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Calculations of plastic collapse load of pressure vessel using FEA*

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Abstract: This paper proposes a theoretical method using finite element analysis (FEA) to calculate the plastic collapse loads of pressure vessels under internal pressure, and compares the analytical methods according to three criteria stated in the ASME Boiler Pressure Vessel Code. First, a finite element technique using the arc-length algorithm and the restart analysis is developed to conduct the plastic collapse analysis of vessels, which includes the material and geometry non-linear properties of vessels. Second, as the mechanical properties of vessels are assumed to be elastic-perfectly plastic, the limit load analysis is performed by employing the Newton-Raphson algorithm, while the limit pressure of vessels is obtained by the twice-elastic-slope method and the tangent intersection method respectively to avoid excessive deformation. Finally, the elastic stress analysis under working pressure is conducted and the stress strength of vessels is checked by sorting the stress results. The results are compared with those obtained by experiments and other existing models. This work provides a reference for the selection of the failure criteria and the calculation of the plastic collapse load.

Key words: Plastic collapse load, Pressure vessel, Finite element analysis (FEA), Design by analysis (DBA)
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INTRODUCTION

The objective of pressure vessel design by analysis (DBA) is to prevent various possible failures and to ensure safe operation of vessels. This is practically realized by limiting the stresses, strains and design loads of vessels within the allowable values after the failure modes of vessels are determined. However, under excessive static internal pressure, the vessels may experience gross plastic deformation and eventually collapse.

Currently, several international codes of preventing the plastic instability in DBA have been developed, such as the ASME (2007) code and the EN 13445-3 (2002) code. Among them, the ASME code proposes three criteria to prevent the plastic instability of vessels: the elastic stress analysis criterion, the limit load analysis criterion and the elastic-plastic stress analysis criterion. In the elastic stress analysis, the calculated stresses are classified into the primary, secondary and peak stresses, which are all limited by introducing the allowable values. The limit load analysis assumes the elastic-perfectly plastic material property and small deformation of vessels, but no gross plastic deformation. The twice-elastic-slope method proposed by the ASME (2007) code and the tangent intersection method proposed by the EN 13445-3 (2002) code are employed to determine the limit loads of vessels based on the derived pressure-deformation parameter curves. It is not easy to select a suitable deformation parameter especially when multiple loads are acting on the vessels. In contrast, the elastic-plastic stress analysis which directly takes into account the actual material and geometry nonlinear properties of vessels, may result in gross plastic deformations before structural plastic collapse. In this case, the mechanical behaviors of vessels and the load carrying capacity of vessels in the elastic-plastic stress analysis are more practical than those in other two methods. Therefore, it is important

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to explore a calculation method for the plastic collapse loads of vessels theoretically.

An important issue in elastic-plastic stress analysis is to calculate the plastic loads of vessel and the base for vessel design under a specified safety coefficient. Researches have been widely carried out and many methods are developed such as the plastic contour method (Payten and Law, 1998), the plastic work criterion (Muscat *et al.*, 2003) and the limit pressure method (Liu *et al.*, 2004). However, the tangent intersection method and the twice-elastic-slope method lead to the approximate results because the true plastic instability point is not reached in the above calculation and the derived stress-strain curve is incomplete. If the latter two methods are not employed, only the combination of pre- and post-collapse behaviors can just capture the plastic collapse point. Many researchers (Turner, 1910; Faupel, 1956; Christopher *et al.*, 2002; ASME, 2007) conducted the theoretical derivation for the plastic collapse loads of the cylindrical vessels, but could not deal with the complicated vessels with large size of nozzles.

In this paper, a theoretical method using the elastic-plastic finite element analysis (FEA) is proposed to calculate the plastic collapse loads of vessels with nozzles, where the material and geometry nonlinear properties of vessels are considered. The arc-length algorithm (Wempner, 1971; Riks, 1979) and the restart analysis are commonly employed to track the complete stress-strain relationships including the post-necking strain softening behaviors of vessels. The numerical results are compared with those obtained by experiments and other existing models.

FINITE ELEMENT ANALYSIS FOR THREE FAILURE CRITERIA

Finite element modeling

The following analysis concentrates on a vessel shown in Fig.1, in which a Cartesian coordinate system is defined. The vessel includes a cylinder, a spherical head and two nozzles of A and B. The design pressure is 18 MPa. The parametric FEA is performed using the finite element software ANSYS-APDL. The 3D eight-node solid element SOLID45 is adopted to mesh the structure. The effects of heads are represented by the equivalent stresses on the head faces of the nozzles. The finite element mesh model is shown in Fig.2, which includes 18648 nodes and 14289 elements.

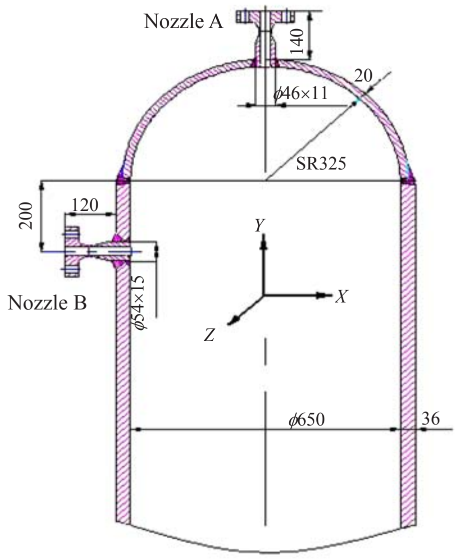

Fig.1 Schematic representation of vessel (unit: mm)

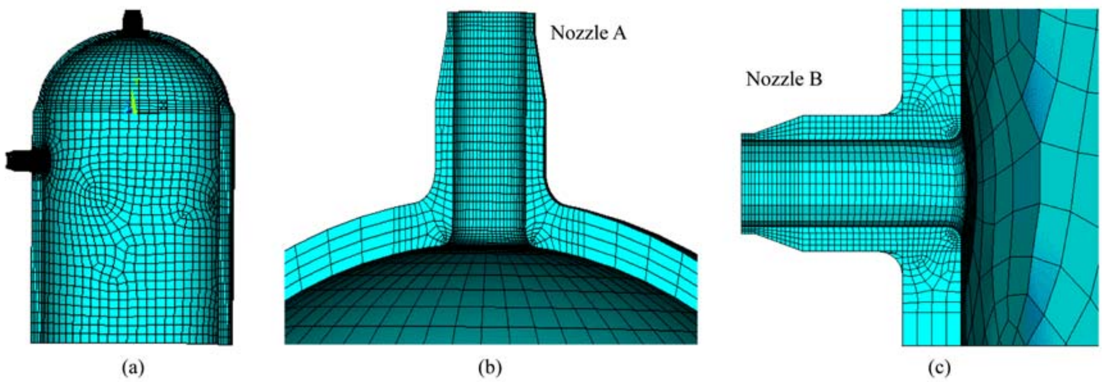

Fig.2 Finite element mesh model. (a) Full model; (b) Local model at nozzle A; (c) Local model at nozzle B

The displacement boundary conditions are

(1) $\Delta Z=0$ at the plane XY;

(2) $\Delta Y=0$ at the bottom head face of the cylinder;

(3) $\Delta X=0$ at any node to eliminate the rigid body displacement of the structure.

The material 16MnR is considered to be isotropic and elastic-plastic. Its mechanical properties are listed in Table 1 (Wang *et al*., 2000).

Table 1 Mechanical properties of 16MnR

Parameter	Value
Young's modulus (MPa)	2.05×10^5
Poisson's ratio	0.3
Yielding strength (MPa)	350
Tensile strength	530
Failure strain	0.248

Elastic-plastic stress analysis

According to the Remberg-Osgood power hardening model, the mechanical properties of the material 16MnR can be expressed as (Wang *et al*., 2000)

$$\varepsilon/\varepsilon_0=\sigma/\sigma_0+(\sigma/\sigma_0)^n, \quad (1)$$

where $\varepsilon_0=1.643\times10^{-3}$, $\sigma_0=300$ MPa and $n=9$ are the characteristic strain, stress and the hardening exponent, respectively.

In the finite element preprocessing, the true stress-strain curve after plastic deformation is fitted by importing 50 discrete stress-strain points to the ANSYS-APDL software. The constitutive relationship of material is shown in Fig.3.

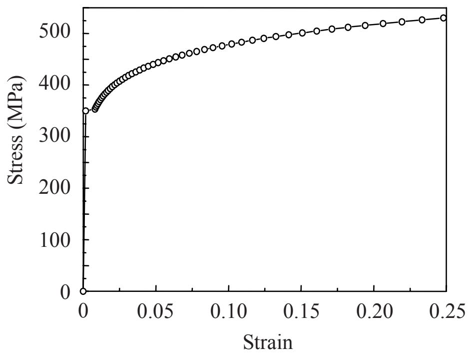

Fig.3 Stress-strain curve of the material 16MnR

The elastic-plastic stress analysis of the structure under internal pressure can be conducted using the Newton-Raphson iterative algorithm. However, for the sudden plastic collapse of the structure, this method cannot further track the load path and becomes invalid because the integrated structural stiffness matrices of the structure at the plastic collapse point are singular, as shown in Fig.4.

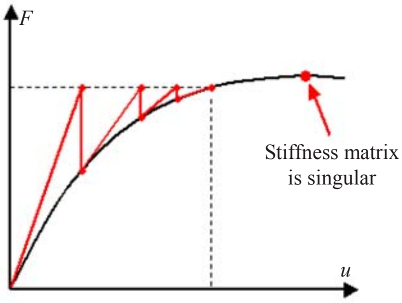

Fig.4 Newton-Raphson iterative algorithm

To solve this problem, the arc-length algorithm is adopted to track the nonlinear post-necking path and to calculate the plastic collapse load of the structure, as shown in Fig.5. This method is originally proposed by Wempner (1971) and Riks (1979), and further improved by Ramm (1981) and Crisfield (1983). To handle zero and negative tangent stiffnesses, the arc-length algorithm multiplies the incremental load by a load factor λ, where λ is between −1 and 1. This extra unknown variable λ alters the finite element equilibrium equation to $\boldsymbol{K}^{\mathrm{T}}\boldsymbol{u}=\lambda\boldsymbol{F}_{\mathrm{a}}-\boldsymbol{F}_{\mathrm{nr}}$, where $\boldsymbol{F}_{\mathrm{a}}$ is the external force and $\boldsymbol{F}_{\mathrm{nr}}$ is the internal non-balanceable force. To deal with this, the arc-length method imposes another constraint, which is stated as

$$\sqrt{\Delta u_n^2+\lambda^2}=R, \quad (2)$$

where Δu is the displacement increment and R is the arc-length radius.

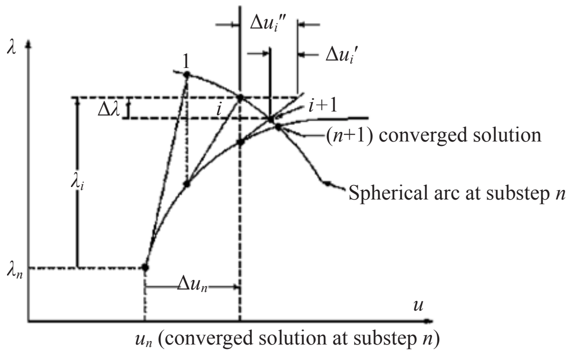

Fig.5 Arc-length algorithm (Riks, 1979)

In the FEA, the arc-length algorithm (Wempner, 1971; Riks, 1979) and the restart analysis are combined to calculate the structural response, where a subroutine is coded using the ANSYS-APDL and integrated into the main program. The flow chart of the FEA is shown in Fig.6. The pressure increment ΔP=0.1 MPa is chosen and the corresponding load step l_s=520 is specified. A text file Para.txt is established to realize the multi-frame restart analysis. The number of load substep near the collapse point is changed to improve the calculation efficiency.

The numerical calculations are implemented on the high-performance computer and the main configurations are: Intel Xeon CPU with 8 processors (the main frequency of each processor is 2.33 GHz) and 12 GB memory. The calculation takes about 10 h.

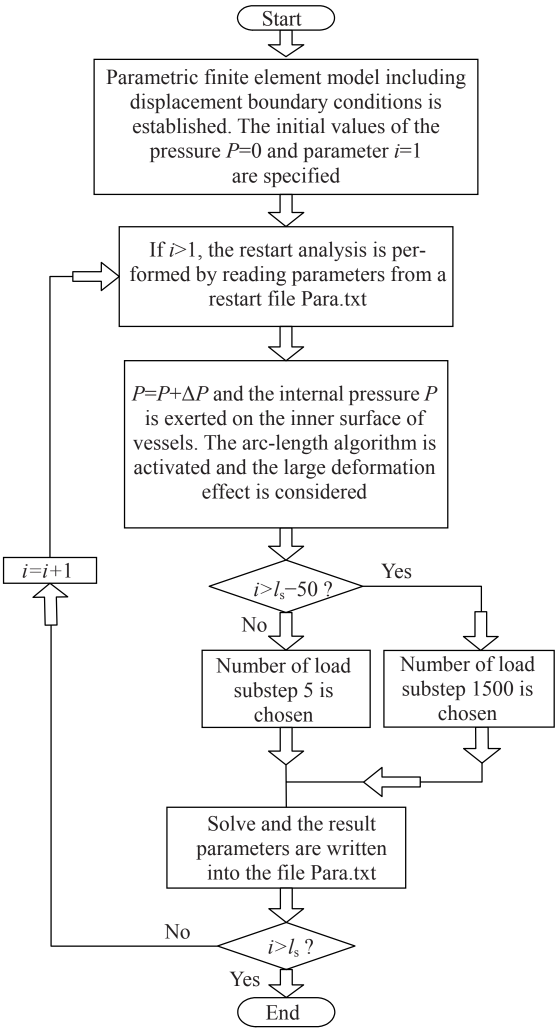

Fig.6 Flow chart of the finite element analysis

Fig.7 shows the pressure-Mises strain curves at the nozzles A, B and far-field region, respectively. It is shown that the pressure first increases and then decreases with increasing strain. The vessel experiences a relatively small deformation stage under the pressure less than about 42 MPa and a large deformation stage under the pressure from about 42 MPa to collapse pressure. After the curve enters into the descending stage, the structure starts to collapse in the form of unstable plastic tension. The three curves in Fig.7 lead to the same value 50.3 MPa for the plastic collapse load. In contrast, the finite element values for the vessels without above nozzles are about 52.3 MPa.

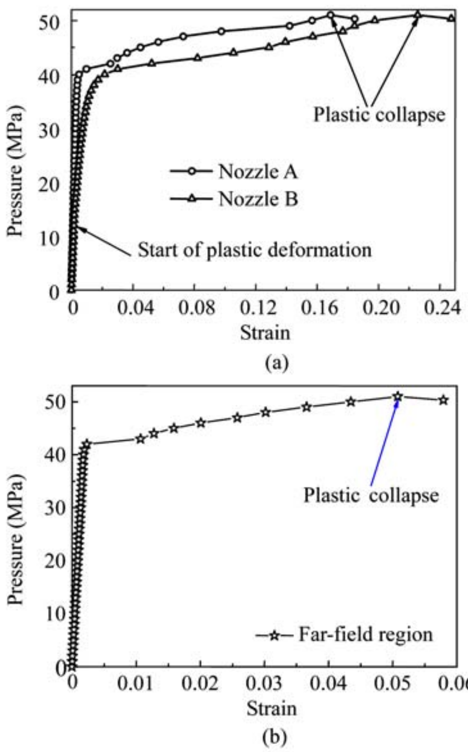

Fig.7 Pressure-Mises strain curves. (a) Nozzles A and B; (b) Far-field region

The calculated values for the plastic collapse load are compared with the theoretical values obtained by Turner (1910), Faupel (1956), Christopher *et al.*(2002) and ASME (2007) code, shown in Table 2, by which the differences between the finite element result of 52.3 MPa and those using other models can be interpreted in two aspects: first, the existing models do not consider the effects of nozzles on the plastic collapse load; second, the expressions in Table 2 are derived based on the empirical estimate and cannot explain the true plastic hardening process of vessels. Therefore, a power hardening model is important for the ductile material 16MnR in this analysis if a rigorous prediction of the plastic collapse load is to include the strain hardening effect (Zhu and Leis, 2006).

Table 2 Comparative plastic collapse loads

Model	Expression	Value (MPa)
Turner (1910)	$\sigma_{uts}\ln(r_o/r_i)$	55.64
Faupel (1956)	$2/\sqrt{3}\sigma_y(2-\sigma_y/\sigma_{uts})\ln(r_o/r_i)$	56.84
Christopher *et al.*(2002)	$2\sigma_{uts}(r_o/r_i-1)/(r_o/r_i+1)$	55.66
ASME	$\sigma_{uts}\left(\frac{r_o/r_i-1}{0.6r_o/r_i+0.4}\right)\ln(r_o/r_i),\ r_o/r_i<1.5$	55.00

r_i: the inner radius of the cylinder; r_o: the outer radius of the cylinder; σ_y: the yielding strength; σ_{uts}: the tensile strength

In order to validate the efficiency of the proposed solution algorithm, the axisymmetric FEA is performed on two vessels designed by us, which includes a cylinder and two normal elliptical heads. The eight-node element PLANE82 is used to mesh the model. The stress-strain curve of the material X5CrNi18-10, the finite element model with boundary conditions and the burst specimen are shown in Figs.8~10, respectively. The yielding and tensile strengths of the material X5CrNi18-10 are σ_y=250 MPa and σ_{uts}=1007.8 MPa, respectively.

The plastic collapse loads using FEA and experiments are listed in Table 3. Results show that the theoretical values are in good agreement with the experimental values. The errors 4.5% and −5.7% mainly come from three aspects: first, the material takes on variability and it would be unlikely that the material in the vessels is uniform; second, the theoretical calculations neglect the effect of welding; third, the pressurization rate of the pneumatic system also affects the experimental burst strength in some extent.

Limit load analysis

In the limit load analysis, a slope of 0.5% in the plastic yielding stage is selected and the bilinear hardening material property is used. After the stress distributions are obtained, the limit load of the structure can be obtained in three steps: (1) the pressure-strain curves at the stress concentration area are mapped; (2) the twice-elastic-slope method and the tangent intersection method are employed respectively to determine the limit loads of each region; (3) by comparison, the minimum load is considered as the limit load.

Fig.11 shows the pressure-Mises strain curves at the nozzles A, B and far-field region, respectively, for the finite element model in Fig.2. Table 4 lists the

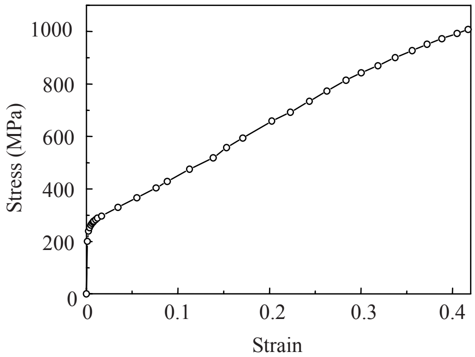

Fig.8 Stress-strain curve of the material X5CrNi18-10

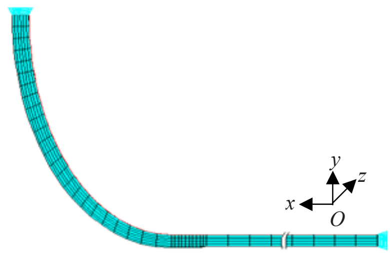

Fig.9 Finite element mesh model with boundary conditions

Fig.10 Specimen after burst

Table 3 Comparison of the theoretical and experimental results

Vessel No.	Vessel size (mm)	FEA (MPa)	Experimental value (MPa)	Error (%)
1	r_i=250, L=1500, t_c=6.5, t_h=8.3	15.5	16.2	4.5
2	r_i=250, L=1500, t_c=12.6, t_h=14.6	29.7	28.0	−5.7

t_c: the thickness of the cylinder; L: the length of the cylinder; t_h: the thickness of the head

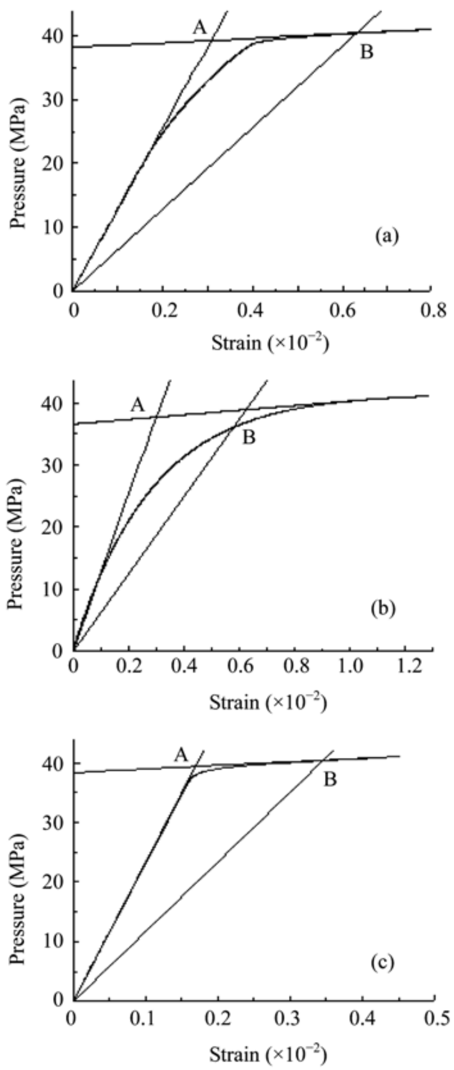

Fig.11 Pressure-Mises stain curves at (a) the joint of the head and Nozzle A; (b) the joint of the cylinder and Nozzle B; (c) the joint of the head and cylinder. Points A and B denote the intersection positions using the tangent intersection method and the twice-elastic-slope method, respectively

Table 4 Calculated limit loads at different positions

Case	Limit loads (MPa)	
	Tangent intersection method	Twice-elastic-slope method
(a) in Fig.11	39.5	37.9
(b) in Fig.11	37.5	35.4
(c) in Fig.11	40.1	38.3

values of the limit loads at different positions. The values obtained using the tangent intersection method are slightly larger than those obtained using the twice-elastic-slope method. By comparison, the limit load of the structure is about 35.4 MPa, which is smaller than that of the plastic collapse load 50.3 MPa. This shows that the elastic-plastic analysis can lead to stronger ability to resist the failure than the limit load analysis because the latter is not true stress analysis due to the assumption of elastic-perfectly plastic material.

The ASME (2007) code limit analysis requires that "… the specified loading do not exceed two-thirds of the lower bound collapse load P_{min}". Thus, the allowable maximum design pressure P is

$$P= 2P_{min}/3, \tag{3}$$

where P_{min}=35.4 MPa. Therefore, the value for the maximum design pressure P reaches 23.6 MPa. By comparison, the case under design pressure 18 MPa is safe.

Elastic stress analysis

In the elastic stress analysis, the material is assumed to be linear-elastic and the plastic hardening is not considered. The elastic stress analysis criterion combines the elastic stress analysis and the plastic failure criterion, and the calculated stress under design pressure is termed as "nominal stress". Based on the principle of "uniform safety tolerance", the stress analysis partitions the stresses according to their different properties in aspects of the stress origin, the stress distributions and their effects on the structure. The method above is also called as "the stress categorization method", which partitions the elastic stresses into the primary stress, secondary stress and peak stress. Then, the sorted stress components are limited using different strength constraint conditions.

Fig.12 shows the distribution of stress intensity under 18 MPa design pressure and the stress category lines. In stress assessment, all average stresses near the nozzles are considered as the local membrane stress and all bending stresses arising from internal pressure near the nozzles are sorted as the secondary stress. Both the ASME (2007) code and the EN 13445-3 (2002) code regulate that the safety coefficient is 2.4 and thus the design stress strength is S_m=min(530/2.4, 350/1.5)=220.8 MPa. Table 5 lists the assessment results showing that the case under design pressure 18 MPa is safe though the stress categorization method cannot directly determine the bearing limit of vessels.

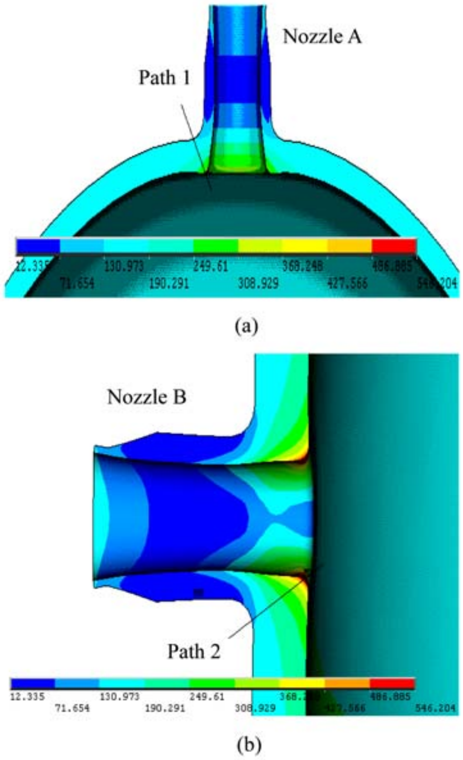

Fig.12 Stress intensity distributions and stress category lines under 18 MPa design pressure at (a) nozzle A and (b) nozzle B

Table 5 Stress strength assessment

Stress category	Stress (MPa)		Stress strength (MPa)	Assessment results
	Path 1	Path 2		
P_L	205.3	255	331.2 (1.5KS_m)	Pass
P_L+P_b	≤297.6	≤425.5	662.4 (3KS_m)	Pass

P_L: local membrane stress; P_b: local bending stress; K: load combination coefficient

CONCLUSION

This paper explores the calculation methods using FEA for three failure criteria proposed by the ASME (2007) code which prevent the plastic instability of vessels. The arc-length algorithm and the restart analysis using FEA are combined to develop a subroutine to calculate the plastic collapse loads of vessels. After the pressure-strain curves are derived, the failure loads are directly obtained in the elastic-plastic stress analysis, and obtained in the limit load analysis by using the tangent intersection method and the twice-elastic- slope method. By comparison, the proposed finite element algorithm for calculating the plastic collapse loads under internal pressure is relatively effective. Also, the complete elastic-plastic stress analysis is more practical and instructive than the limit load analysis and the stress categorization method for the design of vessels though the calculation cost of the complete elastic-plastic stress analysis is comparatively high.

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