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Performance of dynamically loaded journal bearings lubricated with couple stress fluids considering the elasticity of the liner*

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Abstract: To take into account the couple stress effects, a modified Reynolds equation is derived for dynamically loaded journal bearings with the consideration of the elasticity of the liner. The numerical results show that the influence of couple stresses on the bearing characteristics is significant. Compared with Newtonian lubricants, lubricants with couple stresses increase the fluid film pressure, as a result enhance the load-carrying capacity and reduce the friction coefficient. However, since the elasticity of the liner weakens the couple stress effect, elastic liners yield a reduction in the load-carrying capacity and an increase in the friction coefficient. The elastic deformation of the bearing liner should be considered in an accurate performance evaluation of the journal bearing.

Key words: Journal bearings, Couple stress, Elasticity, Lubrication
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INTRODUCTION

In recent years, experiments have shown clear evidence that the small amounts of long-chained additives in a Newtonian fluid can improve lubrication performance. The behavior of additive added oils is no longer Newtonian but non-Newtonian (Williamson *et al.*, 1997; Sharma *et al.*, 2000). Many micro-continuum theories have been developed to explain the peculiar behavior of these kinds of non-Newtonian fluids. Among them, the Stokes theory is considered as the simplest generalization of the classical theory of fluids, which allows for the polar effects such as the presence of couple stresses and body couples. Because of its relative mathematical simplicity, the couple stress model has been applied to the theoretical study of lubrication problems. There are many applications of the couple stress model on static characteristics or pure squeeze film behavior of journal bearings. Mokhiamer *et al.*(1999) and Lin (1998) made studies on the performance of finite journal bearings lubricated with a couple stress fluid. The results showed that the larger the chain length of the additive molecule, the greater was the effects of the couple stresses. Lin *et al.*(2001) theoretically studied the pure squeeze film behavior of a finite partial journal bearing with couple stress lubricants operating under a time-dependent cyclic load. It was concluded that the effects of couple stresses resulted in a decrease in the value of eccentricity of the journal center and the finite partial bearing with a couple stress fluid as the lubricant yielded an increase in the minimum permissible clearance and provided a longer time to prevent the journal-bearing contact. Naduvinamani *et al.*(2002) made a study of static rotor bearing systems lubricated with couple stress fluids to analyze the effect of surface roughness using stochastic approach. It was observed that the surface roughness considerably influenced the static characteristics of the rotor bearing system and that the couple stresses increased the load carrying capacity and decreased the coefficient of friction for both types of roughness patterns. There have been some other

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studies of couple stress effects, such as on short porous journal bearings by Naduvinamani *et al.*(2001), on externally pressurized circular step thrust bearings by Lin (1999), and on dynamically loaded journal bearings by Wang *et al.*(2002). There are many researches of couple stresses effects on characteristics of journal bearings, but the effects of couple stresses on the lubrication performance of dynamically loaded journal bearings considering the elasticity of the liner is deficiency in other research.

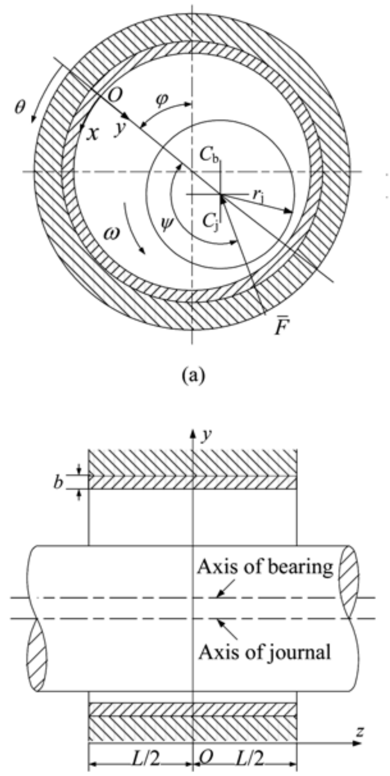
In this paper, a modified Reynolds equation for dynamic loads governing the film pressure is derived considering the elasticity of the liner. The bearing characteristics are presented for different values of the elastic coefficients and couple stress parameters.

MATHEMATICAL MODEL

The physical configuration of the system is shown schematically in Fig.1, where r_j is the radius of the journal, r_b is the radius of the bearing. And $\varepsilon=e/c$, $c=r_b-r_j$, where e is eccentricity of the journal center, ε is eccentricity ratio. Rectangular coordinates (x, y, z) and column coordinates (r, θ, z) are fixed on the bearing. The coordinates center lies on the mid-width of the bearing and is in accord with the bearing center. φ is attitude angle of the line C_jC_b. In dynamic conditions, the magnitude and direction of the load are variable, therefore the angular velocities of the journal and the bearing ω_j and ω_b are variable, too. Let $\omega=\omega_j+\omega_b$. $\bar{F}$ is the dimensionless load-carrying capacity, ψ is the attitude angle of $\bar{F}$.

It is assumed that the body forces and body moments are neglected, and that the fluid film is thin compared with the journal radius. According to the momentum equations and the continuity equation of an incompressible fluid with couple stresses, the velocity components u and w can be derived as follows (Lin *et al.*, 2001):

$$u=(U_j-U_b)\frac{y}{h}+\frac{1}{2\mu}\frac{\partial p}{\partial x}\left\{y(y-h)+2l^2\left[1-\frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\}, \quad (1)$$



(a)

(b)

Fig.1 Physical configuration of a dynamically loaded journal bearing. (a) Main view; (b) Side view

$$w=\frac{1}{2\mu}\frac{\partial p}{\partial z}\left\{y(y-h)+2l^2\left[1-\frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\}, \quad (2)$$

where μ is the viscosity coefficient, η is a new material constant responsible for the couple stress property, $l=\sqrt{\eta/\mu}$. The film thickness at any angle θ for a rigid bearing is given by $h=c+e\cos\theta$.

The film thickness at any angle θ considering the elasticity of the bearing liner is (Naduvinamani *et al.*, 2001)

$$h=c+e\cos\theta+\delta, \quad (3)$$

where $\delta=(pb/E)(1-\upsilon^2)$ with b being bearing liner thickness and υ Poisson's ratio.

Substituting the expressions of u and w into the continuity equation and integrating the equation across the film thickness using boundary conditions, a modified Reynolds equation considering the elasticity of the liner based on couple stress model can be obtained below (Ma *et al.*, 2004),

$$\frac{\partial}{\partial x}\left[f(h,l)\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[f(h,l)\frac{\partial p}{\partial z}\right]=6\mu(U_{\rm j}+U_{\rm b})\frac{\partial h}{\partial x}+12\mu\frac{\partial h}{\partial t}, \tag{4}$$

where $f(h,l)=h^3-12l^2\left[h-2l\tanh(h/(2l))\right]$, $U_{\rm j}=r_{\rm j}\omega_{\rm j}\approx r_{\rm b}\omega_{\rm j}$, $U_{\rm b}=r_{\rm b}\omega_{\rm b}$.

To obtain the modified Reynolds equation in dimensionless form, let the effective angular velocity (Chen *et al.*, 1991) $\omega^*=\omega-2{\rm d}\varphi/{\rm d}t$, and introduce the following dimensionless variables and parameters: $\theta=x/r_{\rm j}$, $\bar{z}=2z/L$, $\lambda=L/(2r_{\rm j})$, $\bar{h}=h/c$, $\bar{l}=l/c$, $q=\frac{2}{\omega^*}\frac{{\rm d}\varepsilon}{{\rm d}t}$, $\bar{p}=\frac{pc^2}{6\mu r_{\rm j}^2\omega^*}$.

Thus, the dimensionless modified Reynolds equation considering the elasticity of the liner is

$$\frac{\partial}{\partial\theta}\left[\bar{f}(\bar{h},\bar{l})\frac{\partial\bar{p}}{\partial\theta}\right]+\left(\frac{1}{\lambda}\right)^2\frac{\partial}{\partial\bar{z}}\left[\bar{f}(\bar{h},\bar{l})\frac{\partial\bar{p}}{\partial\bar{z}}\right]=-\varepsilon\sin\theta+q\cos\theta-C_{\rm e}\frac{\partial\bar{p}}{\partial\theta}, \tag{5}$$

where
$\bar{h}=1+\varepsilon\cos\theta+C_{\rm e}\bar{p}$, $C_{\rm e}=6\mu Ur_{\rm j}\omega^*b(1-\upsilon^2)/(c^3E)$,
$\bar{f}(\bar{h},\bar{l})=\bar{h}^3-12\bar{l}^2\left[\bar{h}-2\bar{l}\tanh(\bar{h}/(2\bar{l}))\right]$,

where q is the dynamic parameter, $C_{\rm e}$ is the elastic coefficient. As the value of q approaches zero, the dimensionless Reynolds equation represents the status of a pure rotation. In the circumferential direction it is assumed that the positive pressure terminates at θ^* where the pressure gradient angle is zero. Then the boundary conditions for the film pressure are

$$\begin{aligned}&\bar{p}\big|_{\bar{z}=\pm1}=0,\ \frac{\partial\bar{p}}{\partial\bar{z}}\Big|_{\bar{z}=0}=0\\&\bar{p}\big|_{\theta=0}=\bar{p}\big|_{\theta=\theta^*}=0,\ \frac{\partial\bar{p}}{\partial\theta}\Big|_{\theta=\theta^*}=0.\end{aligned} \tag{6}$$

The load components in dimensionless form along and perpendicular to the line of centers are calculated by integrating the film pressure acting on the journal surface:

$$\bar{F}_{\rm p}=\int_0^1\int_0^{\theta^*}\bar{p}\cos\theta{\rm d}\theta{\rm d}\bar{z},\quad \bar{F}_{\rm v}=\int_0^1\int_0^{\theta^*}\bar{p}\sin\theta{\rm d}\theta{\rm d}\bar{z}. \tag{7}$$

The dimensionless load-carrying capacity $\bar{F}$ and the attitude angle ψ are

$$\bar{F}=\sqrt{\bar{F}_{\rm p}^2+\bar{F}_{\rm v}^2},\quad \psi=\arctan\left(\bar{F}_{\rm v}/\bar{F}_{\rm p}\right). \tag{8}$$

The shear stress at the journal surface is

$$\tau=\mu\frac{\partial u}{\partial y}\Big|_{y=h}-\eta\frac{\partial^3u}{\partial y^3}\Big|_{y=h}. \tag{9}$$

Substituting Eq.(1) into Eq.(9) and integrating the equation around the journal surface, the dimensionless friction force acting on the journal is derived as

$$\bar{F}_{\rm f}=\int_0^1\int_0^{2\pi}\left(\frac{1}{\bar{h}}\left(1+\frac{c\dot{\varepsilon}\sin\theta-e\dot{\varphi}\cos\theta}{\omega r_{\rm j}}\right)+\frac{\bar{h}}{2}\frac{\partial\bar{p}}{\partial\theta}\right){\rm d}\theta{\rm d}\bar{z}. \tag{10}$$

The friction coefficient can be calculated by

$$C_{\rm f}=\bar{F}_{\rm f}/\bar{F}. \tag{11}$$

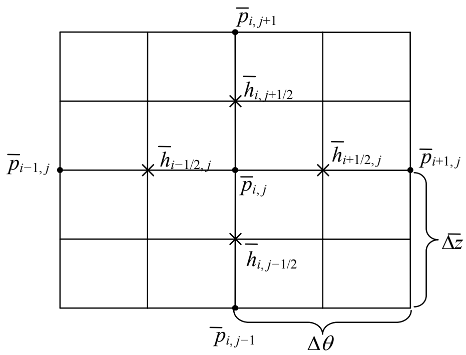
NUMERICAL METHODS

The dimensionless modified Reynolds equation is solved numerically by using a finite difference scheme. Applying the central difference approximation for derivatives, a grid point notation for the film domain is shown in Fig.2. Then Eq.(5) becomes

$$A_1\bar{p}_{i,j}+A_2\bar{p}_{i+1,j}+A_3\bar{p}_{i-1,j}+A_4\bar{p}_{i,j+1}+A_5\bar{p}_{i,j-1}=B_{i,j}, \tag{12}$$

where
$A_1=(\bar{f}_{i+1/2,j}+\bar{f}_{i-1/2,j})/\Delta\theta^2+(\bar{f}_{i,j+1/2}+\bar{f}_{i,j-1/2})/(\lambda^2\Delta\bar{z}^2)$;

$$A_2 = -\bar{f}_{i+1/2,j} \big/ \Delta\theta^2;\ \ A_3 = -\bar{f}_{i-1/2,j} \big/ \Delta\theta^2;$$
$$A_4 = -\bar{f}_{i,j+1/2} \big/ (\lambda^2 \Delta\bar{z}^2);\ \ A_5 = -\bar{f}_{i,j-1/2} \big/ (\lambda^2 \Delta\bar{z}^2);$$
$$B_{i,j} = \varepsilon \sin\theta_i - q\cos\theta_i + C_e(\bar{p}_{i+1,j} - \bar{p}_{i-1,j})/(2\Delta\theta).$$



$\Delta\theta$ and Δz are the variables in the circumferential and the axial directions, respectively; $\bar{p}$ and $\bar{h}$ are the dimensionless pressures and film thicknesses of grid points, respectively

Fig.2 Grid point notation for the film domain

Periodic conditions in Eq.(6) are used to solve the dynamic problem for any given initial conditions. Then the dimensionless squeeze film pressure can be solved numerically. The iterative procedure is terminated when the difference in the successive iterations becomes less than the predefined tolerance as shown below:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left|\bar{p}_{i,j}^{(k)} - \bar{p}_{i,j}^{(k-1)}\right| \Big/ \sum_{i=1}^{m}\sum_{j=1}^{n}\left|\bar{p}_{i,j}^{(k)}\right| \le 10^{-3}. \qquad (13)$$

RESULTS AND DISCUSSION

We select the following parameters for numerical analysis: a ratio of bearing length to journal diameter of λ=1, an eccentricity ratio ε ranging from 0.1 to 0.8, a dynamic parameter q of 0.1, and a couple stress parameter $\bar{l}$ ranging from 0 to 0.4.

Fig.3a shows the pressure distribution in the circumferential direction at the bearing mid-plane for different values of elastic coefficients and couple stress parameters. The pressure distribution along the axial direction at the maximum pressure plane for different couple stress parameters is shown in Fig.3b for both rigid (C_e=0) and elastic (C_e=0.05) liners. It is found that the couple stresses increase the film pressure, and that when the value of couple stress parameter $\bar{l}$ is large, the influence of couple stresses is visibly distinguished. For the same couple stress parameter, the maximum pressure for a bearing with a rigid liner is greater than that for a bearing with an elastic liner and the increase is more pronounced for high values of $\bar{l}$. For the same elastic coefficient, the film pressure is the least when $\bar{l}$=0 (Newtonian lubricant) and the increase is more pronounced for high values of $\bar{l}$.

Figs.4a and 4b present the dimensionless load-carrying capacity for different elastic coefficients and couple stress parameters, respectively. Since the couple stress results in a higher film pressure, the integrated load-carrying capacity is enhanced. For Newtonian and couple stress lubricants, the load-carrying capacity decreases with increasing elastic coefficient with the same eccentricity ratio and couple stress parameters, that is, the load-carrying capacity for rigid liners is greater than that for elastic ones. While the load-carrying capacity increases with increasing couple stress parameter with the same elastic coefficient especially at high eccentricity ratio. Couple stress parameter has more effects on rigid bearing material than on elastic one. The variation of the attitude angle vs eccentricity ratio for different couple stress parameter with rigid and elastic liners is shown in Fig.5a. It is found that the attitude angle increases with increasing couple stress parameter and the attitude angle for elastic liners is smaller than that for rigid ones. Furthermore, the effects are more pronounced for high values of eccentricity ratios and couple stress parameters. Fig.5b displays the friction coefficient as a function of eccentricity ratio for different couple stress parameters. As seen in Fig.5b, the friction coefficient decreases with increasing couple stress parameter and rigid liners yield lower values of the friction coefficient than elastic liners.

CONCLUSION

In this paper the modified Reynolds Eq.(5) is derived for dynamically loaded journal bearings lubricated with couple stress fluids considering the elasticity of the liner. The numerical results show

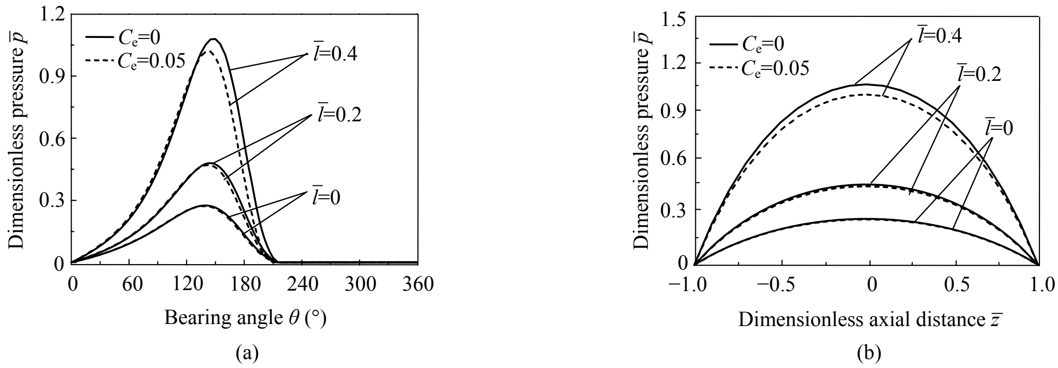


Fig.3 Pressure distribution (a) along the circumferential direction at bearing mid-plane and (b) along the axial direction at the maximum pressure plane for various C_e and $\bar{l}$

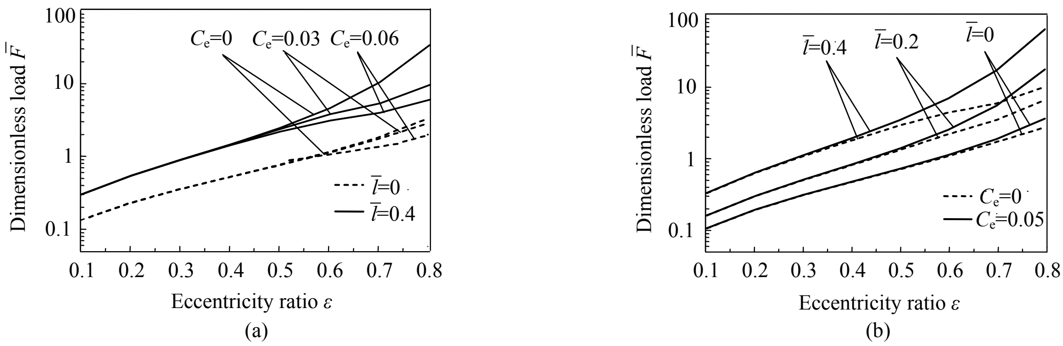


Fig.4 Load-carrying capacity vs eccentricity ratio curve. (a) $\bar{l}$=0 and 0.4 for various C_e; (b) C_e=0 and 0.05 for various $\bar{l}$

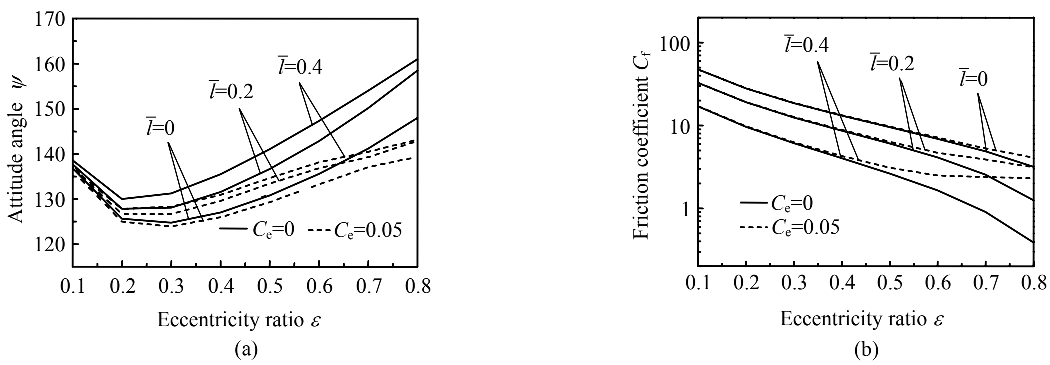


Fig.5 (a) Attitude angle and (b) friction coefficient vs eccentricity ratio curve with C_e=0 and 0.05 for various $\bar{l}$

that the maximum pressure for a bearing with a rigid liner is greater than that for a bearing with an elastic liner, and that the effects are more pronounced at high eccentricity ratio and couple stress parameter. The bearing load-carrying capacity decreases with increasing elastic coefficient but increases with couple stress parameter when lubricated with Newtonian and couple stress fluids, respectively, and the influence of couple stresses on the load-carrying capacity is significantly apparent for high values of eccentricity ratio. Couple stress parameter has more effects on rigid bearing material than on elastic one. The elastic deformation of the bearing liner should be considered if an accurate performance evaluation of the bearing is required.

References

Chen, B.X., Qiu, Z.G., Zhang, H.S., 1991. The Theory of Liquid Lubrication and its Application. Mechanical Industry Press, Beijing, p.193 (in Chinese).

Lin, J.R., 1998. Squeeze film characteristics of finite journal bearings: couple stress fluid model. *Tribology International*, **31**(4):201-207. [doi:10.1016/S0301-679X(98)00022-X]

Lin, J.R., 1999. Static and dynamic characteristics of externally pressurized circular step thrust bearings lubricated with couple stress fluids. *Tribology International*, **32**(4):207-216. [doi:10.1016/S0301-679X(99)00034-1]

Lin, J.R., Yang, C.B., Lu, R.F., 2001. Effects of couple stresses in the cyclic squeeze films of finite partial journal bearings. *Tribology International*, **34**(2):119-125. [doi:10.1016/S0301-679X(00)00147-X]

Ma, Y.Y., Wang, W.H., Cheng, X.H., 2004. A study of dynamically loaded journal bearings lubricated with non-Newtonian couple stress fluids. *Tribology Letters*, **17**(1):69-74. [doi:10.1023/B:TRIL.0000017420.44627.63]

Mokhiamer, U.M., Crosby, W.A., El-Gamal, H.A., 1999. A study of a journal bearing lubricated by fluids with couple stress considering the elasticity of the liner. *Wear*, **224**(2):194-201. [doi:10.1016/S0043-1648(98)00320-2]

Naduvinamani, N.B., Hiremath, P.S., Gurubasavaraj, G., 2001. Squeeze film lubrication of a short porous journal bearing with couple stress fluids. *Tribology International*, **34**(11):739-747. [doi:10.1016/S0301-679X(01)00064-0]

Naduvinamani, N.B., Hiremath, P.S., Gurubasavaraj, G., 2002. Effect of surface roughness on the static characteristics of rotor bearings with couple stress fluids. *Computers and Structures*, **80**(14-15):1243-1253. [doi:10.1016/S0045-7949(02)00085-8]

Sharma, S.C., Jain, S.C., Sah, P.L., 2000. Effect of non-Newtonian behaviour of lubricant and bearing flexibility on the performance of slot-entry journal bearing. *Tribology International*, **33**(7):507-517. [doi:10.1016/S0301-679X(00)00093-1]

Wang, X.L., Zhu, K.Q., Wen, S.Z., 2002. On the performance of dynamically loaded journal bearings lubricated with couple stress fluids. *Tribology International*, **35**(3):185-191. [doi:10.1016/S0301-679X(01)00114-1]

Williamson, B.P., Walters, K., Bates, T.W., Coy, R.C., Milton, A.L., 1997. The viscoelastic properties of multigrade oils and their effect on journal-bearing characteristics. *Journal Non-Newtonian Fluid Mechanics*, **73**(1-2):115-126.