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A novel dynamic equivalence method for grid-connected wind farm

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Abstract: Dynamic equivalence of the wind farm is a fundamental problem in the simulation of a power system connected with wind farms because it is unpractical to model every generator in a wind farm in detail. In this paper, an Equivalence Method based on the Output Characteristics (EMOC) is proposed, with which the wind farm composed of Squirrel-Cage Induction Generators (SCIGs) can be equivalent to one generator. By considering the diversity of wind generators and special operating characteristics of a wind farm, the equivalent generator based on EMOC responds accurately in various faults. No matter whether the wind farm is integrated in grid or just programmed, EMOC can be used to acquire an accurate equivalent generator. Simulation of the dynamic equivalence of an SCIG wind farm validated the method.

Key words: Grid-connected wind farm, Dynamic equivalence, Squirrel-Cage Induction Generator (SCIG), Genetic algorithm
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INTRODUCTION

Wind energy was the fastest growing energy in the past decades. 74.22-GW wind generating capacity had been installed in the world by the end of 2006, and the yearly growth rate of 2006 is 25.7% (GWEC, 2007). Grid-connected wind farm, which is the dominant technology in wind generation, is developing rapidly in many countries and the impact of the wind farms on grid has drawn much attention from the R&D community.

The basic way to consider a wind farm in simulation is to model every generator of the wind farm (Hansen *et al*., 2002; Muljadi and Butterfield, 2004). However, it is unpractical to model every wind generator in system simulation, so an equivalent wind generator model is needed to represent the grid-connected wind farm. The studies on equivalence of induction motors are helpful for this research of wind farm equivalence.

Abdel Hakim and Berg (1976) derived a single equivalent motor by making its impedance at zero speed equal to that of the group with all the individual motors at zero speed. The slip of the equivalent machine is then determined by equating the impedances of the single machine and those of the group with the individual machines running at their specified speeds. Thus, at zero speed and specified speeds of the motors in the group, the slips of the reduced model and the group are exactly equal but at other speeds they cannot be equal, as demonstrated by Abdel Hakim and Berg (1976).

In (Kataoka and Kai, 2000), the single equivalent motor is acquired based on the equivalent circuit and the parameters are calculated at two specified states: the no-load operation state and the locked-rotor state. At no-load operation, the slip of induction motor is approximately 0; at locked-rotor condition, the slip of induction motor is equal to 1. Under these two conditions, the rotor impedances of the motors are calculated.

Rahim and Laldin (1987) have derived the parameters of an equivalent machine by minimizing the difference between the responses of the reduced and complete models, using a weighted least square technique. They used the fifth-order model, including the electromechanical equation, to represent the equivalent machine as well as the individual machines in the complete model.

In (Sriharan *et al*., 1993), the behavior of a group of induction motors during a short period following a disturbance is considered. Each machine is represented by four first-order linear differential equations because the speed may be assumed constant. Thus the complete model of the group is also linear and the linear model reduction method is used. In the original form of this method, the eigenvalues of the complete system, furthest from the origin, are left out to obtain a reduced model using only the dominant eigenvalues nearer to the origin.

In wind farm equivalence, some special characteristics of the wind farm must be considered. Firstly, the output power of a wind farm may vary from zero to the installed capacity, so the equivalent generator should be accurate in a wide operation range of the wind farm. Secondly, every generator of the wind farm has different parameters and operating states, which is difficult to be considered in the equivalence method. Thirdly, there exist various faults in a power system. The equivalent generator should respond to different faults correctly. In this paper, the equivalence for a wind farm composed of Squirrel-Cage Induction Generators (SCIGs) is investigated and an Equivalence Method based on the Output Characteristics (EMOC) of the wind farm is proposed. With this method, an accurate equivalent generator of a wind farm with consideration of the special characteristics could be achieved for the simulation of a power system connected with wind farms.

The structure of the paper is as follows. In Section 2, the wind farm equivalence is analyzed and an optimization model is proposed for equivalence. In Section 3, EMOC is validated by simulation of the dynamic equivalence of a wind farm. Section 4 gives the conclusions.

EQUIVALENCE METHOD

Analysis of wind farm equivalence

The power system connected with a wind farm could be simplified as Fig.1. Bus-1 is the point of common coupling of the wind farm. On the view of system dynamics, the real power, reactive power and voltage at Bus-1 are most concerned. These data reflect the operation characteristics of the grid-connected wind farm, so it is feasible to develop an equivalence method of the wind farm based on the analysis of these data.

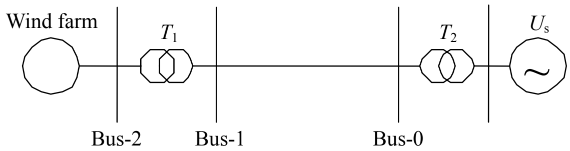

Fig.1 Simplified scheme of the power system connected with a wind farm. T_1 is the step-up transformer of the wind farm, T_2 is the step-up transformer into the grid, and U_s is the equivalent supply of the grid. The buses are numbered as Bus-0 to Bus-2

Wind energy is uncontrollable, so the output power of a wind farm varies from zero to the installed capacity at different wind speeds. As a result, the operating state of a wind farm is changeful. The real output power of a wind farm according to different wind speeds is required in equivalence, as well as the reactive output power and the voltage amplitude at Bus-1. The above data compose the output characteristics of the wind farm at different wind speeds, which are denoted by $\tilde{C}$ in the followings.

The diversity of generators in a wind farm is also an important factor in equivalence. The generators may have different parameters and the operating condition for each generator may be very different. As a result, the individual operating characteristic of all the generators, which has great effect on $\tilde{C}$, is quite different from each other. In order to involve the generator diversity in $\tilde{C}$, we have to develop proper means to get $\tilde{C}$.

$\tilde{C}$ can be acquired through two alternative methods. The first is practical measurement. In this method, $\tilde{C}$ is generated based on the measurements at Bus-1. The real power, reactive power and voltage at different wind speeds are recorded to form $\tilde{C}$. The measured data contain the full information of the wind farm including generator diversity, reactive compensation, line feeders, etc. So there is no need to consider any further detailed model of the wind farm. If the wind farm has been connected to the power system, the measurement method could be utilized. But if the wind farm has not been connected to the grid, we have to use the second method, i.e., theoretical calculation, to acquire $\tilde{C}$, and much detailed information of the wind farm should be taken into account.

In theoretical calculation, a detailed model of the wind farm is needed, where the generator diversity, line feeders and reactive compensation should be properly considered. With this model, the power system connected with the wind farm as shown in Fig.1 is simulated, and the recorded data of Bus-1 at different wind speeds is $\tilde{C}$.

Equivalence method based on $\tilde{C}$

The output characteristics $\tilde{C}$ of a wind farm contain abundant information of the wind farm operation. The equivalent generator of the wind farm is optimized by fitting $\tilde{C}$ in the followings.

The wind generator can be modeled in different ways for different uses. The shafting dynamics of a wind generator is usually approximated by two kinds of models (Tande *et al*., 2004): two-mass model and one-mass model. The two-mass model considers the turbine and generator inertia with a shaft and an ideal gearbox between them. In one-mass model, the masses of generator and turbine are summed up as one mass. In power system simulation, the one-mass model can correctly simulate the dynamic response of the wind farm, so we choose this model in equivalence.

In all kinds of wind turbines, the stall-controlled wind turbine usually matches the SCIG. This type of wind turbines has no blade control system and the captured wind power varies according to the power characteristic of the wind turbine (Ackermann, 2005), as shown in Fig.2.

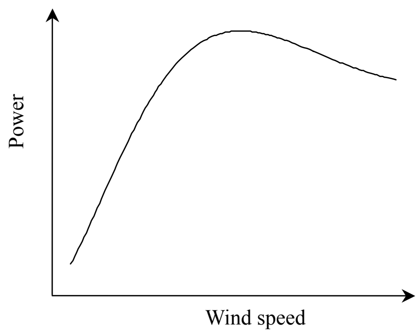

Fig.2 Power characteristic of the stall-controlled wind turbine

According to the analysis, the one-mass model is used in this paper. Applying per-unit values the model of equivalent generator is given by a third-order equation (Feijóo *et al*., 2000):

$$\dot{U} = \dot{E}' + (r_\mathrm{s} + \mathrm{j}x')\dot{I}, \tag{1}$$

$$p\dot{E}' = -\mathrm{j}s\omega\dot{E}' - [\dot{E}' - \mathrm{j}(x - x')\dot{I}]/T_0', \tag{2}$$

$$ps = \frac{1}{J}\left(P_\mathrm{e} - \frac{P_\mathrm{m}}{1-s}\right), \tag{3}$$

$$P_\mathrm{e} = -\mathrm{Re}(\dot{E}'\dot{I}^*), \tag{4}$$

where $\dot{U}$ is the stator voltage phasor, $\dot{I}$ the stator current phasor, $\dot{E}'$ the electric potential phasor, P_m the captured wind power, P_e the electromagnetic power, s the rotor slip, ω the supply angular frequency, T_0' the time constant of the rotor transient, J the inertia of the generator and the turbine, x the synchronous reactance, x' the transient reactance, r_s the stator resistance, p the differential operator.

Four equivalent parameters in the model need to be calculated, i.e., T_0', x, x', J (r_s is ignored for its small impact on the dynamic response of the generator), among which J is an important one when it comes to the dynamic simulation. In equivalence, J represents the overall inertia of the original system, so it is calculated as (Ni *et al*., 2002):

$$J = \sum_{i=1}^{M} J_i, \tag{5}$$

where M is the number of generators in the original system, J_i is the inertia of the ith wind generator.

The other three parameters cannot be calculated directly. They are optimized by fitting $\tilde{C}$.

When the wind farm in Fig.1 is replaced with the equivalent generator described as Eqs.(1)~(4), the output characteristic curves of the equivalent generator, denoted as C, can also be acquired by simulation. If the deviation between C and $\tilde{C}$ is set to be the objective, an optimization model for the equivalent parameters is developed as follows:

Objective:

$$\min f = \frac{1}{N}\sum_{i=1}^{N}\left[(U_i - \tilde{U}_i)^2 + (P_i - \tilde{P}_i)^2 + (Q_i - \tilde{Q}_i)^2\right], \tag{6}$$

Subject to:

$$\dot{U}_i - \dot{E}_i' - \mathrm{j}x'\dot{I}_i = 0,$$

$$-\mathrm{j}s_i\omega\dot{E}_i' - [\dot{E}_i' - \mathrm{j}(x - x')\dot{I}_i]/T_0' = 0,$$

$$\frac{1}{J}\left(P_{ei}-\frac{P_{mi}}{1-s_i}\right)=0,$$
$$P_{ei}+\mathrm{Re}(\dot{E}_i'\dot{I}_i^*)=0,\ \ P_{mi}-g_1(v_{wi},s_i)=0,$$
$$g_2(\dot{U}_i,P_i,Q_i)=0,\ \ P_i+\mathrm{Re}(\dot{U}_i\dot{I}_i^*)=0,$$
$$Q_i+\mathrm{Im}(\dot{U}_i\dot{I}_i^*)=0,$$
$$0<x<x_{\max},\ \ 0<x'<x'_{\max},\ \ 0<T_0'<T'_{0\max},$$

where v_{wi} (i=1,···,N) is one wind speed sample, N is the number of wind speed samples. $\tilde{U}_i$, $\tilde{P}_i$, $\tilde{Q}_i$ and U_i, P_i, Q_i (i=1,···,N) are the components of $\tilde{C}$ and C at different wind speeds v_{wi}, respectively. $g_1(\cdot)$ is used to calculate the captured wind power, and $g_2(\cdot)=0$ is the power flow equation of the grid. x, x' and T_0' constitute the decision variables of the optimization model.

The optimization model is nonlinear and complex, and we solve it by the genetic algorithm (Liu, 2002). The solution, which matches the minimum value of the objective function, is accepted as the best equivalent parameters.

It is obvious that the equivalent generator is only an approximation of the wind farm and the optimized equivalent parameters contain the diversity information of the wind farm. So many other generator models can also be used in the equivalence. For example, if the third-order model as Eqs.(1)~(4) with a parallel capacitor C_p is used, the equivalent parameters x, x', T_0' and C_p need to be optimized. But if the accuracy of a model is high enough, a more complex model is unnecessary.

VALIDATION OF EMOC

Equivalence of an SCIG wind farm

The test system is a Single-Machine Infinite-Bus (SMIB) system as shown in Fig.3. In the test system, the wind farm is composed of 30 wind generators of three different types. The parameters of the three types are quite different, as shown in Table 1. The generators are parallel connected to the collective bus (Bus-2) through output transformers. The voltage of the wind farm is step-up to the grid voltage through the transformer T_1, and the wind farm is connected to the grid at Bus-1. The infinite generator is shown as U_s and the voltage is set to be 1 p.u. The power base in simulation is set to be the capacity of all the wind generators, i.e., 29.5 MW. The per-unit values of the reactance of transformers T_1 and T_2 are both 0.05 p.u. The reactance of a single line is 0.2 p.u. The spatial distribution of the wind speed is assumed to be uniform in the wind farm. The rated wind speed for the original and equivalent generators is set to be 14 m/s.

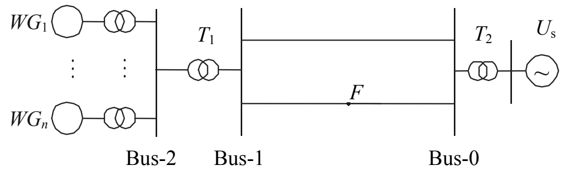

Fig.3 Scheme of the test system. The wind generators are numbered as *WG*$_1$ to *WG*$_n$. *F* denotes the midpoint of one line. The other parameters are the same as those in Fig.1

Table 1 Parameters of the original and equivalent generators*

Gen.	S_n (kW)	J (s)	T_0' (s)	x' (p.u.)	x (p.u.)
1	600	8	3.120	0.206	3.647
2	850	9	1.478	0.116	2.760
3	1500	10	1.621	0.185	1.777
EG	29500	9.305	1.667	0.171	2.241
REG	29500	9.305	1.885	0.161	2.240

* Parameters of the original generators are acquired from the wind generator models in (Tande *et al.*, 2004; Wu *et al.*, 2004) and MATLAB/SIMULINK, respectively; Per-unit values are calculated based on S_n of each generator, and are fit for the 690 V/50 Hz system Gen.: generator, EG: equivalent generator, REG: reference equivalent generator. S_n denotes the rated capacity. J, T_0', x' and x are the parameters of the generator as in Eqs.(1)~(4)

Based on EMOC, the wind farm is equivalent to one wind generator. The minimum value of the objective function in the optimization is 3.21×10^{-8} and the optimized equivalent parameters are shown as EG in Table 1. In order to validate the equivalence model, a reference equivalent generator is acquired based on (Kataoka and Kai, 2000) and the equivalent parameters are shown as REG in Table 1.

Simulation and comparison

Four kinds of dynamic processes are simulated: (1) The wind speed fluctuation (the wind speed increases uniformly from 11 m/s to 14 m/s in 3 s); (2) A short-circuit fault is applied at Bus-1, and disappears after 0.1 s (Fault 1); (3) A short-circuit fault is applied at the point F, and the fault line is cut off after 0.1 s (Fault 2); (4) The voltage of U_s steps up by 0.05 p.u., then backs down to the initial value.

The dynamic responses are evaluated with the indexes named max-error and average-error. The indexes are defined as follows:

$$e_{\max} = \max_{k=1}^{K} \left| \left(R_{\mathrm{f}}(k) - R(k)\right) / R(k) \right|, \quad (7)$$

$$e_{\mathrm{avg}} = \sqrt{\sum_{k=1}^{K} \frac{1}{K} \left[\left(R_{\mathrm{f}}(k) - R(k)\right) / R(k) \right]^2}, \quad (8)$$

where R_{f} and R represent the dynamic responses of the wind farm and the equivalent generator, respectively, K is the number of samples of the dynamic response at different sampling time.

The max- and average-errors are shown in Table 2. The average-errors of the real output power (P_{out}), the reactive output power (Q_{out}) and the voltage (U_{t}) of Bus-1 are less than 0.97%, 1.15% and 0.22%, respectively. The maximum errors appear at the rated wind speed of the wind farm. When the wind speed decreases, the errors decrease as well.

According to Table 2, the largest errors appear in Fault 1, and the response curves in Fault 1 are shown in Fig.4. The real and reactive output powers of the wind farm and the equivalent generator are compared in Figs.4a and 4b, respectively.

The four dynamic processes are simulated with the reference equivalent generator given in Table 1 and the average-errors are calculated and shown in Table 2. As can be seen from Table 2, the equivalent generator based on EMOC is the most accurate in all the dynamic processes.

Comments on EMOC

EMOC can be summarized as four steps. The first and most important step is to generate the output characteristics of the wind farm through measurement or theoretical calculation. If the theoretical calculation is adopted, a detailed model of the wind farm should be considered. The second step is to select the suitable generator model as the equivalent generator and confirm the equivalent parameters needed to be optimized. The next step is to optimize the equivalent parameters according to the output characteristics of the wind farm. Finally, the equivalent generator should be validated in dynamic simulation. If the wind farm has been connected to the grid, the field measurement can be used to validate the equivalent generator.

EMOC does not consider the fault responses of the wind farm. Two reasons are taken into account:

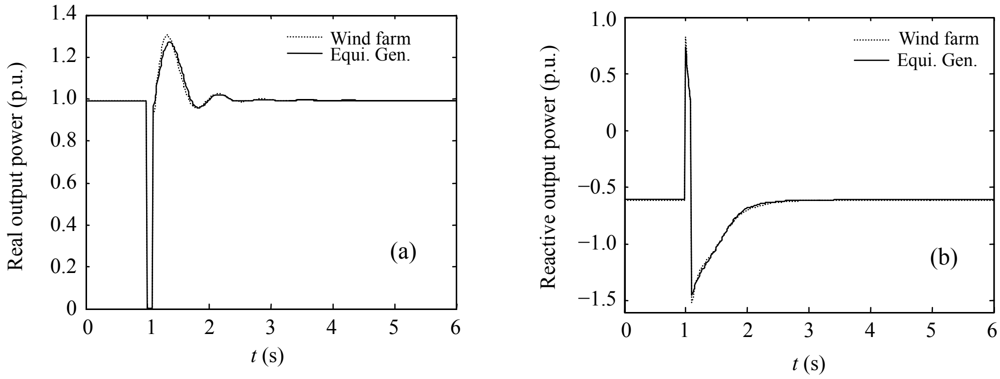

Fig.4 Real (a) and reactive (b) output powers of the wind farm and the equivalent generator in Fault 1

Table 2 Max- and average-errors with the equivalent generator (EG) and the average-error with the reference equivalent generator (REG)

Parameters	P_{out} (%)			Q_{out} (%)			U_{t} (%)		
	$e_{\max}$	e_{avgEG}	e_{avgREG}	$e_{\max}$	e_{avgEG}	e_{avgREG}	$e_{\max}$	e_{avgEG}	e_{avgREG}
Wind speed fluctuation	0.09	0.03	0.15	0.20	0.12	1.56	0.02	0.01	0.11
Fault 1	4.45	0.97	1.27	9.89	1.15	3.51	1.93	0.18	0.36
Fault 2	2.15	0.42	0.66	6.21	0.84	4.16	1.54	0.22	1.02
Small voltage disturbance	0.15	0.04	0.10	1.23	0.10	2.36	0.07	0.01	0.18

$e_{\max}$ and e_{avgEG} are the max- and average-errors of the equivalent generator, respectively. e_{avgREG} is the average-error of the reference equivalent generator. P_{out}, Q_{out} and U_{t} are the real output power, reactive output power and voltage of the wind farm, respectively the fault diversity in a power system and the uncertainty of the output power of a wind farm. For example, if the equivalent generator is based on the response to a bus short-circuit fault, even though the simulation is accurate in this case, it would not respond correctly to a line short-circuit fault. And if the equivalent generator is based on the assumption that the wind farm is at full output power, it would not respond correctly when the output of the wind farm falls off.

CONCLUSION

Dynamic equivalence is a fundamental problem in the simulation of a power system connected with wind farms. In this paper, a new Equivalence Method based on the Output Characteristics (EMOC) of the wind farm is developed for the wind farm composed of SCIGs.

EMOC is not only suitable for the equivalence of the practical wind farm, but also fit for the wind farm in programming. With this method, the model of equivalent generator can be selected flexible according to different research objectives.

EMOC is validated in the dynamic equivalence of an SCIG wind farm. The equivalent generator responds accurately to various dynamic processes, which proves the effectiveness of EMOC in the dynamic equivalence of a grid-connected wind farm.

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