as follows,

$$\bar{u}(k) = f[u(k)] = \sum_{i=1}^m c_i u^i(k). \quad (1)$$

Usually, the first coefficient is defined as  $c_1=1$  (Ding and Chen, 2005).



**Fig.1 The Hammerstein model structure**

The input-output relationship of the system can be described as follows:

$$\begin{aligned} y_m(k) &= G(z^{-1})u(k) \\ &= z^{-\frac{\tau}{T_s}} \frac{B(z^{-1})}{A(z^{-1})} \bar{u}(k) = z^{-d} \frac{B(z^{-1})}{A(z^{-1})} \bar{u}(k), \end{aligned} \quad (2)$$

where  $\tau$  is time delay, and  $T_s$  is sample period.  $G(z^{-1})$  is formally linear transfer function and  $A(z^{-1})$ ,  $B(z^{-1})$  are polynomials in shift operator  $z^{-1}$ ,

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{n_b} z^{-n_b}, \quad (3)$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_{n_a} z^{-n_a}, \quad (4)$$

and  $b_0=1$ .

### LEAST SQUARE ALGORITHM FOR MODIFIED HAMMERSTEIN MODEL

From Eqs.(1)~(4), letting  $d=0$  for simplicity, the

following recursive equation can be derived in inputs and measured outputs,

$$\begin{aligned} y_m(k) &= -\sum_{i=1}^{n_a} a_i y_m(k-i) + \sum_{i=0}^{n_b} b_i \bar{u}(k-i) \\ &= -\sum_{i=1}^{n_a} a_i y_m(k-i) + \sum_{j=0}^{n_b} b_j \sum_{i=1}^m c_i u^i(k-j). \end{aligned} \quad (5)$$

Define the parameter vector  $\theta$  and information vector  $\psi_0(k)$  and  $\psi(k)$  as,

$$\begin{aligned} \theta &= [c_2 \mathbf{b} \quad c_3 \mathbf{b} \quad \cdots \quad c_m \mathbf{b}]^T, \\ \mathbf{a} &= [1 \quad a_1 \quad a_2 \quad \cdots \quad a_{n_a}]^T, \\ \mathbf{b} &= [b_0 \quad b_1 \quad b_2 \quad \cdots \quad b_{n_b}]^T, \\ \psi_0(k) &= \begin{bmatrix} y_m(k) \\ y_m(k-1) \\ y_m(k-2) \\ \vdots \\ y_m(k-n_a) \end{bmatrix}, \quad \psi(k) = \begin{bmatrix} \varphi_2(k) \\ \varphi_3(k) \\ \vdots \\ \varphi_m(k) \end{bmatrix}, \\ \varphi_0(k) &= \begin{bmatrix} u(k) \\ u(k-1) \\ u(k-2) \\ \vdots \\ u(k-n_b) \end{bmatrix}, \quad \varphi_j(k) = \begin{bmatrix} \gamma_j(u(k)) \\ \gamma_j(u(k-1)) \\ \gamma_j(u(k-2)) \\ \vdots \\ \gamma_j(u(k-n_b)) \end{bmatrix}, \\ \gamma_j(u(k)) &= u^j(k), \quad j=2, \dots, m, \end{aligned}$$

then we have

$$\psi_0^T(k) \mathbf{a} - \varphi_0^T(k) \mathbf{b} = \psi_0^T(k) \theta, \quad (6)$$

$$\text{or} \quad X(k) = \psi^T(k) \theta, \quad (7)$$

where vectors  $\mathbf{a}$  and  $\mathbf{b}$  can be obtained by the approximation equation in terms of the driving voltage in Section 4.

Let  $\hat{\theta}$  denote the estimate of  $\theta$  and define the predictive error cost function as,

$$J(\theta) = \sum_{i=k-p+1}^k (X(i) - \psi^T(i) \theta)^2, \quad (8)$$

where  $p$  is data length,  $p \gg (m-1)n_b$ .

Let

$$X(k) = \begin{bmatrix} X(k) \\ X(k-1) \\ \vdots \\ X(k-p+1) \end{bmatrix}, \quad \Phi(k) = \begin{bmatrix} \psi(k) \\ \psi(k-1) \\ \vdots \\ \psi(k-p+1) \end{bmatrix},$$

hence,

$$J_p(\theta) = \|X(k) - \Phi(k) \theta\|^2. \quad (9)$$

Minimizing  $J_p(\theta)$  gives the least-square estimation,

$$\hat{\theta} = [\Phi^T(k) \Phi(k)]^{-1} \Phi(k) X(k). \quad (10)$$

## DRIVING VOLTAGE EFFECT ON DYNAMIC PARAMETERS OF RTWUSM AND APPROXIMATIONS

The hardware configuration of the RTWUSM is shown in Fig.2. The input voltage ( $0 \leq u(k) \leq 3$ ) from D/A is transferred to PWM and control the MOS-FET drive. Then the two-channel half-bridge oscillation inverter generates two-phase alternating voltages  $V_A$  and  $V_B$  which are applied to RTWUSM. The design specifications of RTWUSM are given in Table 1. The circuit of the two-channel half-bridge oscillation inverter is showed in Fig.3. Positive 12 V DC and negative 12 V DC are used to control the revolving direction of the RTWUSM.

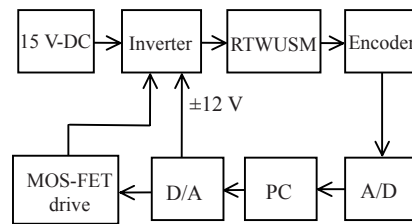


Fig.2 The hardware configuration of the RTWUSM

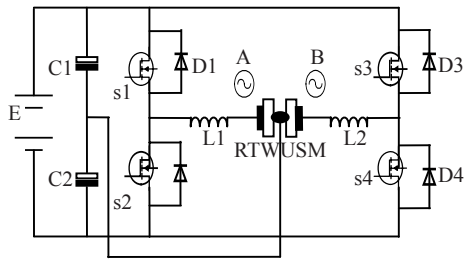
Like other conventional motors, RTWUSM exhibits dynamic behavior that can be represented by the following transfer function with velocity output  $\omega(s)$  against input  $U(s)$  (Bigdeli and Haeri, 2004; Jin et al., 1998),

$$G(s) = \omega(s)/U(s) = e^{-\tau s} K / (Ts + 1), \quad (11)$$

where  $\tau$  is the pure time delay of the system.

**Table 1 Design specifications of RTWUSM**

Parameter	Value
Type	fif60
Drive frequency (kHz)	4.5
Drive voltage (Vrms)	137.4
Rated current (Arms)	0.32
Revolving speed (r/min)	100 <sup>-1</sup>
Rated output power (W)	3
Torque (N·cm)	0~50
Weight (kg)	0.250



**Fig.3 The two-channel half-bridge oscillation inverter**

The motor parameters are time varying due to the drive conditions such as source voltage and load-torque. So we substitute  $T_m[u(k)]$ ,  $K_m[u(k)]$  for the time constant  $T$  and obtain  $K$  to represent the change due to the driving voltage.

Thus the transfer function of RTWUSM Eq.(11) is transformed to the following equation,

$$G_m(s, u) = e^{-\tau s} \frac{K_m[u]}{T_m[u]s + 1}, \quad (12)$$

where  $T_m[u(k)]$  and  $K_m[u(k)]$  are approximated by  $n$ th-order polynomial equations against driving voltage as follows:

$$T_m[u(k)] = \sum_{i=0}^{n_T} \alpha_i u^i(k), \quad (13)$$

$$K_m[u(k)] = \sum_{i=0}^{n_K} \beta_i u^i(k). \quad (14)$$

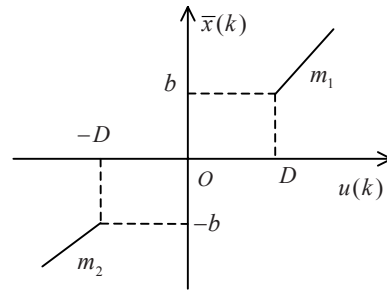
And discretizing continuous transfer function Eq.(12) gives,

$$G_m(z^{-1}) = z^{-d} \frac{b_0}{1 + a_1 z^{-1}}, \quad (15)$$

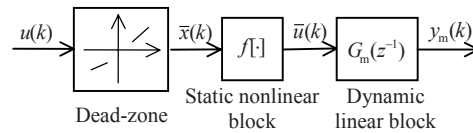
where,  $b_0 = K_m[u(k)] \{1 - \exp(-T_s/T_m[u(k)])\}$ ,  $d = \tau/T_s$ ,  $a_1 = -\exp(-T_s/T_m[u(k)])$ . Then from Eq.(10), the estimated value of  $\theta$  is obtained.

**DEAD-ZONE IDENTIFICATION**

The dead-zone of the RTWUSM is a problem to design controller which has some steady-state tracking error if neglected. It is important to identify and compensate for it in order to improve the control performance. In this section, the dead-zone of the RTWUSM is identified based on modified Hammerstein structure as shown in Figs.4 and 5.



**Fig.4 Discontinuous nonlinearity with dead-zone**



**Fig.5 The modified Hammerstein model structure**

Therein,  $u(k)$  is input voltage fed into RTWUSM.  $\bar{x}(k)$  is the output of nonlinear block combining dead-zone effect, exactly equivalent to the symbol  $u(k)$  in Fig.1 and Eq.(1). The functions  $f[\cdot]$ ,  $G_m(z^{-1})$  and variables  $\bar{u}(k)$ ,  $y_m(k)$  are the same symbols as defined above. The value of  $\bar{x}(k)$  depends on the magnitude and sign of  $u(k)$  and their relation can be described as follows (Voros, 2003; 1997),

$$v(k) = m_1 u(k) + (m_2 - m_1) h(k) u(k), \quad (16)$$

$$z(k) = m_1 u(k) + (m_2 - m_1) h(k) u(k) - D \operatorname{sgn}[u(k)]$$

$$[\nu(k)/u(k)] + b \operatorname{sgn}[u(k)], \quad (17)$$

$$\bar{x}(k) = m_1 u(k) + (m_2 - m_1) h(k) u(k) - D \operatorname{sgn}[u(k)] \frac{\nu(k)}{u(k)}$$

$$+b \operatorname{sgn}[u(k)] + 0.5\{1 + \operatorname{sgn}[D - |u(k)|]\}z(k), \quad (18)$$

where  $m_1, m_2$  are corresponding segment slopes,  $D$  is dead-zone parameter and  $b$  is preload constant. The switching function  $h(k)$  is defined as follows,

$$h(k) = h[u(k)] = \begin{cases} 0, & \text{if } u(k) \geq 0; \\ 1, & \text{if } u(k) < 0. \end{cases} \quad (19)$$

Then the input-output form of the modified Hammerstein model can be described as follows,

$$y_m(k) = \bar{u}(k-d) + \sum_{i=0}^{n_b} b_i \bar{u}(k-d-i) - \sum_{i=1}^{n_a} a_i y_m(k-i), \quad (20)$$

where  $\bar{u}(k) = f[\bar{x}(k)]$ .

Substituting Eq.(18) into Eq.(20) gives,

$$\begin{aligned} y_m(k) = & f\{m_1 u(k-d) + (m_2 - m_1)h(k-d)u(k-d) - \\ & D \operatorname{sgn}[u(k-d)]\frac{v(k-d)}{u(k-d)} + b \operatorname{sgn}[u(k-d)] - \\ & 0.5\{1 + \operatorname{sgn}[D - |u(k-d)|]\}z(k-d)\} + \\ & \sum_{i=0}^{n_b} b_i \bar{u}(k-d-i) - \sum_{i=1}^{n_a} a_i y_m(k-i), \end{aligned} \quad (21)$$

i.e.,

$$\begin{aligned} y_m(k) = & \sum_{i=1}^4 \xi_i [m_1, m_2, D, b] \cdot \Gamma_i[u, h, v, z] + \Gamma_0[u, h, v, z] \\ & + \sum_{i=0}^{n_b} b_i \bar{u}(k-d-i) - \sum_{i=1}^{n_a} a_i y_m(k-i), \end{aligned} \quad (22)$$

where  $f[\cdot]$  is given by Eq.(10),  $\bar{u}(k-d-i)$  is obtained from Eq.(1).  $a_i, b_i$  can be estimated from Eq.(10).

Define the parameter and information vectors as,

$$\boldsymbol{\theta}_{DZ} = [\xi_1 \quad \xi_2 \quad \xi_3 \quad \xi_4]^T, \quad (23)$$

$$\boldsymbol{\varphi}(k) = [\Gamma_1 \quad \Gamma_2 \quad \Gamma_3 \quad \Gamma_4]^T. \quad (24)$$

Then modified output,

$$\begin{aligned} y_c(k) = & y_m(k) - \sum_{i=0}^{n_b} b_i \bar{x}(k-d-i) + \\ & \sum_{i=1}^{n_a} a_i y_m(k-i) - \Gamma_0[u, h, v, z]. \end{aligned} \quad (25)$$

Then we have,

$$y_c(k) = \boldsymbol{\varphi}(k)^T \boldsymbol{\theta}_{DZ}. \quad (26)$$

Define the predictive error cost function as,

$$J_2(\boldsymbol{\theta}_{DZ}) = \sum_{k=1}^l (y_c(k) - \hat{\boldsymbol{\varphi}}(k)^T \boldsymbol{\theta}_{DZ})^2, \quad (27)$$

where  $\hat{\boldsymbol{\varphi}}(k)$  is the estimation of  $\boldsymbol{\varphi}(k)$  for recursive algorithm.

Minimizing the cost function  $J_2(\boldsymbol{\theta}_{DZ})$  gives the least-square estimation  $\hat{\boldsymbol{\theta}}_{DZ}$  for  $\boldsymbol{\theta}_{DZ}$ . The recursive algorithm is written as follows,

$$\hat{\boldsymbol{\theta}}_{DZ}(k) = \hat{\boldsymbol{\theta}}_{DZ}(k-1) + \mathbf{K}(k) \{\hat{y}_c(k) - \hat{\boldsymbol{\varphi}}(k)^T \hat{\boldsymbol{\theta}}_{DZ}(k-1)\}, \quad (28)$$

$$\mathbf{K}(k) = \mathbf{P}(k-1) \hat{\boldsymbol{\varphi}}(k) \{\hat{\boldsymbol{\varphi}}^T(k) \mathbf{P}(k-1) \hat{\boldsymbol{\varphi}}(k) + 1\}^{-1}, \quad (29)$$

$$\mathbf{P}(k) = [1 - \mathbf{K}(k) \hat{\boldsymbol{\varphi}}(k)] \mathbf{P}(k-1), \quad (30)$$

$$\mathbf{P}(0) = \alpha^2 \mathbf{I}, \quad \hat{\boldsymbol{\theta}}_{DZ}(0) = \boldsymbol{\varepsilon}.$$

The internal variables  $v(k), z(k)$  and  $\bar{x}(k)$  as well as  $y_c(k)$  can be estimated by Eq.(16)~(18) and Eq.(25) in each recursion with previous estimation of  $\hat{\boldsymbol{\theta}}_{DZ}$ .

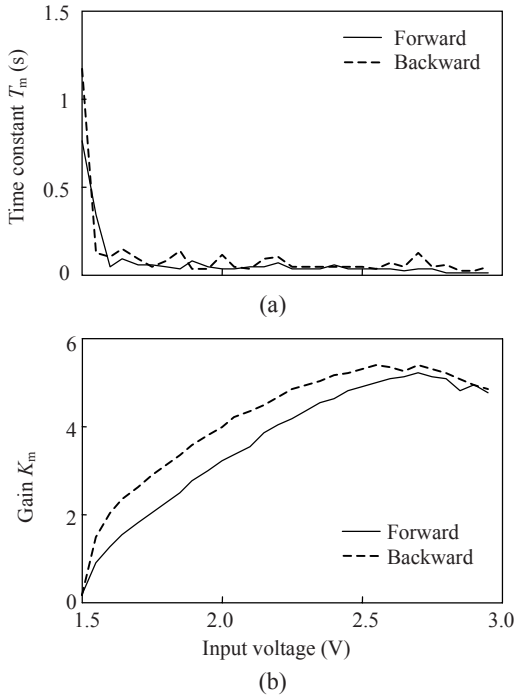
The estimated values for dead-zone parameters  $m_1, m_2$ , and  $b$  against input driving voltage  $u(k)$  are obtained by solving the equations group  $\xi[m_1, m_2, D, b]$  using  $\hat{\boldsymbol{\theta}}_{DZ}$ .

## EXPERIMENTS AND SIMULATIONS

The incremental coder has been used to measure the angle position with sample period  $T_s=0.05$  s. After the analysis of the experiment measurements, we get the system pure time delay  $\tau=0.05$  s, namely  $d=1$  for discrete form of transfer function.

The varying wave of dynamic parameters  $T_m$  and  $K_m$  against driving voltage is shown in Figs.6a and 6b when the motor is in the forward and backward direction.

Here, a 6th-order polynomial equation is used to approximate the effect of driving voltage on time constant from Eq.(13), and  $T_{mFw}$  and  $T_{mBw}$  denote



**Fig.6 Driving voltage effect on time constant  $T_m$  (a) and on Gain  $K_m$  (b)**

forward and backward direction respectively, i.e.

$$T_{mFw}[u(k)] = 2028.3u^{-6} + 5671.4u^{-5} - 6547.6u^{-4} + 4003u^{-3} - 1370u^{-2} + 249.4u^{-1} - 18.9, \quad (31)$$

$$T_{mBw}[-u(k)] = -1648[-u]^{-6} - 4576.6[-u]^{-5} - 5250.9[-u]^{-4} - 3200[-u]^{-3} - 1097.8[-u]^{-2} - 202.1[-u]^{-1} - 15.6. \quad (32)$$

And a 3rd-order polynomial equation is used to approximate the effect of driving voltage on Gain  $K_m$ .  $K_{mFw}$  and  $K_{mBw}$  denote forward and backward direction respectively. We get,

$$K_{mFw}[u(k)] = -1.0203u^3 + 4.0340u^2 + 0.7359u - 6.2248, \quad (33)$$

$$K_{mBw}[-u(k)] = -0.8924[-u]^3 - 9.5479[-u]^2 - 31.64 - 28.1305[-u], \quad (34)$$

where the negative sign before  $u(k)$  in Eqs.(32) and (34) denotes the input voltage ( $0 \leq u(k) \leq 3$ ) used to control RTWUSM to revolve in negative direction.

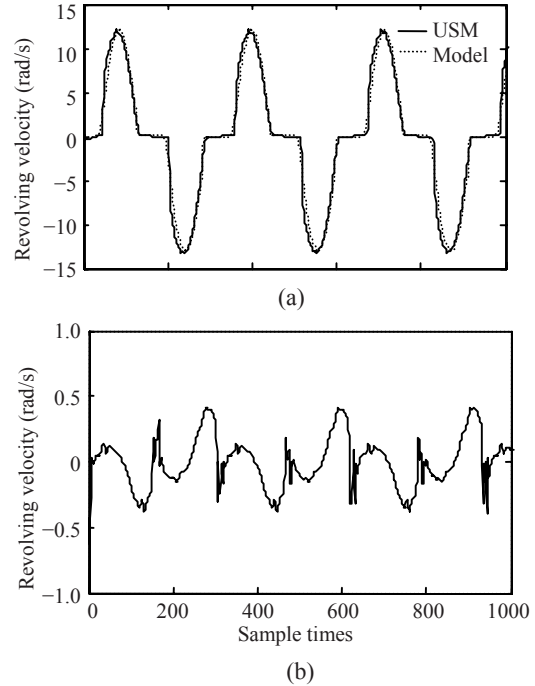
Taking  $u(k) = 2.2 + 0.2\sin(0.2T_s k) + 0.2\sin(1.5T_s k) + 0.35\sin(10T_s k)$  to get the identification data for positive and negative directions, letting  $m=3$  in Eq.(1), the estimated value of  $\theta$  can be obtained from Eq.(10).

$$\text{Forward direction, } \begin{cases} c_2 = -0.0442, \\ c_3 = 0.0157; \end{cases}$$

$$\text{Backward direction, } \begin{cases} c_2 = 0.0476, \\ c_3 = 0.0171. \end{cases}$$

Taking slope and cosine signal as input voltages, based on the proposed identification method for dead-zone parameters of RTWUSM from Eq.(16)~(30), letting  $m_1=1$ ,  $m_2=1$  and  $b=1$  for simplicity, we can get  $D=1.4936$ .

Taking  $u(k) = 2.5\sin(0.4T_s k)$  V as the input voltages for the validation of the developed model, the validation results and prediction error are shown in Fig.7.



**Fig.7 Model validation (a) and prediction error (b) with  $u(k) = 2.5\sin(0.4T_s k)$**

The simulation results showed that the proposed model based on Hammerstein structure presents good agreement with the RTWUSM.

## CONCLUSION

In this paper, an identification method for RTWUSM is proposed based on modified Hammer-

stein model structure. The effect of the driving voltage on the parameters of the motor is investigated. The dynamic nonlinearities of the RTWUSM due to the driving voltage are focused on and transformed to the effect on parameters of the dynamic linear block in the Hammerstein structure. Polynomial equations are implemented to approximate the effect of the driving voltage based on the experimental results. The dead-zone of the RTWUSM is identified through direct application of recursive Least Square Algorithm based on the above approximation methods.

Experimental results showed that the proposed identification method based on Hammerstein model yields satisfactory modelling results. The model is mathematically simple and feasible for design of controller based on it.

The proposed method can easily be expanded to be used for modelling the RTWUSM with regard to the effect due to the load-torque, temperature rise, etc. Furthermore, intelligent identification tools like neural networks can be incorporated to approximate the effect of the driving voltage on the parameters of the Hammerstein model. It is very promising in engineering for the modelling of the USM.

## References

- Aoyagi, M., Tomikawa, Y., Takano, T., 1996. Simplified equivalent circuit of the ultrasonic motor and its applications. *Ultrasonics*, **34**(2-5):275-278. [doi:10.1016/0041-624X(95)00071-A]
- Bigdeli, N., Haeri, M., 2004. Modeling of an Ultrasonic Motor Based on Hammerstein Model Structure. 8th Control, Automation, Robotics and Vision Conference, ICARCV-04, **2**(6-9):1374-1378. [doi:10.1109/ICARCV.2004.1469047]
- Ding, F., Chen, T., 2005. Identification of Hammerstein nonlinear ARMAX systems. *Automatica*, **41**(9):1479-1489. [doi:10.1016/j.automatica.2005.03.026]
- Frangi, A., Corigliano, A., Binci, M., Faure, P., 2005. Finite element modeling of a rotating piezoelectric ultrasonic motor. *Ultrasonics*, **43**(9):747-755. [doi:10.1016/j.ultras.2005.04.005]
- Hagood IV, N.W., McFarland, A.J., 1995. Modeling of a piezoelectric rotary ultrasonic motor. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, **42**(2):210-224. [doi:10.1109/58.365235]
- Jeong, S.H., Lee, H.K., Kim, Y.J., Kim, H.H., Lim, K.J., 1997. Vibration Analysis of the Stator in Ultrasonic Motor by FEM. Proceedings of the 5th International Conference on Properties and Applications of Dielectric Materials, ICPADM-97. Korea, p.1091-1094. [doi:10.1109/ICPADM.1997.616637]
- Jin, L., Zhu, M., Zhao, C., 1998. Position and speed controlling for traveling wave ultrasonic motors. *Journal of Experimental Mechanics*, **13**(2):263-266 (in Chinese).
- Kagawa, Y., Tsuchiya, T., Kataoka, T., Yamabuchi, T., Furukawa, T., 1996. Finite element simulation of dynamic responses of piezoelectric actuators. *Journal of Sound and Vibration*, **191**(4):519-538. [doi:10.1006/jsvi.1996.0138]
- Sashida, T., 1993. An Introduction to Ultrasonic Motors. Oxford Science Publication, Oxford.
- Schmidt, J.P., Hagedorn, P., Miao, B.Q., 1996. A note on the contact problem in an ultrasonic travelling wave motor. *Int. J. Non-Linear Mechanics*, **31**(6):915-924. [doi:10.1016/S0020-7462(96)00112-6]
- Senjyu, T., Kashiwagi, T., Uezato, K., 2001. Position control of ultrasonic motors using MRAC with dead-zone compensation. *IEEE Transactions on Industrial Electronics*, **48**(6):1278-1285. [doi:10.1109/41.969411]
- Senjyu, T., Nakamura, M., Urasaki, N., Sekine, H., Funabashi, T., 2006. Mathematical Model of Ultrasonic Motors for Speed Control. Twenty-First Annual IEEE Applied Power Electronics Conference and Exposition, APEC 06, p.290-295. [doi:10.1109/APEC.2006.1620553]
- Tomikawa, Y., Yaginuma, M., Hirose, S., Takano, T., 1991. An equivalent circuit expression of an ultrasonic motor and measurement of its elements. *Jpn. J. Appl. Phys.*, **30**(Part 1, 9B):2398-2401. [doi:10.1143/JJAP.30.2398]
- Ueha, S., Tomikawa, Y., 1993. Ultrasonic Motors: Theory and Applications. Clarendon Press, Oxford.
- Voros, J., 1997. Parameter identification of discontinuous Hammerstein systems. *Automatica*, **33**(6):1141-1146. [doi:10.1016/S0005-1098(97)00009-5]
- Voros, J., 2003. Recursive identification of Hammerstein systems with discontinuous nonlinearities containing dead-zones. *IEEE Transactions on Automatic Control*, **48**(12):2203-2206. [doi:10.1109/TAC.2003.820146]
- Wallaschek, J., 1998. Contact mechanics of piezoelectric ultrasonic motors. *Smart Mater. Struct.*, **7**(3):369-381. [doi:10.1088/0964-1726/7/3/011]
- Wang, S.Y., 2004. A finite element model for the static and dynamic analysis of a piezoelectric bimorph. *International Journal of Solids and Structures*, **41**(15):4075-4096. [doi:10.1016/j.ijsolstr.2004.02.058]
- Yang, M., Que, P., 2001. Performances estimation of a rotary travelling-wave ultrasonic motor based on two-dimension analytical model. *Ultrasonics*, **39**(2):115-120. [doi:10.1016/S0041-624X(00)00053-6]
- Zhu, M., 2004. Contact analysis and mathematical modeling of traveling wave ultrasonic motors. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **51**(6):668-679. [doi:10.1109/TUFFC.2004.1308725]