Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Determination of rock resistant coefficient based on Mohr-Coulomb criterion for underwater tunnel

Zhong-ren TU[†1,2], Qiang YANG[1], Qi-ming SHEN[2], Xiao-wen WANG[2]
([1]*State Key Laboratory of Hydroscience and Engineering, Tsinghua University, Beijing 100084, China*)
([2]*School of River and Ocean Engineering, Chongqing Jiaotong University, Chongqing 400074, China*)
[†]E-mail: tuzrtj@126.com; tuzr@mail.tsinghua.edu.cn
Received July 20, 2007; revision accepted May 28, 2008

Abstract: According to the load-structure method, the wall rock with lining can bear the load caused by the surrounding rock, and the rock resistant coefficient (RRC) is a key parameter for evaluating the capacity of this wall rock. Based on the Mohr-Coulomb yield criterion, this paper develops a formula for calculating the RRC, which has been applied to the real engineering project, such as Xiamen Xiang'an East Passage Underwater Tunnel Project. The fact shows that this formula is helpful for designers to determine the RRC value.

Key words: Rock resistant coefficient (RRC), Mohr-Coulomb yield criterion, Rock mechanics, Underwater tunnel
doi:10.1631/jzus.A071390 **Document code:** A **CLC number:** U45; TU9

INTRODUCTION

Geometry has an inherent ability of resisting load, and the ability can be quantified by a coefficient named as subgrade reaction coefficient (SRC) in soil mechanics or rock resistant coefficient (RRC) in rock mechanics. Various mathematical models and methods, such as modified beam on elastic foundation (MBEF) method by Liao (1995) and extended kalman filter (EKF) method by Kobayashi *et al*.(2007) based on in situ horizontal load test, had been proposed to determine the SRC to estimate the horizontal SRC. Other scholars such as Honjo *et al*.(1999) and Badoni and Makris (1996) researched different methods to determine the SRC.

Based on complex potential function and conformal mapping representation, a closed-form elastic solution for stresses and displacement around tunnel was presented by Exadaktylos and Stavropoulou (2002). According to the tunnel design codes in China, the local deformation theory and the load-structure method are used to direct the tunnel lining design. The RRC is a vital parameter in the process of lining thickness calculation because it affects the estimating of the stress in the lining greatly. Practically, the designers determine the RRC value by referring to a table in which all levels and types rock mass information are collected, and by using Xu (1993)'s elastics mechanics based formula or Qian (1955)'s formula. Using finite ring model and different yield criteria and constitutive equations, Cai and Cai (2004) and Lü (1981) developed several RRC calculation formulae for the water conveyance tunnel. Based on the unified strength theory, Xu and Yu (2004) obtained another formula, which could take the inter-principal stress into consideration. Since the formulae mentioned above are feasible only if the rock mass is in elastic state, a new RRC calculation formula is derived based on Mohr-Coulomb criterion which can be applied to rock mass in both elastic state and plastic state.

FUNDAMENTAL EQUATIONS

Failure condition

The formula deductive process for RRC adopts the Mohr-Coulomb criterion as failure condition, which can be written as follows:

$$\tau = c - \sigma \cot\varphi, \tag{1}$$

where τ is shearing strength, σ is normal stress, c and φ are cohesion and internal friction angle of rock mass, respectively.

Constitutive equation (in polar coordinate)

Since rock mass may be in different stress states, the constitutive equations for elastic stage and plastic stage are adopted as follows.

1. Elastic stage

$$\begin{cases} \varepsilon_r = [\sigma_r - \mu_{\mathrm{d}}(\sigma_\theta + \sigma_z)]/E_{\mathrm{d}},\ \gamma_{r\theta} = \tau_{r\theta}/E_{\mathrm{d}}, \\ \varepsilon_\theta = [\sigma_\theta - \mu_{\mathrm{d}}(\sigma_z + \sigma_r)]/E_{\mathrm{d}},\ \gamma_{\theta z} = \tau_{\theta z}/E_{\mathrm{d}}, \\ \varepsilon_z = [\sigma_z - \mu_{\mathrm{d}}(\sigma_r + \sigma_\theta)]/E_{\mathrm{d}},\ \gamma_{zr} = \tau_{zr}/E_{\mathrm{d}}. \end{cases} \tag{2}$$

2. Plastic stage (total strain theory)

$$\begin{cases} \varepsilon_r^{\mathrm{p}} = \psi[\sigma_r - (\sigma_\theta + \sigma_z)/2]/(3G_{\mathrm{d}}), \\ \varepsilon_\theta^{\mathrm{p}} = \psi[\sigma_\theta - (\sigma_z + \sigma_r)/2]/(3G_{\mathrm{d}}), \\ \varepsilon_z^{\mathrm{p}} = \psi[\sigma_z - (\sigma_r + \sigma_\theta)/2]/(3G_{\mathrm{d}}), \\ \gamma_{r\theta}^{\mathrm{p}} = \psi\tau_{r\theta}/G_{\mathrm{d}},\ \varepsilon_{\theta z}^{\mathrm{p}} = \psi\tau_{\theta z}/G_{\mathrm{d}},\ \varepsilon_{zr}^{\mathrm{p}} = \psi\tau_{zr}/G_{\mathrm{d}}, \end{cases} \tag{3}$$

where ψ is plasticity function and equals zero in elastic deformation stage.

ELASTIC AND PLASTIC STRESS ANALYSIS OF INNER CYLINDER BOUNDARY

Plastic stress analysis

According to the theoretical model, if the uniformed pressure around the inner boundary reaches the rock mass plastic yield strength, the plastic zone will appear nearby the inner circle and its shape is annular, as shown in Fig.1.

In Fig.1, r_2, r_3 and p_2 are the plastic zone inner radius, plastic outer radius and inner uniformed pressure, respectively.

Usually the inner pressure p_2 is less than the initial stress in the rock mass, hence we obtain $\sigma_\theta < \sigma_r < 0$.

The Mohr circle locates at the left side of τ coordinate axis (Y-axis), and σ_θ is the minimum stress. The Mohr-Coulomb yield law can be written as

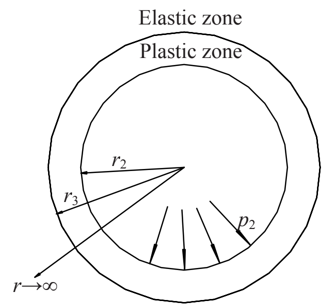

Fig.1 Theoretical model with elastic and plastic zones

$$(\sigma_\theta - \sigma_r)/2 = (\sigma_r - c\cot\varphi)\sin\varphi/(1-\sin\varphi), \tag{4}$$

which is the yield condition for the plastic zone.

Introducing Eq.(4) into the balance differential equation, separating variable and applying variable integral, we have

$$\ln(-\sigma_r + c\cot\varphi) = 2\sin\varphi \ln r/(1-\sin\varphi) + B. \tag{5}$$

Stress boundary in the plastic zone inner radius can be written as

$$\sigma_{r|r=r_2} = -p_2,$$

and the integral constant B in Eq.(5) can be solved as

$$B = \ln(p_0 + c\cot\varphi) - 2\sin\varphi \ln r_2/(1-\sin\varphi).$$

We get the stress in the plastic

$$\begin{cases} \sigma_\theta = -(p_0 + c\cot\varphi)(1+\sin\varphi)(r/r_2)^{2\sin\varphi/(1-\sin\varphi)} \\ \qquad \cdot (1-\sin\varphi)^{-1} + c\cot\varphi, \\ \sigma_r = -(p_0 + c\cot\varphi)(r/r_2)^{2\sin\varphi/(1-\sin\varphi)} + c\cot\varphi. \end{cases} \tag{6}$$

Outer radius r_3 of plastic zone stress

The interface between elastic zone and plastic zone is a circle of radius r_3 with the radial stress σ_{r_3}. The whole elastic zone can be treated as a thick cylinder with an inner radius r_3 and an outer radius approaching to ∞. The stress expression in elastic zone can be obtained from elastic plane strain formula as

$$\sigma_r = \sigma_{r_3} r_3^2/r^2,\ \ \sigma_\theta = -\sigma_{r_3} r_3^2/r^2. \tag{7}$$

Introducing Eq.(6) and Eq.(7) into the contact condition

$$(\sigma_r+\sigma_\theta)^{\rm p}\Big|_{r=r_3}=(\sigma_r+\sigma_\theta)^{\rm e}\Big|_{r=r_3},$$

the outer radius r_3 for the plastic zone can be obtained as

$$r_3=r_2\left[c\cot\varphi(1+\sin\varphi)/(p_2+c\cot\varphi)\right]^{(1-\sin\varphi)/(2\sin\varphi)},\quad(8)$$

and radial pressure p_2 can be written as

$$p_2=\left[c\cot\varphi(1+\sin\varphi)\right](r_2/r_3)^{2\sin\varphi/(1-\sin\varphi)}-c\cot\varphi.\quad(9)$$

Eq.(9) is the necessary and sufficient condition between inner pressure and rock mass parameters for the plastic zone existing around the tunnel rock mass.

Plastic zone radial displacement u_r

The relation between σ_z, σ_r and σ_θ for elastic and plastic plane strain condition can be written as follows:

$$\sigma_z=(\sigma_r+\sigma_\theta)/2,\quad(10)$$

and the elastic strain in the plastic zone can be written in the form of

$$\varepsilon_r^{\rm e}=[\sigma_r-\mu_{\rm d}(\sigma_\theta+\sigma_z)]/E_{\rm d}.\quad(11)$$

Combining Eq.(10) and Eq.(11), we obtain another expression for the elastic strain $\varepsilon_r^{\rm e}$ in the plastic zone

$$\varepsilon_r^{\rm e}=[(1-\mu_{\rm d}/2)\sigma_r-3\mu_{\rm d}\sigma_\theta/2]/E_{\rm d}.\quad(12)$$

Similarly, we can obtain the elastic tangential strain expression in the plastic zone

$$\varepsilon_\theta^{\rm e}=[(1-\mu_{\rm d}/2)\sigma_\theta-3\mu_{\rm d}\sigma_r/2]/E_{\rm d}.\quad(13)$$

Plastic strain can be expressed by radial stress σ_r tangential stress σ_θ, shear modulus $G_{\rm d}$, and plasticity function ψ.

$$\begin{cases}\varepsilon_r^{\rm p}=\psi(\sigma_r-\sigma_\theta)/(4G_{\rm d}),\\ \varepsilon_\theta^{\rm p}=\psi(\sigma_\theta-\sigma_r)/(4G_{\rm d}).\end{cases}\quad(14)$$

Now we obtain the total strain in plastic zone, which is the sum of the elastic strain and the plastic strain, i.e.,

$$\begin{cases}\varepsilon_r=\varepsilon_r^{\rm e}+\varepsilon_r^{\rm p}={\rm d}u/{\rm d}r=[(1-\mu_{\rm d}/2)\sigma_r-3\mu_{\rm d}\sigma_\theta/2]/E_{\rm d}\\ \qquad+\psi(\sigma_r-\sigma_\theta)/(4G_{\rm d}),\\ \varepsilon_\theta=\varepsilon_\theta^{\rm e}+\varepsilon_\theta^{\rm p}=u/r=[(1-\mu_{\rm d}/2)\sigma_\theta-3\mu_{\rm d}\sigma_r/2]/E_{\rm d}\\ \qquad+\psi(\sigma_\theta-\sigma_r)/(4G_{\rm d}).\end{cases}\quad(15)$$

Introducing Eq.(15) into the displacement compatibility relation

$${\rm d}\varepsilon_\theta/{\rm d}r+(\varepsilon_\theta-\varepsilon_r)/r=0,$$

we obtain Eq.(16)

$$\frac{1}{E_{\rm d}}\left[(1-2\mu_{\rm d})\frac{{\rm d}\sigma_\theta}{{\rm d}r}-\frac{3}{2}\mu_{\rm d}\frac{{\rm d}\sigma_r}{{\rm d}r}\right]+\frac{\psi}{4G_{\rm d}}\left(\frac{{\rm d}\sigma_\theta}{{\rm d}r}-\frac{{\rm d}\sigma_r}{{\rm d}r}\right)+\frac{\sigma_\theta-\sigma_r}{4G_{\rm d}}\frac{{\rm d}\psi}{{\rm d}r}+\left(\frac{1+\mu_{\rm d}}{E_{\rm d}}+\frac{\psi}{2G_{\rm d}}\right)\frac{\sigma_\theta-\sigma_r}{r}=0.\quad(16)$$

From the stress equilibrium equation

$${\rm d}\sigma_r/{\rm d}r+(\sigma_r-\sigma_\theta)/r=0,$$

we can easily know that

$$(\sigma_\theta-\sigma_r)/r={\rm d}\sigma_r/{\rm d}r.\quad(17)$$

Firstly introducing Eq.(17) into Eq.(16), we have

$$\left(\frac{1-\mu_{\rm d}/2}{E_{\rm d}}+\frac{\psi}{4G_{\rm d}}\right)\frac{\rm d}{{\rm d}r}(\sigma_\theta+\sigma_r)+\frac{\sigma_\theta-\sigma_r}{4G_{\rm d}}\frac{{\rm d}\psi}{{\rm d}r}=0.\quad(18)$$

Then introducing Eq.(6) into Eq.(18), we obtain the displacement compatibility differential equation in the form of plasticity function ψ, i.e.,

$$\frac{{\rm d}\psi}{{\rm d}r}+\frac{2}{(1-\sin\varphi)r}\psi+\frac{4G_{\rm d}(2-\mu_{\rm d})}{E_{\rm d}(1-\sin\varphi)}\frac{1}{r}=0.\quad(19)$$

Solving Eq.(19), we can write the expression of plasticity function ψ as

$$\psi = \mathrm{e}^{-\int \frac{2}{1-\sin\varphi}\frac{1}{r}\mathrm{d}r}\left[B - \int \frac{4G_\mathrm{d}(2-\mu_\mathrm{d})}{E_\mathrm{d}(1-\sin\varphi)}\frac{1}{r}\mathrm{e}^{\int \frac{2}{1-\sin\varphi}\frac{1}{r}\mathrm{d}r}\mathrm{d}r\right]$$
$$= Br^{-\frac{2}{1-\sin\varphi}} - 2G_\mathrm{d}(2-\mu_\mathrm{d})/E_\mathrm{d}, \quad (20)$$

where B is the integral constant which can be determined from the contact boundary condition.

As mentioned above, the plasticity function ψ equals zero in the elastic stage. If displacing r with r_3 in Eq.(20), the integral constant B can be written as

$$B = 2G_\mathrm{d}(2-\mu_\mathrm{d})r_3^{2/(1-\sin\varphi)}/E_\mathrm{d}.$$

Now we have the last expression of plasticity function ψ as

$$\psi = 2G_\mathrm{d}(2-\mu_\mathrm{d})[(r_3/r)^{2/(1-\sin\varphi)} - 1]/E_\mathrm{d}. \quad (21)$$

Introducing Eq.(6) and Eq.(21) into the second equation of Eq.(15), we have the equation of the displacement in the plastic zone as

$$u_r = r\varepsilon_\theta = \frac{r}{E_\mathrm{d}}\Big\{-(p_2 + c\cot\varphi)(r/r_2)^{2\sin\varphi/(1-\sin\varphi)}\Big[(1-2\mu_\mathrm{d})$$
$$+2\sin\varphi(1-\mu_\mathrm{d}/2)(r_3/r)^{2/(1-\sin\varphi)}/(1-\sin\varphi)\Big]$$
$$+(1-2\mu_\mathrm{d})c\cot\varphi\Big\}. \quad (22)$$

Using Eq.(22), the displacement at the plastic zone inner boundary is

$$u_{r_2} = \frac{r_2}{E_\mathrm{d}}\Big\{-(p_2 + c\cot\varphi)\Big[(1-2\mu_\mathrm{d})$$
$$+2\sin\varphi(1-\mu_\mathrm{d}/2)(r_3/r_2)^{2/(1-\sin\varphi)}/(1-\sin\varphi)\Big] \quad (23)$$
$$+(1-2\mu_\mathrm{d})c\cot\varphi\Big\}.$$

DEDUCING OF RRC (K)

In practical construction process, the crack zone between the plastic zone and the tunnel lining appears because of various causes such as excavation disturbing or blasting, etc. The depth of this crack ring is r_2–r_1, shown as Fig.2.

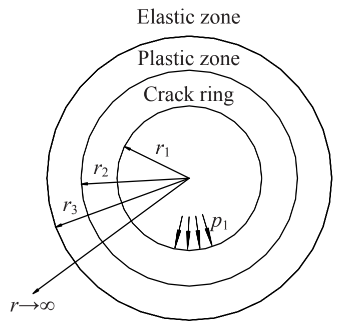

Fig.2 Theoretical analysis model with crack ring

For the simplest situation, we assume the rock mass in the crack ring can only bear pressure but not the tension. So the stress balance equation is changed into the form of Eq.(24):

$$\mathrm{d}\sigma_r/\mathrm{d}r + \sigma_r/r = 0. \quad (24)$$

Integrating and solving Eq.(24) with $\sigma_{r|r=r_1} = -p_1$ as the stress definite condition, we obtain

$$\sigma_r = -p_1 r_1/r. \quad (25)$$

Using the stress boundary condition $\sigma_{r|r=r_2} = -p_2$, we obtain the relation between p_1 and p_2 as

$$p_2 = p_1 r_1/r_2. \quad (26)$$

Combining the geometric equation and constitutive equation of the elastic theory, we have

$$\mathrm{d}u_r = -\frac{1-\nu_0^2}{E_0}p_1 r_1\frac{\mathrm{d}r}{r}, \quad (27)$$

where E_0 and ν_0 are elastic modulus and Poisson's ratio, respectively.

Using displacement at the plastic zone inner boundary as the particular solution condition to solve Eq.(27) and integrating Eq.(27), we have the expression of displacement in crack zone as

$$u_r = (1-\nu_0^2)p_1 r_1 \ln(r_2/r)/E_0 + r_2\{-(p_2 + c\cot\varphi) \cdot[(1-2\mu_d) + 2(1-\mu_d/2)(r_3/r_2)^{2/(1-\sin\varphi)}\sin\varphi \cdot(1-\sin\varphi)^{-1}] + (1-2\mu_d)c\cot\varphi\}/E_d. \quad (28)$$

Then in the location of $r=r_1$, the radial displacement can be written as

$$u_{r_1} = (1-\nu_0^2)p_1 r_1 \ln(r_2/r_1)/E_0 + \{-(p_2 + c\cot\varphi) \cdot[(1-2\mu_d) + 2(1-\mu_d/2)(r_3/r_2)^{2/(1-\sin\varphi)}\sin\varphi \cdot(1-\sin\varphi)^{-1}] + (1-2\mu_d)c\cot\varphi\}r_2/E_d. \quad (29)$$

Inducing Eq.(26) into Eq.(9), we have

$$(r_3/r_2)^{\frac{2}{1-\sin\varphi}} = \left[c\cot\varphi(1-\sin\varphi)/(c\cot\varphi + p_1 r_1/r_2)\right]^{\frac{1}{\sin\varphi}}. \quad (30)$$

By introducing Eq.(30) into Eq.(29), we can easily obtain the radial displacement at the inner boundary of crack ring as

$$u_{r_1} = \frac{1-\nu_0^2}{E_0}p_1 r_1 \ln\frac{r_2}{r_1} + \frac{r_2}{E_d}\left\{-(p_2 + c\cot\varphi) \cdot\left[(1-2\mu_d) + (1-\frac{1}{2}\mu_d)\left(\frac{c\cot\varphi(1-\sin\varphi)}{c\cot\varphi + p_1 r_1/r_2}\right)^{1/\sin\varphi} \cdot\frac{2\sin\varphi}{1-\sin\varphi}\right] + (1-2\mu_d)c\mathrm{ctg}\varphi\right\}. \quad (31)$$

Now RRC (K) can be calculated according to the Winkler theorem:

$$K = p_1/u_{r_1} = \left\{\frac{1-\nu_0^2}{E_0}r_1 \ln\frac{r_2}{r_1} + \frac{r_2}{E_d p_1}\left\{-(1-2\mu_d)\frac{p_1 r_1}{r_2} - \left(\frac{p_1 r_1}{r_2} + c\cot\varphi\right)\cdot\left(1-\frac{1}{2}\mu_d\right) \cdot\left[\frac{c\cot\varphi(1-\sin\varphi)}{c\cot\varphi + p_1 r_1/r_2}\right]^{1/\sin\varphi}\right\}\frac{2\sin\varphi}{1-\sin\varphi}\right\}^{-1} \quad (32)$$

and Eq.(32) is the RRC (K) calculation expression, which is based on the Mohr-Coulomb criterion.

Engineering application

Xiamen Xiang'an East Passage Undersea Tunnel, with 5.9 km in length, as the first domestic multi-hole submarine tunnel, consists of a downstream tunnel and a upstream one, and a service tunnel.

1. Transect geometrical parameters

Main tunnel: 16.38 m×12.35 m;

Service tunnel: 6.50 m×6.05 m.

The equivalent radii for the main tunnel and service tunnel are 8.2 m and 3.3 m, respectively.

2. Geological mechanics parameters

The geological mechanics parameters are shown in Table 1.

Table 1 Parameters of tunnel wall rock

Rock level	Gravity γ (kN/m^3)	Young's modulus E (GPa)	Poisson ratio ν	Friction angle φ (°)	Friction coefficent f
W_4	18.0	0.1	0.48	25	0.3
W_3	19.0	1.0	0.46	30	0.4
W_2	25.0	25.0	0.29	50	0.5
W_1	26.5	40.0	0.20	70	0.6
W	26.0	25.0	0.28	55	0.5

3. Calculation results

Introducing geometrical parameters and geological mechanics parameters into Eq.(32), the RRC value corresponding to different rock levels can be calculated. The results are shown in Table 2.

Table 2 RRC calculation result (MPa/m)

Rock level		Eq.(32) result	Road tunnel code result
W_1	Upstream tunnel	2162	500~2800
	Service tunnel	5372	
	Downstream tunnel	2162	
W_3	Upstream tunnel	315	200~500
	Service tunnel	405	
	Downstream tunnel	315	
W_4	Upstream tunnel	73	100~200
	Service tunnel	177	
	Downstream tunnel	73	

CONCLUSION

The idea of load-structure fully reflects the harmony and rationality between the safety and economy considerations during the underground structure designing process. And RRC (K) is a key quantity parameter embodying the idea for surrounding rock mass nearby the tunnel structure. In this paper, RRC (*K*) is deduced based on Mohr-Coulomb yield criterion.

Eq.(32) was applied to Xiamen Xiang'an East Passage Undersea Tunnel. Compared with the road tunnel design code, Eq.(32) can narrow the RRC value range for different levels of rock mass.

References

Badoni, D., Makris, N., 1996. Nonlinear response of single plies under lateral inertial and seismic loads. *Soil Dynamics and Earthquake Engineering*, **15**(1):29-43. [doi:10.1016/0267-7261(95)00027-5]

Cai, X.H., Cai, Y.P., 2004. Structure Stress Calculation for Hydraulic Pressure Tunnel. China Water Power Press, Beijing, p.124-135 (in Chinese).

Exadaktylos, G.E., Stavropoulou, M.C., 2002. A closed-form elastic solution for stresses and displacements around tunnel. *International Journal of Rock Mechanics and Mining Science*, **39**(7):905-916. [doi:10.1016/S1365-1609(02)00079-5]

Honjo, Y., Sako, K., Kikuchi, Y., 1999. Comparison of several regulation procedures in inverse analysis of horizontal subgrade reaction coefficient of piles. *Journal of Applied Mechanics*, **2**:73-81.

Kobayashi, N., Shibata, T., Kikuchi, Y., Murakami, A., 2007. Estimation of horizontal subgrade reaction coefficient by inverse analysis. *Computers and Geotechnics*, in press. [doi:10.1016/j.Compgeo.2007.11.002]

Liao, S.S.C., 1995. Estimating the coefficient of subgrade reaction for plane strain condition. *ICE Proceedings Geotechnical Engineering*, **113**(3):166-181. [doi:10.1680/igeng.1995.27812]

Lü, Y.N., 1981. A new formula of rock resistant factor "*K*" in hydraulic pressure tunnel. *Chinese Journal of Geotechnical Engineering (Yan Tu Gong Cheng Xue Bao)*, **3**(1):70-80 (in Chinese).

Qian, L.X., 1955. Calculation method of elastic resistant coefficient "*K*" in hydraulic pressure tunnel. *China Civil Engineering Journal (Tu Mu Gong Cheng Xue Bao)*, **2**(4):369-380 (in Chinese).

Xu, S.Q., Yu, M.H., 2004. Calculation of rock resistant factor in tunnel considering intermediate principal stress effect. *Chinese Journal of Rock Mechanics and Engineering (Yan Shi Li Xue Yu Gong Cheng Xue Bao)*, **23**(Supplement 1):4303-43059 (in Chinese).

Xu, Z.Y., 1993. Rock Mechanics. China Water Power Press, Beijing, p.259-276 (in Chinese).