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Dynamical output feedback stabilization for neutral systems with mixed delays*

Wei QIAN[†1,2], Guo-jiang SHEN[†‡1], You-xian SUN[1]
([1]State Key Lab of Industrial Control Technology, Zhejiang University, Hangzhou 310027, China)
([2]Department of Computer Science and Technology, Henan Polytechnic University, Jiaozuo 454000, China)
[†]E-mail: way.qian@yahoo.com.cn; gjshen@iipc.zju.edu.cn
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Abstract: This paper is concerned with the issue of stabilization for the linear neutral systems with mixed delays. The attention is focused on the design of output feedback controllers which guarantee the asymptotical stability of the closed-loop systems. Based on the model transformation of neutral type, the Lyapunov-Krasovskii functional method is employed to establish the delay-dependent stability criterion. Then, through the controller parameterization and some matrix transformation techniques, the desired parameters are determined under the delay-dependent design condition in terms of linear matrix inequalities (LMIs), and the desired controller is explicitly formulated. A numerical example is given to illustrate the effectiveness of the proposed method.

Key words: Neutral systems, Mixed delays, Output feedback stabilization, Linear matrix inequality (LMI)
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INTRODUCTION

Time delays are often encountered in various engineering systems, such as nuclear reactors, chemical engineering systems, biological systems, population dynamic models, and so on. In some practical systems, the model can be described by a functional differential equation of neutral type, which contains delays both in the state and in the state derivative. In many cases, time delays are the source of instability and degradation in control performance of many control systems (Niculescu, 2001). Therefore, the stability analysis and the synthesis of controllers for neutral systems are of theoretical and practical importance.

It is well known that the choice of an appropriate Lyapunov-Krasovskii functional is crucial for deriving stability condition and obtaining a solution to control problems. The general form of a functional leads to the complicated Riccati type differential equations or inequalities. Special forms of Lyapunov-Krasovskii functionals lead to simpler delay-independent and less conservative delay-dependent Riccati equations or linear matrix inequalities (LMIs), see e.g. (Choi and Chung, 1997; Jeung *et al*., 1998; Kharitonov, 1999; Xue and Qiu, 2000; Fridman and Shaked, 2001; Gu, 2001; He *et al*., 2004).

Recently, increasing attention has been paid to problems of observation, output feedback stabilization and design of observer-based controllers for systems with state delay. The output feedback stabilization problem for discrete-time systems was addressed in (Gao *et al*., 2004), and the robust H_∞ control problem via output feedback controllers was solved by (Xu and Chen, 2004). All the results mentioned above treated the case of retarded systems. In the more general case of neutral systems, many efforts mainly focused on the state feedback control problem for systems with discrete or distributed delays (Niculescu, 2000; Lien *et al*., 2000; Mahmoud, 2000),

‡ Corresponding author
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and the obtained results cannot be directly extended to the output feedback cases. To the best of our knowledge, few results have been achieved on the delay-dependent output feedback stabilization problem for neutral systems (Fridman and Shaked, 2002).

In this paper, we consider the output feedback control problem for neutral type systems with mixed delays. By using a "neutral type" model transformation and constructing an appropriate Lyapunov-Krasovskii functional, a delay-dependent stability criterion is given. Then by parameterization of the controller, a sufficient condition for the solvability of this problem is obtained in terms of LMIs, which can be efficiently handled by using standard algorithms. The desired output feedback controller can be constructed by solving the given LMI. A numerical example is also presented to illustrate the applicability of the developed results.

Notation: Throughout this paper, $\boldsymbol{X}^{\mathrm{T}}$ denotes the transpose of matrix $\boldsymbol{X}$; C_0 denotes the set of all continuous functions from $[-r, 0]$ to $\mathbb{R}^n$; $\boldsymbol{X}>\boldsymbol{0}$ denotes that $\boldsymbol{X}$ is a positive definite matrix; the symmetric terms in a symmetric matrix are denoted by *, e.g., $\begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y} \\ * & \boldsymbol{Z} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{Y} \\ \boldsymbol{Y}^{\mathrm{T}} & \boldsymbol{Z} \end{bmatrix}$.

PROBLEM FORMULATION

Consider the following neutral systems with mixed delays:

$$\begin{cases} \dot{\boldsymbol{x}}(t) - \boldsymbol{N}\dot{\boldsymbol{x}}(t-\tau) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{A}_1\boldsymbol{x}(t-h) \\ \qquad\qquad + \boldsymbol{A}_2\int_{t-d}^{t}\boldsymbol{x}(\sigma)\mathrm{d}\sigma + \boldsymbol{B}\boldsymbol{u}(t), \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t), \quad \boldsymbol{x}(t) = \boldsymbol{\varphi}(t), \quad t \in [-r, 0], \end{cases} \tag{1}$$

where $\boldsymbol{x}(t)\in\mathbb{R}^n$, $\boldsymbol{u}(t)\in\mathbb{R}^m$, $\boldsymbol{y}(t)\in\mathbb{R}^h$ are the state, control input and measured output, respectively; τ, h, $d>0$ are the constant time delays of the system, $r=\max\{\tau, h, d\}$; $\boldsymbol{\varphi}(\cdot)\in C_0$ is the initial condition function; $\boldsymbol{A}$, $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{N}$ are the known real constant matrices of appropriate dimensions.

For system Eq.(1), we consider the following full-order dynamical output feedback controller:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}_{\mathrm{c}}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_{\mathrm{c}}\boldsymbol{y}(t), \\ \boldsymbol{u}(t) = \boldsymbol{C}_{\mathrm{c}}\hat{\boldsymbol{x}}(t) + \boldsymbol{D}_{\mathrm{c}}\boldsymbol{y}(t), \end{cases} \tag{2}$$

where $t\geq 0$, $\hat{\boldsymbol{x}}(t)\in\mathbb{R}^n$ is the controller state, and $\boldsymbol{A}_{\mathrm{c}}$, $\boldsymbol{B}_{\mathrm{c}}$, $\boldsymbol{C}_{\mathrm{c}}$, $\boldsymbol{D}_{\mathrm{c}}$ are matrices to be determined. Applying this controller in Eq.(2) to system Eq.(1) results in the following closed-loop system:

$$\dot{\boldsymbol{\xi}}(t) - \bar{\boldsymbol{N}}\dot{\boldsymbol{\xi}}(t-\tau) = \bar{\boldsymbol{A}}\boldsymbol{\xi}(t) + \bar{\boldsymbol{A}}_1\boldsymbol{\xi}(t-h) + \bar{\boldsymbol{A}}_2\int_{t-d}^{t}\boldsymbol{\xi}(\sigma)\mathrm{d}\sigma, \tag{3}$$

where $t\geq 0$, and

$$\boldsymbol{\xi}(t) = \begin{bmatrix} \boldsymbol{x} \\ \hat{\boldsymbol{x}} \end{bmatrix}, \quad \bar{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}+\boldsymbol{B}\boldsymbol{D}_{\mathrm{c}}\boldsymbol{C} & \boldsymbol{B}\boldsymbol{C}_{\mathrm{c}} \\ \boldsymbol{B}_{\mathrm{c}}\boldsymbol{C} & \boldsymbol{A}_{\mathrm{c}} \end{bmatrix},$$

$$\bar{\boldsymbol{A}}_1 = \begin{bmatrix} \boldsymbol{I}_n \\ \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{0} \end{bmatrix}, \quad \bar{\boldsymbol{A}}_2 = \begin{bmatrix} \boldsymbol{I}_n \\ \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{A}_2 & \boldsymbol{0} \end{bmatrix},$$

$$\bar{\boldsymbol{N}} = \begin{bmatrix} \boldsymbol{I}_n \\ \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{N} & \boldsymbol{0} \end{bmatrix}.$$

The objective of this paper is to design a controller in the form of Eq.(2) such that the closed-loop system Eq.(3) is asymptotically stable.

Here, we define an operator $\mathsf{G}\boldsymbol{\xi}_t : C_0 \rightarrow \mathbb{R}^n$ as

$$\begin{aligned} \mathsf{G}\boldsymbol{\xi}_t &= \boldsymbol{\xi}(t) - \bar{\boldsymbol{N}}\boldsymbol{\xi}(t-\tau) + \bar{\boldsymbol{A}}_1\int_{t-h}^{t}\boldsymbol{x}(\sigma)\mathrm{d}\sigma \\ &\quad + \bar{\boldsymbol{A}}_2\int_{t-d}^{t}(\sigma - t + d)\boldsymbol{x}(\sigma)\mathrm{d}\sigma \end{aligned} \tag{4}$$

with the operator, and then the transformed closed-loop system is

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathsf{G}\boldsymbol{\xi}_t) = \hat{\boldsymbol{A}}\boldsymbol{\xi}(t), \quad t \geq 0, \tag{5}$$

where $\hat{\boldsymbol{A}} = \bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}_1 + d\bar{\boldsymbol{A}}_2$.

Before moving on, we introduce two lemmas, which are essential for deriving our main results.

Lemma 1 (Yue *et al.*, 2003) For given positive scalars α_1, α_2, α_3, where $\alpha_1+\alpha_2+\alpha_3<1$, if a symmetric positive-definite matrix $\boldsymbol{M}\in\mathbb{R}^{2n\times 2n}$ exists such that the following LMI Eq.(6) holds, then the operator $\mathsf{G}\boldsymbol{\xi}_t$ is stable (Hale and Verduyn, 1993).

$$\begin{bmatrix} \bar{N}^{\mathrm{T}} M \bar{N} - \alpha_1 M & h \bar{N}^{\mathrm{T}} M \bar{A}_1 & d \bar{N}^{\mathrm{T}} M \bar{A}_2 \\ h \bar{A}_1^{\mathrm{T}} M \bar{N} & h^2 \bar{A}_1^{\mathrm{T}} M \bar{A}_1 - \alpha_2 M & dh \bar{A}_1^{\mathrm{T}} M \bar{A}_2 \\ d \bar{A}_2^{\mathrm{T}} M \bar{N} & dh \bar{A}_2^{\mathrm{T}} M \bar{A}_1 & \mathcal{X} \end{bmatrix} < \mathbf{0}, \quad (6)$$

where $\mathcal{X} = d^2 \bar{A}_2^{\mathrm{T}} M \bar{A}_2 - 3\alpha_3 M / d^2$.

Lemma 2 (Gu, 2000) For any scalars α, β with $\alpha>\beta$, a symmetric positive-definite constant matrix M, and a vector-valued function w: $[\beta, \alpha]\rightarrow\mathbb{R}^n$ such that the integration concerned is well-defined, then

$$\left(\int_\beta^\alpha w(\sigma)\mathrm{d}\sigma\right)^{\mathrm{T}} M \left(\int_\beta^\alpha w(\sigma)\mathrm{d}\sigma\right) \le (\alpha-\beta)\int_\beta^\alpha w^{\mathrm{T}}(\sigma) M w(\sigma)\mathrm{d}\sigma.$$

Remark 1 Clearly, during the transformation of the original system and the construction of the output-feedback control, this LMI condition is not affected by the control. Hence it is a necessary criterion for the existence of the concerned controller.

MAIN RESULTS

In this section, an LMI-based approach is developed to solve the dynamical output feedback stabilization problem formulated in the previous section.

Theorem 1 For given positive scalars h, d, α_1, α_2, α_3, where $\alpha_1+\alpha_2+\alpha_3<1$, the closed-loop system Eq.(3) is asymptotically stable, if there exist symmetric positive-definite matrices $P = \begin{bmatrix} P_1 & P_3 \\ P_3^{\mathrm{T}} & P_2 \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$, $S_i \in \mathbb{R}^{n\times n}$ (i=1, 2, 3) satisfying inequalities Eqs.(6) and (7).

$$\begin{bmatrix} \mathcal{A} & \mathcal{Y} & h\mathcal{Y} & \mathcal{Y} & \begin{bmatrix} N^{\mathrm{T}} \\ 0 \end{bmatrix} & h\begin{bmatrix} A_1^{\mathrm{T}} \\ 0 \end{bmatrix} & d^2\begin{bmatrix} A_2^{\mathrm{T}} \\ 0 \end{bmatrix} \\ * & -S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -hS_2 & 0 & 0 & 0 & 0 \\ * & * & * & -S_3 & 0 & 0 & 0 \\ * & * & * & * & -S_1^{-1} & 0 & 0 \\ * & * & * & * & * & -hS_2^{-1} & 0 \\ * & * & * & * & * & * & -2S_3^{-1} \end{bmatrix} < \mathbf{0}. \quad (7)$$

where $\mathcal{A} = \hat{A}^{\mathrm{T}} P + P\hat{A}$, $\mathcal{Y} = \hat{A}^{\mathrm{T}} \begin{bmatrix} P_1^{\mathrm{T}} & P_3 \end{bmatrix}^{\mathrm{T}}$.

Proof Choose the following Lyapunov-Krasovskii functional candidate

$$V(\xi_t) = V_1(\xi_t) + V_2(\xi_t) + V_3(\xi_t) + V_4(\xi_t), \quad (8)$$

where

$$V_1(\xi_t) = (\mathrm{G}\xi_t)^{\mathrm{T}} P(\mathrm{G}\xi_t),$$

$$V_2(\xi_t) = \int_{t-\tau}^{t} \xi^{\mathrm{T}}(\theta)\begin{bmatrix} N & \mathbf{0} \end{bmatrix}^{\mathrm{T}} S_1 \begin{bmatrix} N & \mathbf{0} \end{bmatrix}\xi(\theta)\mathrm{d}\theta,$$

$$V_3(\xi_t) = \int_{-h}^{0}\int_{t+\theta}^{t} \xi^{\mathrm{T}}(\sigma)\begin{bmatrix} A_1 & \mathbf{0} \end{bmatrix}^{\mathrm{T}} S_2 \begin{bmatrix} A_1 & \mathbf{0} \end{bmatrix}\xi(\sigma)\mathrm{d}\sigma\mathrm{d}\theta,$$

$$V_4(\xi_t) = d^2\int_{0}^{d}\int_{t-\theta}^{t} \left\{(\sigma - t + \theta)\xi^{\mathrm{T}}(\sigma)\begin{bmatrix} A_2 & \mathbf{0} \end{bmatrix}^{\mathrm{T}} \cdot S_3 \begin{bmatrix} A_2 & \mathbf{0} \end{bmatrix}\xi(\sigma)\right\}\mathrm{d}\sigma\mathrm{d}\theta.$$

Calculating the derivative of $V_1(\xi_t)$ along the trajectory of the closed-loop system yields

$$\dot{V}_1(\xi_t) = 2\left(\frac{\mathrm{d}(\mathrm{G}\xi_t)}{\mathrm{d}t}\right)^{\mathrm{T}} P(\mathrm{G}\xi_t) = \xi^{\mathrm{T}}(t)(\hat{A}^{\mathrm{T}} P + P\hat{A})\xi(t) + \eta_1 + \eta_2 + \eta_3.$$

Recalling that for any vectors a, b and any matrix $S>0$ of appropriate dimensions,

$$2a^{\mathrm{T}}b \le a^{\mathrm{T}}Sa + b^{\mathrm{T}}S^{-1}b.$$

Then, for any matrices $S_i>0$ (i=1, 2, 3), we have

$$\begin{aligned} \eta_1 &= -2\xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}} P\bar{N}\xi(t-\tau) \\ &\le \xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}}\begin{bmatrix} P_1^{\mathrm{T}} & P_3 \end{bmatrix}^{\mathrm{T}} S_1^{-1}\begin{bmatrix} P_1 & P_3 \end{bmatrix}\hat{A}\xi(t) \\ &\quad + \xi^{\mathrm{T}}(t-\tau)\begin{bmatrix} N & \mathbf{0} \end{bmatrix}^{\mathrm{T}} S_1\begin{bmatrix} N & \mathbf{0} \end{bmatrix}\xi(t-\tau), \\ \eta_2 &= 2\xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}} P\bar{A}_1\int_{t-h}^{t}\xi(\sigma)\mathrm{d}\sigma \\ &\le h\xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}}\begin{bmatrix} P_1^{\mathrm{T}} & P_3 \end{bmatrix}^{\mathrm{T}} S_2^{-1}\begin{bmatrix} P_1 & P_3 \end{bmatrix}\hat{A}\xi(t) \\ &\quad + \frac{1}{h}\int_{t-h}^{t}\xi^{\mathrm{T}}(\sigma)\begin{bmatrix} A_1 & \mathbf{0} \end{bmatrix}^{\mathrm{T}}\mathrm{d}\sigma S_2\int_{t-h}^{t}\begin{bmatrix} A_1 & \mathbf{0} \end{bmatrix}\xi(\sigma)\mathrm{d}\sigma, \\ \eta_3 &= 2\xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}} P\int_{t-d}^{t}(\sigma - t + d)\bar{A}_2\xi(\sigma)\mathrm{d}\sigma \\ &\le \xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}}\begin{bmatrix} P_1^{\mathrm{T}} & P_3 \end{bmatrix}^{\mathrm{T}} S_3^{-1}\begin{bmatrix} P_1 & P_3 \end{bmatrix}\hat{A}\xi(t) \\ &\quad + \int_{t-d}^{t}(\sigma - t + d)\xi^{\mathrm{T}}(\sigma)\begin{bmatrix} A_2 & \mathbf{0} \end{bmatrix}^{\mathrm{T}}\mathrm{d}\sigma \\ &\quad \cdot S_3\int_{t-d}^{t}(\sigma - t + d)\begin{bmatrix} A_2 & \mathbf{0} \end{bmatrix}\xi(\sigma)\mathrm{d}\sigma \\ &\le \xi^{\mathrm{T}}(t)\hat{A}^{\mathrm{T}}\begin{bmatrix} P_1^{\mathrm{T}} & P_3 \end{bmatrix}^{\mathrm{T}} S_3^{-1}\begin{bmatrix} P_1 & P_3 \end{bmatrix}\hat{A}\xi(t) \end{aligned}$$

$$+d\int_{t-d}^{t}(\sigma-t+d)^2\boldsymbol{\xi}^{\mathrm{T}}(\sigma)\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}\xi(\sigma)\mathrm{d}\sigma$$

$$\le\boldsymbol{\xi}^{\mathrm{T}}(t)\hat{\boldsymbol{A}}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{P}_3\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3^{-1}\begin{bmatrix}\boldsymbol{P}_1 & \boldsymbol{P}_3\end{bmatrix}\hat{\boldsymbol{A}}\boldsymbol{\xi}(t)$$

$$+d^2\int_{t-d}^{t}(\sigma-t+d)\boldsymbol{\xi}^{\mathrm{T}}(\sigma)\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}\xi(\sigma)\mathrm{d}\sigma.$$

Applying Lemma 2, we have

$$\dot{V}_3(\xi_t)=h\boldsymbol{\xi}^{\mathrm{T}}(t)\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_2\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}\xi(t)$$

$$-\int_{t-h}^{t}\xi^{\mathrm{T}}(\theta)\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_2\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}\xi(\theta)\mathrm{d}\theta$$

$$\le h\boldsymbol{\xi}^{\mathrm{T}}(t)\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_2\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}\xi(t)$$

$$-\frac{1}{h}\int_{t-h}^{t}\xi^{\mathrm{T}}(\theta)\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\mathrm{d}\theta\boldsymbol{S}_2\int_{t-h}^{t}\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}\xi(\theta)\mathrm{d}\theta.$$

Calculating the derivative of $V_2(\boldsymbol{\xi}_t)$ and $V_4(\boldsymbol{\xi}_t)$ leads to

$$\dot{V}_2(\xi_t)=\boldsymbol{\xi}^{\mathrm{T}}(t)\begin{bmatrix}\boldsymbol{N} & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_1\begin{bmatrix}\boldsymbol{N} & \boldsymbol{0}\end{bmatrix}\xi(t)-\boldsymbol{\xi}^{\mathrm{T}}(t-\tau)$$

$$\cdot\begin{bmatrix}\boldsymbol{N} & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_1\begin{bmatrix}\boldsymbol{N} & \boldsymbol{0}\end{bmatrix}\xi(t-\tau),$$

$$\dot{V}_4(\xi_t)=\frac{d^4}{2}\xi^{\mathrm{T}}(t)\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}\xi(t)$$

$$-d^2\int_0^d\int_{t-\theta}^{t}\xi^{\mathrm{T}}(\sigma)\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}\xi(\sigma)\mathrm{d}\sigma\mathrm{d}\theta$$

$$=\frac{d^4}{2}\xi^{\mathrm{T}}(t)\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}\xi(t)$$

$$-d^2\int_{t-d}^{t}(\sigma-t+d)\boldsymbol{\xi}^{\mathrm{T}}(\sigma)\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}\xi(\sigma)\mathrm{d}\sigma.$$

It is easy to obtain

$$\dot{V}(\xi_t)\le\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\mathcal{Z}}\xi(t), \tag{9}$$

where

$$\boldsymbol{\mathcal{Z}}=\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\hat{\boldsymbol{A}}+\hat{\boldsymbol{A}}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{P}_3\end{bmatrix}^{\mathrm{T}}\left(\boldsymbol{S}_1^{-1}+h\boldsymbol{S}_2^{-1}+\boldsymbol{S}_3^{-1}\right)$$

$$\cdot\begin{bmatrix}\boldsymbol{P}_1 & \boldsymbol{P}_3\end{bmatrix}\hat{\boldsymbol{A}}+\begin{bmatrix}\boldsymbol{N} & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_1\begin{bmatrix}\boldsymbol{N} & \boldsymbol{0}\end{bmatrix}$$

$$+h\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_2\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{0}\end{bmatrix}+\frac{d^4}{2}\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_3\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{0}\end{bmatrix}.$$

Thus, from the Schur complement (Gu *et al.*, 2003), if the matrix inequality Eq.(7) holds, then $\dot{V}(\xi_t)<0$, which means the closed-loop system Eq.(3) is asymptotically stable. This completes the proof.

Remark 2 To solve the matrix inequality Eq.(7), the cone complementary linearization iterative algorithm proposed in (El Ghaoui *et al.*, 1997; Gao and Wang, 2003; Palhares *et al.*, 2005) can be used. We can introduce three new variables $\boldsymbol{W}_1$, $\boldsymbol{W}_2$, $\boldsymbol{W}_3$ and propose the following nonlinear minimization problem involving LMI conditions.

$$\min(\mathrm{tr}(\boldsymbol{S}_1\boldsymbol{W}_1+\boldsymbol{S}_2\boldsymbol{W}_2+\boldsymbol{S}_3\boldsymbol{W}_3))$$

$$\text{s.t.}\quad \boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2,\boldsymbol{S}_3>\boldsymbol{0},\ \begin{bmatrix}\boldsymbol{W}_1 & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{S}_1\end{bmatrix}\ge\boldsymbol{0},$$

$$\begin{bmatrix}\boldsymbol{W}_2 & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{S}_2\end{bmatrix}\ge\boldsymbol{0},\ \begin{bmatrix}\boldsymbol{W}_3 & \boldsymbol{I}\\ \boldsymbol{I} & \boldsymbol{S}_3\end{bmatrix}\ge\boldsymbol{0}.$$

$$\begin{bmatrix}\boldsymbol{\mathcal{A}} & \boldsymbol{\mathcal{Y}} & h\boldsymbol{\mathcal{Y}} & \boldsymbol{\mathcal{Y}} & \begin{bmatrix}\boldsymbol{N}^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix} & h\begin{bmatrix}\boldsymbol{A}_1^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix} & d^2\begin{bmatrix}\boldsymbol{A}_2^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix}\\ * & -\boldsymbol{S}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -h\boldsymbol{S}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & -\boldsymbol{S}_3 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & -\boldsymbol{W}_1 & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & * & -h\boldsymbol{W}_2 & \boldsymbol{0}\\ * & * & * & * & * & * & -2\boldsymbol{W}_3\end{bmatrix}<\boldsymbol{0}.$$

where $\boldsymbol{\mathcal{A}}=\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\hat{\boldsymbol{A}}$, $\boldsymbol{\mathcal{Y}}=\hat{\boldsymbol{A}}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{P}_3\end{bmatrix}^{\mathrm{T}}$.

If the solution of the above minimization problem is $3n$, it follows from Theorem 1 that the closed-loop system Eq.(3) is asymptotically stable. A similar iterative algorithm to solve this nonlinear convex optimization problem can be obtained by adopting the algorithms in (Gao and Wang, 2003; Palhares *et al.*, 2005) with some modifications.

In this paper, we use another method. Given positive scalars $\lambda_i>0$ (i=1, 2, 3) and the additional linear restrictions $\boldsymbol{S}_i<\lambda_i\boldsymbol{I}_n$ (i=1, 2, 3), the matrix inequality Eq.(6) can be converted to the following LMI:

$$\begin{bmatrix}\boldsymbol{\mathcal{A}} & \boldsymbol{\mathcal{Y}} & h\boldsymbol{\mathcal{Y}} & \boldsymbol{\mathcal{Y}} & \lambda_1\begin{bmatrix}\boldsymbol{N}^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix} & h\lambda_2\begin{bmatrix}\boldsymbol{A}_1^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix} & d^2\lambda_3\begin{bmatrix}\boldsymbol{A}_2^{\mathrm{T}}\\ \boldsymbol{0}\end{bmatrix}\\ * & -\boldsymbol{S}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & -h\boldsymbol{S}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & -\boldsymbol{S}_3 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & -\boldsymbol{S}_1 & \boldsymbol{0} & \boldsymbol{0}\\ * & * & * & * & * & -h\boldsymbol{S}_2 & \boldsymbol{0}\\ * & * & * & * & * & * & -2\boldsymbol{S}_3\end{bmatrix}<\boldsymbol{0}, \tag{10}$$

where $\boldsymbol{\mathcal{A}}=\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\hat{\boldsymbol{A}}$, $\boldsymbol{\mathcal{Y}}=\hat{\boldsymbol{A}}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{P}_1^{\mathrm{T}} & \boldsymbol{P}_3\end{bmatrix}^{\mathrm{T}}$.

For systems without delays, there are mainly two approaches used for determining the parameters of a full-order compensator. One is the elimination of parameters (Iwasaki and Skelton, 1994), the other is the parameterization method (Gahinet and Apkarian, 1994). Both of them depend on matrix operation and transformation. In this paper, parameterization method is exploited with introducing a set of parameters as follows:

$$\boldsymbol{\Phi} := \{\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{R} \in \mathbb{R}^{n\times n}, \boldsymbol{U} \in \mathbb{R}^{m\times n}, \boldsymbol{V} \in \mathbb{R}^{n\times h}, \boldsymbol{W} \in \mathbb{R}^{m\times h}\}, \tag{11}$$

where $\boldsymbol{X}>\boldsymbol{0}$, $\boldsymbol{Y}>\boldsymbol{0}$.

Denoting $\tilde{\boldsymbol{A}} := \boldsymbol{A} + \boldsymbol{A}_1 + d\boldsymbol{A}_2$ and $\boldsymbol{Z}=\boldsymbol{X}-\boldsymbol{Y}^{-1}$, the parameterized form of the compensator is given as

$$\begin{bmatrix} \boldsymbol{D}_{\mathrm{c}} & \boldsymbol{C}_{\mathrm{c}} \\ \boldsymbol{B}_{\mathrm{c}} & \boldsymbol{A}_{\mathrm{c}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{B} & -\boldsymbol{Y}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{W} & \boldsymbol{U} \\ \boldsymbol{V} & \boldsymbol{R}-\boldsymbol{Y}\tilde{\boldsymbol{A}}\boldsymbol{X} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_h & -\boldsymbol{C}\boldsymbol{X}\boldsymbol{Z}^{-1} \\ \boldsymbol{0} & \boldsymbol{Z}^{-1} \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{W} & (-\boldsymbol{W}\boldsymbol{C}\boldsymbol{X}+\boldsymbol{U})\boldsymbol{Z}^{-1} \\ \boldsymbol{B}\boldsymbol{W}-\boldsymbol{Y}^{-1}\boldsymbol{V} & \begin{matrix}(-\boldsymbol{B}\boldsymbol{W}\boldsymbol{C}\boldsymbol{X}+\boldsymbol{B}\boldsymbol{U}+\boldsymbol{Y}^{-1}\boldsymbol{V}\boldsymbol{C}\boldsymbol{X} \\ -\boldsymbol{Y}^{-1}\boldsymbol{R}+\tilde{\boldsymbol{A}}\boldsymbol{X})\boldsymbol{Z}^{-1}\end{matrix} \end{bmatrix}, \tag{12}$$

$$\boldsymbol{P}^{-1}(\boldsymbol{\Phi}) = \boldsymbol{Q}(\boldsymbol{\Phi}) = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{Z} \\ \boldsymbol{Z} & \boldsymbol{Z} \end{bmatrix}. \tag{13}$$

Therefore, we have $\boldsymbol{P}(\boldsymbol{\Phi}) = \begin{bmatrix} \boldsymbol{Y} & -\boldsymbol{Y} \\ -\boldsymbol{Y} & \boldsymbol{Z}^{-1}\boldsymbol{X}\boldsymbol{Y} \end{bmatrix}$.

Substituting the parameterized compensator Eq.(11) to the closed-loop system Eq.(4), the parameterized form of the closed-loop system Eq.(3) is obtained with the coefficient matrices as

$$\hat{\boldsymbol{A}}(\boldsymbol{\Phi}) = \begin{bmatrix} \tilde{\boldsymbol{A}}+\boldsymbol{B}\boldsymbol{W}\boldsymbol{C} & (-\boldsymbol{B}\boldsymbol{W}\boldsymbol{C}\boldsymbol{X}+\boldsymbol{B}\boldsymbol{U})\boldsymbol{Z}^{-1} \\ \boldsymbol{B}\boldsymbol{W}\boldsymbol{C}-\boldsymbol{Y}^{-1}\boldsymbol{V}\boldsymbol{C} & \begin{matrix}(-\boldsymbol{B}\boldsymbol{W}\boldsymbol{C}\boldsymbol{X}+\boldsymbol{B}\boldsymbol{U}+\boldsymbol{Y}^{-1}\boldsymbol{V}\boldsymbol{C}\boldsymbol{X} \\ -\boldsymbol{Y}^{-1}\boldsymbol{R}+\tilde{\boldsymbol{A}}\boldsymbol{X})\boldsymbol{Z}^{-1}\end{matrix} \end{bmatrix}. \tag{14}$$

Theorem 2 For given positive scalars h, d, α_1, α_2, α_3, where $\alpha_1+\alpha_2+\alpha_3<1$, if there exist scalars $\lambda_i>0$ (i=1, 2, 3), matrices $\boldsymbol{S}_i>\boldsymbol{0}$ (i=1, 2, 3) and a parameter set Eq.(11) satisfying the matrix inequality Eq.(6) and the following LMIs:

$$\begin{bmatrix} \boldsymbol{\Gamma}_1+\boldsymbol{\Gamma}_1^{\mathrm{T}} & \boldsymbol{\Gamma}_2^{\mathrm{T}} & h\boldsymbol{\Gamma}_2^{\mathrm{T}} & \boldsymbol{\Gamma}_2^{\mathrm{T}} & \boldsymbol{\Gamma}_3^{\mathrm{T}} & h\boldsymbol{\Gamma}_4^{\mathrm{T}} & d^2\boldsymbol{\Gamma}_5^{\mathrm{T}} \\ * & -\boldsymbol{S}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -h\boldsymbol{S}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & -\boldsymbol{S}_3 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{S}_1 & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & -h\boldsymbol{S}_2 & \boldsymbol{0} \\ * & * & * & * & * & * & -2\boldsymbol{S}_3 \end{bmatrix} < \boldsymbol{0}, \tag{15a}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{I}_n \\ \boldsymbol{I}_n & \boldsymbol{Y} \end{bmatrix} > \boldsymbol{0}, \tag{15b}$$

$$\boldsymbol{S}_i \le \lambda_i \boldsymbol{I}_n,\ i=1, 2, 3, \tag{15c}$$

In LMI Eq.(15a), $\boldsymbol{\Gamma}_i$ (i=1, 2, ..., 5) are all functions of $\boldsymbol{\Phi}$, i.e., $\boldsymbol{\Gamma}_i=\boldsymbol{\Gamma}_i(\boldsymbol{\Phi})$. And

$$\boldsymbol{\Gamma}_1(\boldsymbol{\Phi}) = \begin{bmatrix} \tilde{\boldsymbol{A}}\boldsymbol{X}+\boldsymbol{B}\boldsymbol{U} & \tilde{\boldsymbol{A}}+\boldsymbol{B}\boldsymbol{W}\boldsymbol{C} \\ \boldsymbol{R} & \boldsymbol{Y}\tilde{\boldsymbol{A}}+\boldsymbol{V}\boldsymbol{C} \end{bmatrix},$$
$$\boldsymbol{\Gamma}_2(\boldsymbol{\Phi}) = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{Y}\tilde{\boldsymbol{A}}+\boldsymbol{V}\boldsymbol{C} \end{bmatrix},$$
$$\boldsymbol{\Gamma}_3(\boldsymbol{\Phi}) = \lambda_1\begin{bmatrix} \boldsymbol{N}\boldsymbol{X} & \boldsymbol{N} \end{bmatrix},\ \boldsymbol{\Gamma}_4(\boldsymbol{\Phi}) = \lambda_2\begin{bmatrix} \boldsymbol{A}_1\boldsymbol{X} & \boldsymbol{A}_1 \end{bmatrix},$$
$$\boldsymbol{\Gamma}_5(\boldsymbol{\Phi}) = \lambda_3\begin{bmatrix} \boldsymbol{A}_2\boldsymbol{X} & \boldsymbol{A}_2 \end{bmatrix}.$$

Then there exists a full-order dynamical output feedback controller in the form of Eq.(2) such that the closed-loop system Eq.(3) is asymptotically stable. In this case, the parameters of the desired controller are given in Eq.(12).

Proof Denoting $\boldsymbol{T}(\boldsymbol{\Phi}) := \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{Y} \\ \boldsymbol{0} & -\boldsymbol{Y} \end{bmatrix}$, then it follows

$$\boldsymbol{T}^{\mathrm{T}}(\boldsymbol{\Phi})\boldsymbol{Q}(\boldsymbol{\Phi})\boldsymbol{T}(\boldsymbol{\Phi}) = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{Y} & -\boldsymbol{Y} \end{bmatrix} \begin{bmatrix} \boldsymbol{X} & \boldsymbol{Z} \\ \boldsymbol{Z} & \boldsymbol{Z} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{Y} \\ \boldsymbol{0} & -\boldsymbol{Y} \end{bmatrix}$$
$$= \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \boldsymbol{Y} & -\boldsymbol{Y} \end{bmatrix} \begin{bmatrix} \boldsymbol{X} & \boldsymbol{I}_n \\ \boldsymbol{Z} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{I}_n \\ \boldsymbol{I}_n & \boldsymbol{Y} \end{bmatrix}. \tag{16}$$

So the LMI Eq.(15b) is equivalent to $\boldsymbol{P}(\boldsymbol{\Phi})>\boldsymbol{0}$.

Then substituting the parameterized functional matrix Eq.(13) and the parameterized closed-loop system matrix Eq.(14) into the matrix inequality Eq.(10), we obtain inequality Eq.(17):

$$\begin{bmatrix} \hat{A}^{\mathrm{T}}(\Phi)P + P\hat{A}(\Phi) & \hat{A}^{\mathrm{T}}(\Phi)\begin{bmatrix} Y \\ -Y \end{bmatrix} & h\hat{A}^{\mathrm{T}}(\Phi)\begin{bmatrix} Y \\ -Y \end{bmatrix} & \hat{A}^{\mathrm{T}}(\Phi)\begin{bmatrix} Y \\ -Y \end{bmatrix} & \lambda_1\begin{bmatrix} N^{\mathrm{T}} \\ 0 \end{bmatrix} & h\lambda_2\begin{bmatrix} A_1^{\mathrm{T}} \\ 0 \end{bmatrix} & d^2\lambda_3\begin{bmatrix} A_2^{\mathrm{T}} \\ 0 \end{bmatrix} \\ * & -S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -hS_2 & 0 & 0 & 0 & 0 \\ * & * & * & -S_3 & 0 & 0 & 0 \\ * & * & * & * & -S_1 & 0 & 0 \\ * & * & * & * & * & -hS_2 & 0 \\ * & * & * & * & * & * & -2S_3 \end{bmatrix} < 0. \quad (17)$$

The similarity transformation matrix is constructed as

$$L = \mathrm{diag}\{T^{\mathrm{T}}(\Phi)Q(\Phi), I_{2n}, I_{2n}, I_{2n}, I_{2n}, I_{2n}, I_{2n}\}. \quad (18)$$

Pre- and post-multiplying both sides of inequality Eq.(17) by L and L^{T} respectively, and noticing

$$T^{\mathrm{T}}(\Phi)\hat{A}(\Phi)Q(\Phi)T(\Phi)$$
$$= \begin{bmatrix} I_n & 0 \\ Y & -Y \end{bmatrix}\begin{bmatrix} \tilde{A}X + BU & \tilde{A} + BWC \\ \tilde{A}X + BU - Y^{-1}R & BWC - Y^{-1}VC \end{bmatrix}$$
$$= \Gamma_1(\Phi),$$
$$\begin{bmatrix} P_1(\Phi) & P_3(\Phi) \end{bmatrix}\hat{A}(\Phi)Q(\Phi)T(\Phi)$$
$$= \begin{bmatrix} Y & -Y \end{bmatrix}\begin{bmatrix} \tilde{A}X + BU & \tilde{A} + BWC \\ \tilde{A}X + BU - Y^{-1}R & BWC - Y^{-1}VC \end{bmatrix}$$
$$= \Gamma_2(\Phi),$$
$$\lambda_1\begin{bmatrix} N & 0 \end{bmatrix}Q(\Phi)T(\Phi) = \lambda_1\begin{bmatrix} N & 0 \end{bmatrix}\begin{bmatrix} X & I_n \\ Z & 0 \end{bmatrix} = \Gamma_3(\Phi),$$
$$\lambda_2\begin{bmatrix} A_1 & 0 \end{bmatrix}Q(\Phi)T(\Phi) = \Gamma_4(\Phi),$$
$$\lambda_3\begin{bmatrix} A_2 & 0 \end{bmatrix}Q(\Phi)T(\Phi) = \Gamma_5(\Phi),$$

then we can obtain LMI Eq.(15a). This completes the proof.

Remark 3 Strictly speaking, the synthesis condition is not a convex restriction for all parameters; there are some parameters of the functional left to be specified. This makes the concerned problem more conservative, though the parameters in the controller formulation are retained in the convex form.

Remark 4 In this paper, the considered system is a more general case. The design of controllers for simple time-delay systems, such as systems without distributed delay, can be seen as a special case.

AN ILLUSTRATIVE EXAMPLE

In this section, we provide a numerical example to illustrate the effectiveness of the proposed method in the previous section.

Consider the neutral system Eq.(1) with parameters as follows:

$$N = \begin{bmatrix} 0.1 & 0 \\ -0.1 & 0 \end{bmatrix},\ A = \begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, C = \begin{bmatrix} -1 \\ 1 \end{bmatrix}^{\mathrm{T}},\ h=0.6,\ d=0.2,\ \tau=1.$$

Choosing the auxiliary variables $\lambda_1=10$, $\lambda_2=5$, $\lambda_3=10$, $\alpha_1=0.2$, $\alpha_2=0.4$, $\alpha_3=0.4$, by the Matlab LMI Control Toolbox, a solution can be obtained as

$$X = \begin{bmatrix} 7.7123 & -0.893 \\ -0.893 & 0.8980 \end{bmatrix},\ Y = \begin{bmatrix} 3.9745 & 4.9239 \\ 4.9239 & 7.7637 \end{bmatrix},$$
$$R = \begin{bmatrix} -1.8225 & 1.9529 \\ 1.6501 & -1.7837 \end{bmatrix},\ U = \begin{bmatrix} -9.2645 & 11.6651 \end{bmatrix},$$
$$V = \begin{bmatrix} -1.8970 \\ -8.7352 \end{bmatrix},\ W = 4.7129.$$

Therefore, by Theorem 2, a desired output feedback control law can be constructed as

$$A_{\mathrm{c}} = \begin{bmatrix} -0.9831 & 21.5424 \\ -1.0419 & -19.3784 \end{bmatrix},\ B_{\mathrm{c}} = \begin{bmatrix} 0.4351 \\ -0.8746 \end{bmatrix},$$
$$C_{\mathrm{c}} = \begin{bmatrix} 5.0899 & 13.4059 \end{bmatrix},\ D_{\mathrm{c}} = 4.7129.$$

Fig.1 presents the state response of the closed-loop system Eq.(3) with the initial condition $x^{\mathrm{T}}(t)=[1\ -2\ -1\ 2]$. It can be seen from this figure that the closed-loop system is asymptotically stable under the dynamical output feedback controller.

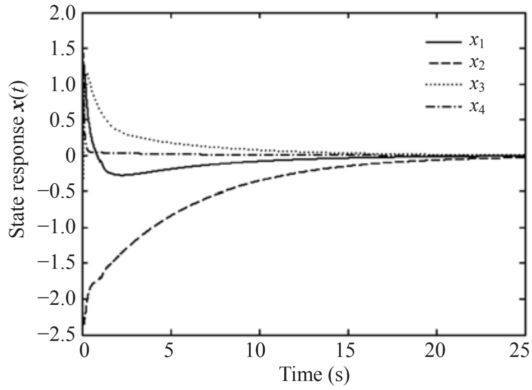

Fig.1 State trajectory of the closed-loop system

CONCLUSION

In this paper, the stabilization problem for neutral systems with discrete and distributed delays is studied. A novel LMI-based approach to designing a dynamical output feedback controller, which ensures the asymptotical stability of the resulting closed-loop system, is proposed. Through the model transformation of neutral type and construction of a proper Lyapunov-Krasovskii functional, a delay-dependent stability criterion is established in terms of LMIs. Then the controller parameterization and matrix transformation are employed to solve the nonconvex feasibility problem, and a sufficient condition for the solvability of the desired controller is obtained in terms of LMIs. The numerical example shows the effectiveness and applicability of the proposed approach.

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