Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Concept modeling of tapered thin-walled tubes

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Received Mar. 6, 2008; Revision accepted May 14, 2008; Crosschecked Oct. 29, 2008

Abstract: This paper presents a method to create concept models for the tapered thin-walled tubes using beam elements and spring elements. Developed concept tapered beam models with different taper angles and cross sections are compared with those detailed models through impact analyses. Important crash results are recorded and compared, and the relatively good agreement is achieved between these analyses. Concept modeling steps are illustrated in detail, and a general concept modeling method for such thin-walled tubes is summarized and presented.

Key words: Tapered tube, Concept model, Axial crushing, Impact analysis, Beam element, Spring element
doi:10.1631/jzus.A0820159 **Document code:** A **CLC number:** O342; TU311

INTRODUCTION

Thin-walled tubes, especially these with rectangular or circular cross sections, have been widely used in automotive structures and trains for increasing the energy absorption efficiency, weight reduction, strength, and manufacturability. Compared with traditional straight tubes, the tapered thin-walled tubes are preferred since they are more likely to provide a desirable constant mean crush load-deflection response during axial crushing (Reid and Reddy, 1986). Meanwhile, tapered tubes can withstand oblique impact loads as well as axial loads, and therefore has stable crush load and deformation response compared with the straight tubes.

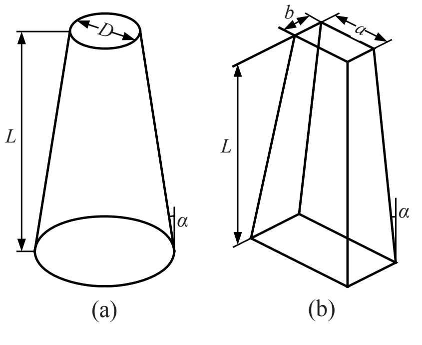
Due to the wide applications of the tapered thin-walled tubes, their load-deformation response and energy absorption performance under axial loading, as well as other important characteristics, have been studied (Paavola and Salonen, 1999; Wang *et al*., 2000, Lee and Oh, 2000; Nagel and Thambiratnam, 2005; Gupta *et al*., 2006; Liao and Zhong, 2008). Such studies considered various tapered tubes with rectangular or circular cross sections, which had different taper angles. All the computer analyses and simulations involved in those studies are performed through shell-element models (detailed models), which achieved computational accuracy by high fidelity to the physical geometry. However, in modeling the thin-walled tubes, users always want to use beam-element models, which can be used for computer simulation quickly and efficiently, and also allow users to have a quick and rough idea about the behavior of several alternative designs. The idea of using such beam-element models, called concept models, have been demonstrated in previous literature and applied for quasi-static and impact analyses (Prater *et al*., 2002; 2005; Liu and Day, 2006).

In previous study, beam elements are used to create concept models for the straight thin-walled tubes, which have constant cross section. Nevertheless, few studies have discussed generating concept models for the tapered thin-walled tubes. This paper aims at creating concept models for the tapered thin-walled tubes, which can be used for axial crushing analysis. In developing such models, the modeling methodology put forward by Liu and Day (2006) is referenced, the developed concept models are used for the axial crush analyses and the crushing force, and displacement and absorbed energy are compared with those from the detailed models to validate the concept models. Finally, a method to develop the concept models for the tapered thin-walled tubes is summarized and presented. The developed concept models compose of beam and spring elements, where the beam elements represent the basic profile and cross-sectional information of the tapered tubes and the spring elements are to simulate their axial buckling behaviors. All the modeling and crash analyses involved in this paper are carried out using LS-DYNA.

PROBLEM DESCRIPTION

A general tapered tube may have circular or rectangular cross section, in which its length, the dimension of its small end, and the taper angle are the most important parameters in fully defining its configuration (Fig.1). This paper starts to develop concept models for circular tapered tubes and then follows the same approach to create the concept models for rectangular tapered tubes. From Fig.1, it was assumed that the length of the tapered tubes was 300 mm, and the wall thickness was 1.5 mm. For the circular tube, the diameter of its small end was 60 mm, and for the rectangular tube, the dimension of its small end was 80 mm×40 mm. Since practically the taper angle α varies between 5°~15° (Nagel and Thambiratnam, 2005), three models were taken for concept modeling and analyses for each of circular and rectangular tubes with taper angles 5°, 10°, and 15°. Other dimensions of the tapered tubes can be calculated based on the given parameters, and detailed geometries of concerned circular and rectangular tubes are listed in Tables 1 and 2.

The basic idea of generating concept tapered tube model is to divide a tapered tube into several segments, use beam elements with different cross-sectional information to model those segments, and finally assemble those beam segments together. In order to simulate the buckling behavior in the concept models, several translational springs are defined in each segment. The positions and characteristics for such springs are determined from the existing collapse theories for circular tubes (Abramowicz and Jones, 1986) and rectangular tubes (Abramowicz and Wierzbicki, 1989). In modeling each segment, the beam elements are assumed to have constant cross section, which is defined as the median cross section of that segment (Fig.2). Basically, the essence of the modeling method is to use several straight concentric tubes with different cross sections to approximate an entire tapered tube. Here a question arises, that is how many segments a tapered tube should be divided into to guarantee the accuracy of the modeling approximation. From Figs.1 and 2, it can be seen that for a small α, the cross section of that tapered tube varies gradually and might be approximated with less segments. Specifically, if α equals 0, the tapered tube turns to a straight tube and can be exactly represented with “one” segment. On the contrary, a large α means that the cross section varies evidently and more segments are required for closely approximating. In



(a) (b)

Fig.1 Common tapered thin-walled tubes. (a) Circular tube; (b) Rectangular tube

Table 1 Dimensions of circular tapered thin-walled tubes

α (°)	Model	D_t (mm)	D_b (mm)	D_{m1} (mm)	D_{m2} (mm)	D_{m3} (mm)	Length (mm)
5	Detailed	60	112				300
	1-seg	60	112	86			300
	2-seg	60	112	73	99		150
	3-seg	60	112	69	86	103	100
10	Detailed	60	166				300
	1-seg	60	166	113			300
	2-seg	60	166	86.5	139		150
	3-seg	60	166	77.5	113	148	100
15	Detailed	60	221				300
	1-seg	60	221	140.5			300
	2-seg	60	221	100	180		150
	3-seg	60	221	87	140.5	194	100

D_t: diameter of top circular section; D_b: diameter of bottom circular section; D_{m1}, D_{m2} and D_{m3} are mean diameter of circular section in segments 1, 2 and 3, respectively

Table 2 Dimensions of rectangular tapered thin-walled tubes of the 3-seg concept model

α (°)	$a_t \times b_t$	$a_b \times b_b$
5	80 mm×40 mm	126 mm×63 mm
10	80 mm×40 mm	174 mm×87 mm
15	80 mm×40 mm	224 mm×112 mm

$a_t \times b_t$: the dimension of top rectangular section; $a_b \times b_b$: the dimension of bottom rectangular section this paper, concept models of tapered tubes with α of 5°, 10°, and 15° are created, and for each α the tapered tube is divided into 1, 2, and 3 segments during concept modeling in order to find the correlation between the number of segments and α in qualified concept models.

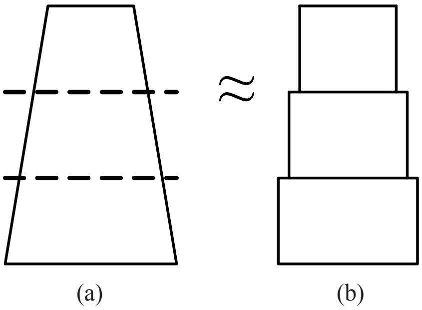


Fig.2 Approximation of (a) tapered tube for (b) concept modeling

After determining the number of straight tubes for the concept models, the modeling method for straight thin-walled beams (Liu and Day, 2006) is applied to creating concept models for each straight tube. The final concept model for an entire tapered tube can be built up by assembling all the concept straight tubes together.

This paper starts from circular tapered tubes. 9 concept models are developed for the circular tapered tubes with α of 5°, 10°, and 15° (each circular tapered tube is approximated with 1, 2, and 3 segments and concept models are created for each of them). The developed concept models are used for impact analyses and the results are compared with those from the existing detailed models to validate such concept models as well as the modeling method. Afterwards, this modeling method is employed to develop concept models for the rectangular tapered tubes and the same impact analyses are carried out for verification. The cross sectional information of the approximated circular tube models is also listed in Table 1 (D_{m1}, D_{m2}, D_{m3}).

CONCEPT MODELING FOR CIRCULAR TUBES

LS-DYNA is used to create the detailed and concept models for the tapered tubes and to simulate the impact analyses and energy absorption. The detailed models are modeled using 4-node Belytschko-Tsay shell element, which has 5 integration points through the thickness to model bending. It has been verified in previous researches that the detailed tapered tube models created in this way can be well used for computer analysis and to accurately simulate the real impact tests (Pasquino and Marotti de Sciarra, 1992; Paavola and Salonen, 1999; Lee and Oh, 2000; Goldfeld *et al.*, 2005). Hughes-Liu beam element with cross section integration is used for generating the concept models. During the analyses (Fig.3), the base of each model (the large end for tapered tubes) is fully constrained to a supporting rigid plate using rigid elements.

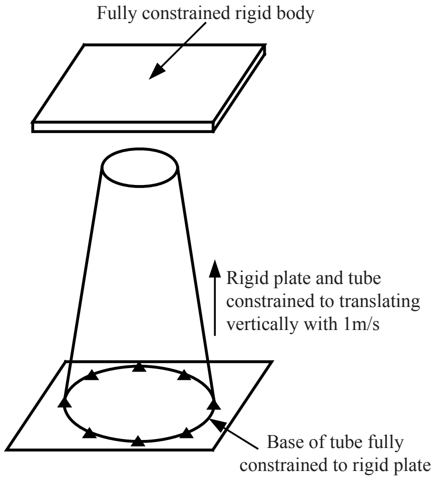


Fig.3 Impact analyses for circular tapered tubes

In defining the spring elements for the circular tubes, Eqs.(1) to (3) are used to determine the position and axial resistance (F-δ curve) for such springs (Goldfeld *et al.*, 2005),

$$P_m/M_0=2.523(2R/t)^{1/2}+15.09, \tag{1}$$
$$H=1.84R(t/(2R))^{1/2}, \tag{2}$$
$$M_0=(\sigma_0 t^2)/4=(0.92\sigma_u t^2)/4, \tag{3}$$

where P_m is mean crushing force; M_0 is fully plastic moment per unit length of section wall; R is radius of the circular cross section; t is wall thickness; H is half length of one plastic fold during the axial buckling of thin-walled tubes; σ_0 is energy equivalent flow stress; σ_u is ultimate stress of material.

Next, the positions and characteristics of the spring elements can be determined for each concept models. Table 3 lists the P_m and H values for each straight circular tube segment, which are calculated from the sectional radius of those tubes (Table 1) and the material properties.

Table 3 Mean crushing forces and plastic fold length of straight circular tube segments

α (°)	Model	P_{m1} (kN)	H_1 (mm)	P_{m2} (kN)	H_2 (mm)	P_{m3} (kN)	H_3 (mm)
5	1-seg	47.8	10.4				
	2-seg	44.3	9.6	51.0	11.2		
	3-seg	43.2	9.4	47.8	10.4	52.0	11.4
10	1-seg	54.3	12.0				
	2-seg	47.9	10.5	59.8	13.3		
	3-seg	45.5	9.9	54.3	12.0	61.6	13.7
15	1-seg	60.1	13.4				
	2-seg	51.3	11.3	67.6	15.1		
	3-seg	48.0	10.5	60.1	13.4	70.0	15.7

P_{m1}, P_{m2} and P_{m3} are mean crushing forces in segments 1, 2 and 3, respectively; H_1, H_2 and H_3 are half lengths of plastic folds in segments 1, 2 and 3, respectively

With the calculated P_m and H, the F-δ curve of such springs can be plotted as Fig.4. With this property, the spring begins to deform when the crushing force reaches P_m. After its deformation reaches $2H$, the spring fails and stops deforming.

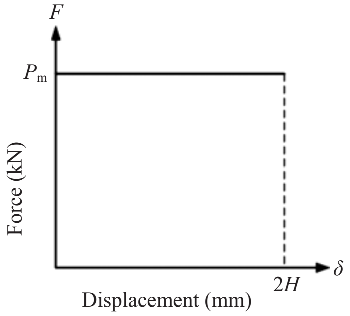


Fig.4 *F-δ* curve of spring elements for straight circular tubes (Liu, 2008)

The developed springs then can be used with the beam elements to create the concept models for the tapered tubes. According to Liu and Day (2006)'s method, in modeling the straight tube segments, each segment is divided into a certain number of parts with equal length $2H$. Beam elements with proper cross-sectional information are used to model those parts, and the defined spring elements are employed to connect those parts together to finish the concept model for the straight tube segment. In this paper, an entire concept tapered circular tube model may include 1 to 3 straight tube segments that are modeled following this method.

For example, when creating a concept model composed of 3 straight tube segments for the tapered tube with α of 5°, according to Table 1, those segments have cross-sectional diameters 69, 86, and 103 mm, respectively. Thus, beam elements with proper cross sections will be used to model these segments and 3 types of springs will be determined and used in each segment. The axial resistances of those springs can be derived based on the values listed in Table 3 and Fig.3. Since each segment has a length 100 mm, in the 3-seg model, segment 1 should have 5 springs because the corresponding length for each plastic fold in real straight tube is 2×9.4=18.8 mm, however, segments 2 and 3 only have 4 springs each because the lengths of real plastic folds are 2×10.4=20.8 mm and 2×11.4=22.8 mm (Fig.5).

The positions of those spring elements are shown in Fig.5 and the same type of springs are equally spaced along the corresponding segment. Fig.5 also shows and compares all the concept models for the tapered tube with α of 5°. Concept models for the other two tapered circular tubes are created following the same way.

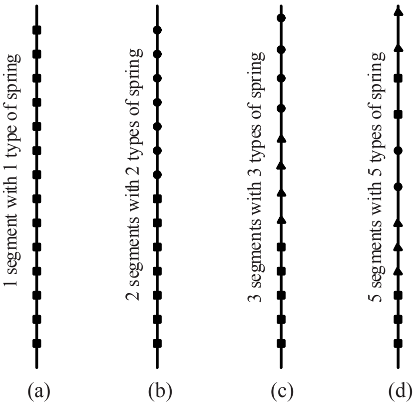


Fig.5 Concept models for 5° tapered tube. (a) 1-seg model; (b) 2-seg model: (c) 3-seg model; (d) 5-seg model

ANALYSES AND RESULTS

The developed concept models and the detailed models then are used for the impact analyses and energy absorption. During the analyses, the supporting rigid plate is constrained to translate vertically over a pre-defined displacement such that the free end of the model is axially crushed onto a fully fixed rigid body. The velocity of the supporting rigid plate is 1 m/s to simulate the quasi-static axial crushing. Contact within each tube as the walls buckle during the axial crushing is modeled using the 3D self-contact interaction option available in LS-DYNA (Hallquist, 1993). All surface contact is treated as frictionless.

The material of the tapered tube models is hot rolled mild steel of yield strength σ_y=305 MPa, Young's modulus E=205 GPa, Poisson's ratio υ=0.3, and density ρ=7700 kg/m^3. The material non-linearity is included using the actual stress-strain curve obtained from a standard tensile tests in order to accurately define the post-yield material response in the FE model. The true stress-plastic strain data points presented in (Nagel and Thambiratnam, 2005) are used in the models, as shown in Table 4.

After the analyses, impact force, displacement, and absorbed energy obtained from those models are recorded and compared to evaluate the developed concept models. Those results are plotted through Figs.6~8 and also listed in Table 5. The deformed configurations for both detailed and concept models are displayed in Fig.9 (tapered tube with α=5°).

Table 4 Stress-strain data points used for mild steel in the numerical simulations

No	σ_t (MPa)	ε_p	No	σ_t (MPa)	ε_p
1	305	0	4	425	0.10
2	344	0.02	5	450	0.14
3	386	0.05	6	470	0.19

σ_t: true stress; ε_p: plastic strain

DISCUSSIONS

From Figs.6~8 it can be found that for the tapered circular tubes, their concept models can properly predict the trend of the displacements and absorbed energies of the detailed models. However, the concept models cannot reflect the crushing force histories but only estimate the mean crushing force during the crash. Different phenomena may be

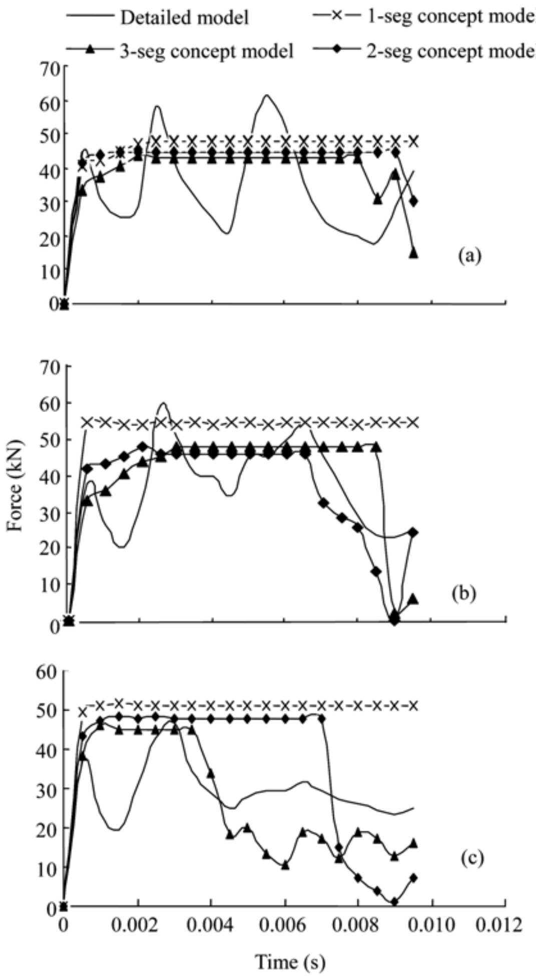


Fig.6 Crushing forces for tapered circular tubes. (a) α=5°; (b) α=10°; (c) α=15°

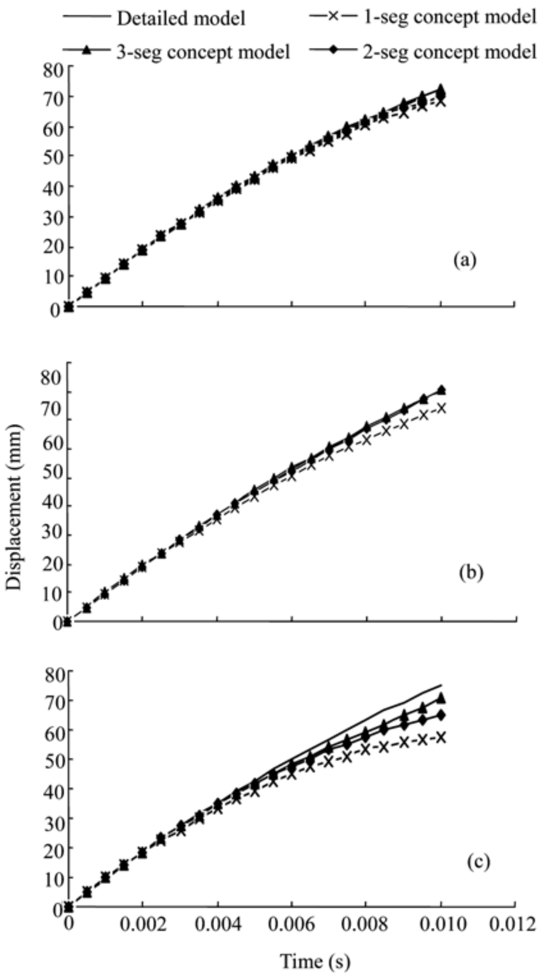


Fig.7 Displacements at the end of tapered circular tubes. (a) α=5°; (b) α=10°; (c) α=15°

Table 5 Comparison of crash results from detailed and concept tapered circular tubes

α (°)	Model	Mean crushing force (kN)	Displacement (mm)	Absorbed energy (kJ)
5	Detailed	32.8	71.9	2.66
	1-seg	44.5 (35.7%)*	68.4 (−4.9%)	3.12 (17.2%)
	2-seg	41.2 (26.3%)	70.0 (−2.6%)	3.02 (13.5%)
	3-seg	36.7 (11.9%)	72.4 (0.7%)	2.72 (2.3%)
10	Detailed	36.0	80.1	3.10
	1-seg	51.6 (43.3%)	74.1 (−7.5%)	3.98 (28.3%)
	2-seg	37.6 (4.4%)	80.9 (1.0%)	3.20 (3.2%)
	3-seg	36.7 (1.9%)	80.6 (0.7%)	3.13 (1.0%)
15	Detailed	28.2	75.1	2.37
	1-seg	48.6 (72.3%)	57.7 (−23.2%)	3.40 (43.5%)
	2-seg	35.1 (24.5%)	64.9 (−10.2%)	3.21 (35.4%)
	3-seg	25.8 (−8.5%)	70.9 (−5.6%)	2.57 (8.4%)

$$^{*}\text{Difference}=\frac{\text{the value of concept model} - \text{the value of detailed model}}{\text{the value of detailed model}}\times 100\%$$

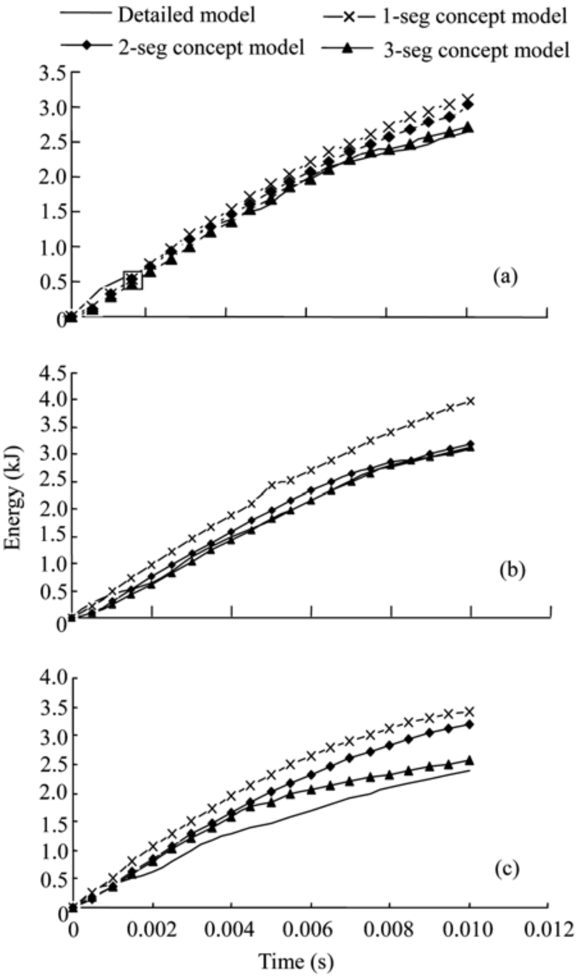


Fig.8 Absorbed energies for tapered circular tubes. (a) α=5°; (b) α=10°; (c) α=15°

Fig.9 Deformed configurations of detailed and concept tapered tube models (α=5°)

observed when validating concept models for the tapered rectangular tubes. Comparing the 1-seg, 2-seg, and 3-seg concept models, it is obvious that the 3-seg concept models best simulate the axial buckling of the detailed tapered circular tubes, which is because the 3-seg concept models approximate the real tapered tubes most closely. Meanwhile, on comparing the final results, it is concluded that the 3-seg concept models are accurate enough in predicting the crash response of the detailed models. Therefore, there is no need to use shorter segments to approximate the real tapered tubes because the added benefit cannot justify the added modeling labor. On the other hand, if a real tapered tube is approximated by more straight segments and used for concept modeling (such as 5-seg model or 7-seg model), the ratio of the cross-sectional dimension and the length of each segment will be too high and the beam element may not be appropriate to model such segments. We will see this in the concept models for the tapered tube with α=15°. From Table 5 it can be seen that the concept models for the tapered tube with α=15° yield relatively large difference compared with the concept models for the tubes with α=5° and α=10°. This is because the three approximated straight segments for the tapered tube with α=15° have higher ratios of the cross-sectional diameters and their lengths and therefore cannot be considered as long, slender members. Thus, using beam elements to model such segments may lose some of accuracy in the concept models, which means that the presented modeling idea and method are only appropriate for those tapered tubes with small taper angle (α≤15°).

Also from Fig.6, it can be seen that even the mean crushing forces obtained from the concept models well correlated to those from the detailed models, the concept models failed to correctly simulate the whole crushing force histories and even missed some peak forces, which might cause an unsafe design (Fig.6a). The reason is that the applied Eqs.(1)~(3) were derived based on predicting the mean crushing force instead of the peak crushing force. Apparently, in order to improve the concept models, new system of formulae needs to be derived and employed for the concept modeling so that the concept circular tube models can correctly predict the peak crushing force.

Generally, the 3-seg concept models can correctly simulate the axial crash of the real tapered circular tubes and therefore can replace the detailed models in the impact analysis. From Table 5 it is found that the differences between the crash results yielded from the 3-seg concept and detailed models are below 15%. The advantage of the concept models lies in the modeling process. Unlike the detailed models, it is not necessary to restart a new computer model if using concept models. A new concept model with different cross-sectional types or dimensions can be easily remodeled by assigning proper cross-sectional information and defining new spring elements based on a previously created concept model. This advantage can save lots of modeling labor and, moreover, because the concept model is only composed of beam and spring elements, it can remarkably save computer expense when it is used to simulate the response of complicated structures such as vehicles and aircrafts. The impact velocity used in this paper is 1 m/s in order to simulate a quasi-static axial crushing. However, in a dynamic axial crushing with velocity >10 m/s, the concept model also shows a good agreement with the detailed model.

Nonetheless, the concept model for such tapered tube is only an approximation of the real model. Compared with the detailed model, the concept model adopts two approximations. Firstly, it approximates an entire tapered tube that has various cross sections with the combination of several straight tube segments that has mean cross sections; Secondly, the concept model assumes such straight segments as beam members. Consequently, the concept model developed following such idea may deviate from the real model more or less due to those approximations, which is obvious for the tapered tube with α≥15°.

In next section, the developed modeling method will be applied to generate the concept models for the tapered rectangular tubes, and the developed concept models only have three segments and three types of springs.

MODELING AND ANALYSES FOR RECTANGULAR TUBES

As concluded in the previous sections, the 3-seg concept model with three types of spring is accurate enough to replace a detailed tapered thin-walled beam model with a length of 300 mm. This conclusion is then verified by creating and validating three 3-seg concept models for three tapered thin-walled rectangular tubes with the dimensions listed in Table 2. In generating these concept models, the same modeling method illustrated before is applied, and Table 6 presents the cross-sectional dimensions for each approximated straight tube segment that are determined based on the cross sections of real tapered tubes and will be used for concept modeling.

Table 6 Cross-sectional dimensions of approximated straight rectangular tube segments of the 3-seg concept model

α (°)	$a_1 \times b_1$	$a_2 \times b_2$	$a_3 \times b_3$
5	87.5 mm×43.75 mm	102.5 mm×51.25 mm	118 mm×59 mm
10	95.5 mm×47.75 mm	126.5 mm×63.25 mm	158 mm×79 mm
15	104 mm×52 mm	152 mm×76 mm	200 mm×100 mm

$a_1 \times b_1$, $a_2 \times b_2$ and $a_3 \times b_3$ are dimensions of rectangular tubes in segments 1, 2 and 3, respectively

Compared with the concept modeling of the tapered circular tubes, the only difference in modeling the rectangular tubes is to determine the characteristics of the springs. Unlike the circular tube, collapse theories about axial crushing of the thin-walled rectangular beams are applied to determining P_m and H (Abramowicz and Wierzbicki, 1989) below:

$$P_\mathrm{m}=9.56\sigma_0 t^{5/3}C^{1/3}, \tag{4}$$

$$H=0.98(tC^2)^{1/3}, \tag{5}$$

where C is the average edge length of the cross section's depth a and width b, $C=(a+b)/2$. Table 7 lists P_m and H values for each straight rectangular tube segment, which are calculated from the sectional geometries of those tubes (Table 6) and the material properties.

After developing the concept models, the same impact analyses are performed on both the developed concept models and the detailed models. Analyses results are displayed through Figs.10~12 and also listed in Table 8.

Table 7 Mean crushing forces and plastic fold length of straight rectangular tube segment of the 3-seg concept model

α (°)	P_m1 (kN)	H_1 (mm)	P_m2 (kN)	H_2 (mm)	P_m3 (kN)	H_3 (mm)
5	31.2	18.3	32.9	20.3	34.5	22.3
10	32.2	19.3	35.3	23.3	38.0	27.1
15	33.1	20.5	37.6	26.4	41.2	31.7

P_m1, P_m2 and P_m3 are mean crushing forces in segments 1, 2 and 3, respectively; H_1, H_2 and H_3 are half lengths of plastic folds in segments 1, 2 and 3, respectively

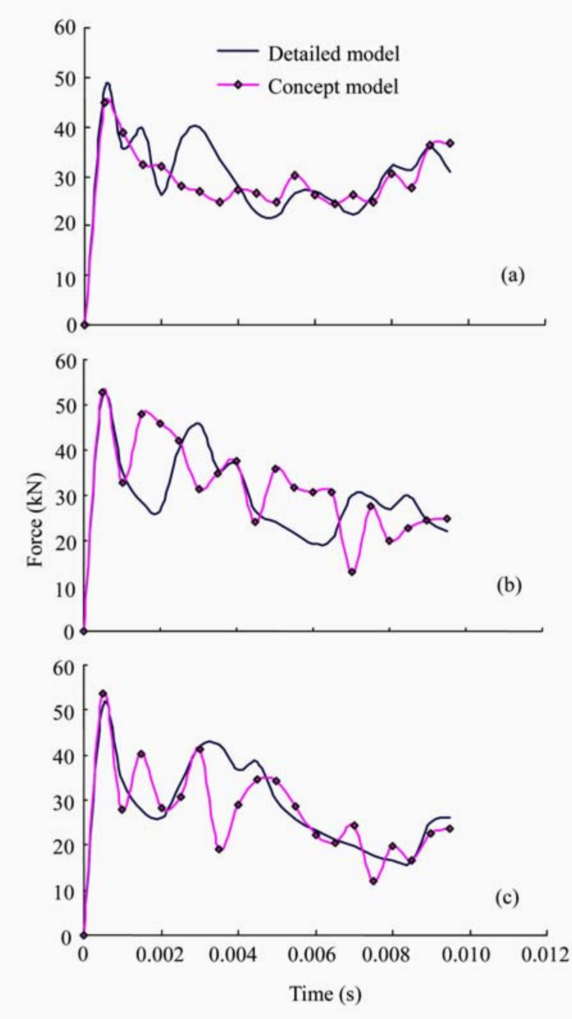


Fig.10 Crushing forces for tapered rectangular tubes. (a) α=5°; (b) α=10°; (c) α=15°

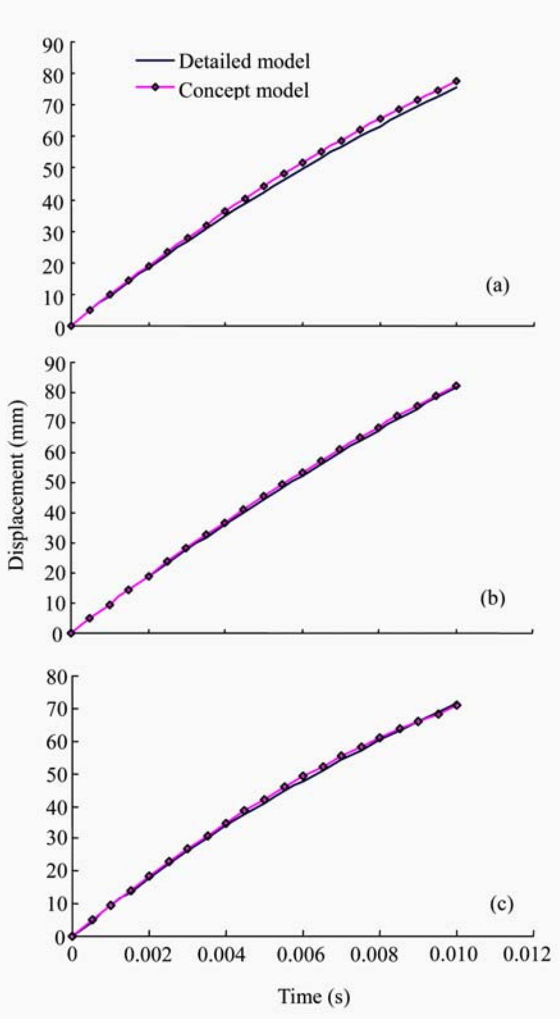


Fig.11 Displacements at the end of tapered rectangular tubes. (a) α=5°; (b) α=10°; (c) α=15°

Table 8 Comparison of crash results from detailed and concept tapered rectangular tubes

α (°)	Model	Peak crushing force (kN)	Displacement (mm)	Absorbed energy (kN)
5	Detailed	47.7	75.4	2.55
	Concept	44.8 (−6.1%)*	77.6 (2.9%)	2.58 (1.2%)
10	Detailed	51.6	81.6	2.68
	Concept	52.8 (2.3%)	82.2 (0.7%)	2.75 (2.6%)
15	Detailed	50.7	71.4	2.37
	Concept	53.5 (5.5%)	70.8 (−0.8%)	2.55 (7.6%)

$$^{*}\text{Difference} = \frac{\text{the value of concept model} - \text{the value of detailed model}}{\text{the value of detailed model}} \times 100\%$$

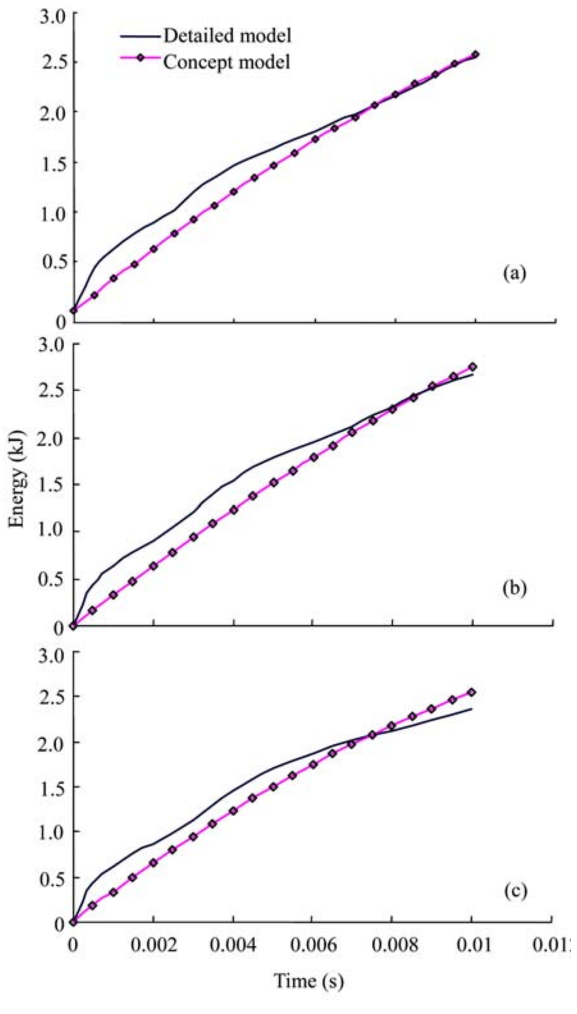


Fig.12 Absorbed energies for tapered rectangular tubes. (a) α=5°; (b) α=10°; (c) α=15°

From Figs.10~12 and Table 8, it is found that the differences between the results from the concept and detailed models are below 10%. Unlike the concept models for the tapered circular tubes, the concept models for the rectangular tubes can properly reflect the crushing force histories yielded from the detailed models. Thus, the peak crushing forces, instead of the mean crushing forces, are listed and compared in Table 8. Similar phenomena can also be observed in (Liu and Day, 2006). The reason of improvements in predicting the crushing forces of the rectangular tubes may be because the applied equations (Eqs.(4) and (5)) can better describe the crush characteristics of the rectangular tubes than the equations for the circular tubes (Eqs.(1) and (2)).

Similar to the tapered circular tubes, it is concluded that the concept models for the tapered rectangular tubes can correctly simulate the crash behavior of the detailed models and therefore can replace the existing detailed models for impact analyses and energy absorption. Compared with the detailed models, such concept models require less modeling labor and can greatly save computer resources. Therefore, the idea of creating concept models for tapered tubes has been verified and the concept models developed in this paper has been validated. The presented modeling method can then be applied for practical product design and simulation.

CONCLUSION

The modeling method presented in this paper can be used to develop the concept models for most tapered thin-walled tubes efficiently and correctly. The developed concept models can be applied for computer simulation and analysis, and can save considerable modeling labor and computer expense compared to the detailed models. Furthermore, the validation of such concept models reveals an important characteristic of the tapered tubes, that is, a tapered thin-walled tube during the crash can be assumed as the sum of several straight thin-walled tubes.

Also, the concept model can save a lot of computer resources during computer simulation because such concept model is a simplification of the detailed model. However, in creating a concept model for the impact analysis, the impact response of the detailed model has to be well understood, and current collapse theories and mathematic equations are required to determine the characteristics of the spring elements. Thus, compared with the detailed models, the presented concept models are quite "simple" in their profiles while really "complex" in the theories and mathematics behind them.

FUTURE WORK

The concept models presented in this paper faithfully simulate the axial crushing behavior of the tapered thin-walled tubes. In the future, the presented modeling method should be improved for the application to the cases of oblique impacts and tapered tubes with filler material.

Meanwhile, the concept models can be successfully used for low-velocity impact analyses (<15 m/s). For high-velocity impact cases, the high impact velocity does affect the buckling behavior of the tube through originating an elastic-plastic stress wave that propagates along square and circular tubes (Karagiozova and Jones, 2004; 2008). Karagiozova and Jones investigated the influence of the impact velocity on the axial crushing response of the tube by considering the transient deformation process and created finite element models to correctly simulate the response of the tube during an axial impact analysis with higher velocity. The methods and models presented in their works must be adopted into the concept modeling so that the concept models will also be able to simulate the high-velocity impact analysis.

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