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Finite particle method for kinematically indeterminate bar assemblies*

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Abstract: This study presents a structural analysis algorithm called the finite particle method (FPM) for kinematically indeterminate bar assemblies. Different from the traditional analysis method, FPM is based on the combination of the vector mechanics and numerical calculations. It models the analyzed domain composed of finite particles. Newton's second law is adopted to describe the motions of all particles. A convected material frame and explicit time integration for the solution procedure is also adopted in this method. By using the FPM, there is no need to solve any nonlinear equations, to calculate the stiffness matrix or equilibrium matrix, which is very helpful in the analysis of kinematically indeterminate structures. The basic formulations for the space bar are derived, following its solution procedures for bar assemblies. Three numerical examples are analyzed using the FPM. Results obtained from both the straight pretension cable and the suspension cable assembly show that the FPM can produce a more accurate analysis result. The motion simulation of the four-bar space assembly demonstrates the capability of this method in the analysis of kinematically indeterminate structures.

Key words: Finite particle method (FPM), Vector mechanics, Convected material frame, Explicit time integrations, Kinematically indeterminate bar assemblies

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INTRODUCTION

In contrast to kinematically determinate structures, kinematically indeterminate structures are special structures that consist of internal mechanism (Pellegrino and Calladine, 1986). Internal mechanism can be divided into infinitesimal mechanism (Type I) and finite mechanism (Type II). Type I, which is just movement trend, can be stiffened by self-stress. This kind of structures can usually be imposed by pre-stress, such as tensegrity, cable dome, cable truss. Type II can undergo non-strain motion, which exists in deployable structures (Chen and You, 2005; Luo and Mao, 2007).

For these kinematically indeterminate structures, the stiffness matrix may not have full rank. The displacement-based methods, such as the finite element method (FEM), which are the most efficient for statically determinate or redundant assemblies, become invalid. Although this problem can be solved numerically with special consideration, a literature survey shows that many researchers have paid attention to some other numerical methods. By using the generalized inverse, Tanaka and Hangai (1986) proposed a method called the generalized incremental method to analyze unstable truss structures. Zhao *et al.*(2006) developed this method and analyzed the deployable space structures with the generalized inverse theory. Many researchers paid attention to the force method in the analysis of kinematiclly indeterminate structure (Pellegrino *et al.*, 1992; Luo and Lu, 2006). But the above-mentioned methods are less convenience in terms of the matrix operation. The computation requires a large amount of computer memory and hence becomes computationally expensive.

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Different from the traditional method based on the classical mechanics and variational principles, Ting *et al.*(2004) proposed a new mechanics concept of structural analysis, the combination of the vector mechanics and numerical calculations. This new mechanics concept models the analyzed domain composed by finite particles instead of a continuous mathematical body. Newton's second law is adopted to describe the motions of all particles. A convected material frame (Shih *et al.*, 2004) and explicit time integration for the solution procedure is also included in this concept. Based on this concept, Wu *et al.*(2006) made a nonlinear motion analysis of 2D structures. Wang *et al.*(2005) explored the possibilities of vector form analysis of space truss structure in large elastic-plastic deformation. Later, Wang *et al.*(2006a) made a nonlinear analysis reticulated space truss structures. Furthermore, they (Wang *et al.*, 2006b) applied this concept to the simulation of the progressive failure and collapse of structure under earthquake loading.

This study presents a new analysis method called the finite particle method (FPM) based on the above mechanics concept. It can simulate the motion behavior of the kinematically indeterminate structures, and produce a more accurate analysis results. The layout of the paper is as follows. In Section 2, the basic formulations for the space bar are derived, which are followed by solution procedures of this method for bar assemblies in Section 3. A few examples are given to demonstrate the capabilities and accuracy of this method in the analysis of kinematically indeterminate structures in Section 4. Section 5 concludes the paper with discussion and some suggestions for future work.

FUNDAMENTALS OF THE FINITE PARTICLE METHOD

FPM is based on the vector mechanics and numerical calculations. The vector mechanics proposed by Ting *et al.*(2004) and Ting (2007) is a kind of physical mechanics. It models the analyzed domain to be composed by finite particles instead of mathematical functions and continuous bodies in traditional mechanics. The structure mass is assumed to be represented by each particle. Particles in the structure are connected by elements. Elements have no mass. Thus, they are in static equilibrium subjected to external forces and moments. Following these assumptions, the structure can be described by a set of discrete particles and elements as shown in Fig.1.

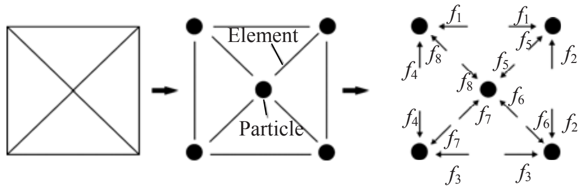

Fig.1 Simplified model of the structure

The motion of the arbitrary particle α follows Newton's second law:

$$\boldsymbol{M}_\alpha \ddot{\boldsymbol{d}}_\alpha = \boldsymbol{F}_\alpha^{\text{ext}} - \boldsymbol{F}_\alpha^{\text{int}}, \tag{1}$$

where $\boldsymbol{M}_\alpha$ is the mass value, $\boldsymbol{d}_\alpha$ is the displacement vector, $\ddot{\boldsymbol{d}}_\alpha$ is the acceleration vector, $\boldsymbol{F}_\alpha^{\text{ext}}$ and $\boldsymbol{F}_\alpha^{\text{int}}$ are the external force and internal force of particle α, respectively.

If the damping force is taken into consideration, the motion equation can be expressed as

$$\boldsymbol{M}_\alpha \ddot{\boldsymbol{d}}_\alpha = \boldsymbol{F}_\alpha^{\text{ext}} - \boldsymbol{F}_\alpha^{\text{int}} - \boldsymbol{F}_\alpha^{\text{dmp}}, \tag{2}$$

where the damping force $\boldsymbol{F}_\alpha^{\text{dmp}} = \mu \boldsymbol{M}_\alpha \dot{\boldsymbol{d}}_\alpha$, μ is the damping factor, which is the same as the definition in the dynamic relaxation method (Lewis, 1984).

In this study, we pay attention to space bar assemblies that only have three translation degrees of freedom on the node. Motion equations for space bar assemblies are

$$\begin{bmatrix} m_\alpha & 0 & 0 \\ 0 & m_\alpha & 0 \\ 0 & 0 & m_\alpha \end{bmatrix} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}_\alpha = \begin{bmatrix} f_x^{\text{ext}} \\ f_y^{\text{ext}} \\ f_z^{\text{ext}} \end{bmatrix}_\alpha - \begin{bmatrix} f_x^{\text{int}} \\ f_y^{\text{int}} \\ f_z^{\text{int}} \end{bmatrix}_\alpha - \begin{bmatrix} f_x^{\text{dmp}} \\ f_y^{\text{dmp}} \\ f_z^{\text{dmp}} \end{bmatrix}_\alpha . \tag{3}$$

From Eq.(3), it can be found that every particle is in a dynamic equilibrium state under the internal force and the external force of the particle. Many methods can be employed to find solutions for Eq.(3). To avoid the iteration in the solution procedure, the explicit time integrations are suggested here. If a simple central difference is adopted, the velocity and acceleration can be approximated as

$$\dot{\boldsymbol{d}}_n = \frac{1}{2\Delta t}(\boldsymbol{d}_{n+1} - \boldsymbol{d}_{n-1}), \tag{4}$$

$$\ddot{\boldsymbol{d}}_n = \frac{1}{\Delta t^2}(\boldsymbol{d}_{n+1} - 2\boldsymbol{d}_n + \boldsymbol{d}_{n-1}), \tag{5}$$

where $\boldsymbol{d}_{n+1}$, $\boldsymbol{d}_n$ and $\boldsymbol{d}_{n-1}$ are the displacements of an arbitrary particle at steps $n+1$, n and $n-1$, respectively; Δt is the constant time increment. Substituting Eqs.(4) and (5) into Eq.(2) yields

$$\begin{aligned}\boldsymbol{d}_{n+1} = &\left(\frac{2}{2+\mu\Delta t}\right)\frac{\Delta t^2}{m_\alpha}(\boldsymbol{F}_n^{\mathrm{ext}} - \boldsymbol{F}_n^{\mathrm{int}}) \\ &+\left(\frac{4}{2+\mu\Delta t}\right)\boldsymbol{d}_n - \left(\frac{2-\mu\Delta t}{2+\mu\Delta t}\right)\boldsymbol{d}_{n-1}.\end{aligned} \tag{6}$$

Eq.(6) is a simple and explicit formula, from which displacements of structures can be determined.

$\boldsymbol{F}_\alpha^{\mathrm{ext}}$ is the summation of the external forces acting on particle α, which can be either physical forces or equivalent force defined by mathematical concepts. The equivalent external force can be obtained by the principle of virtual work. The resulting element external forces have the same form to the results in the FEM which are omitted here.

$\boldsymbol{F}_\alpha^{\mathrm{int}}$ is the summation of the internal forces exerted by the elements connecting with the particle α. To calculate the internal forces, consider a 3D bar shown in Fig.2. The displacement vector of the end nodes 1 and 2 of this element at time t_a and t_b ($=t_a+\Delta t$) is defined as ($\boldsymbol{x}_1^a, \boldsymbol{x}_2^a$) and ($\boldsymbol{x}_1^b, \boldsymbol{x}_2^b$) respectively, as shown in Fig.2a. If the configuration of the element at t_a is chosen as the reference configuration, the relative displacement at nodes 1 and 2 between time t_a and t_b are $\mathrm{d}\boldsymbol{x}_1 = \boldsymbol{x}_1^b - \boldsymbol{x}_1^a$ and $\mathrm{d}\boldsymbol{x}_2 = \boldsymbol{x}_2^b - \boldsymbol{x}_2^a$, as shown in Fig.2b. Then taking the motions of node 1 as the rigid body translations of the element, the relative displacements of node 2 to node 1 between the reference configuration and the current configuration is $\mathrm{d}\boldsymbol{u}_2 = \mathrm{d}\boldsymbol{x}_2 - \mathrm{d}\boldsymbol{x}_1$. Since the internal forces are related to the deformation only, it is necessary to remove rigid body translations and rotations from the relative displacements. A simple kinematical formulation, the convected material frame, is suggested here. First, assume the element 1′2′ at t_b is subjected to a fictitious translation ($-\mathrm{d}\boldsymbol{x}_1$) and a fictitious reversed rotation ($-\Delta\theta$). Then the element 1′2′ is displaced to the position 1″2″, as shown in Fig.3a. The internal force is obtained in this configuration. The relative rigid body displacement increment $\mathrm{d}\boldsymbol{u}_2^{\mathrm{r}}$ can be obtained as follows (Goldstein *et al.*, 2002):

$$\mathrm{d}\boldsymbol{u}_2^{\mathrm{r}} = -(\boldsymbol{R}^{\mathrm{T}} - \boldsymbol{I})\mathrm{d}\boldsymbol{x}', \tag{7}$$

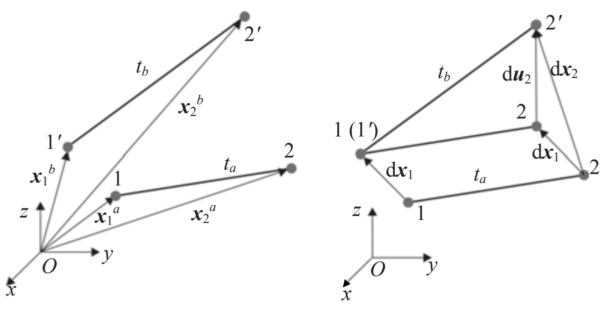

Fig.2 3D bar. (a) Displacements of particles at the two ends of the element; (b) Relative displacements of particles

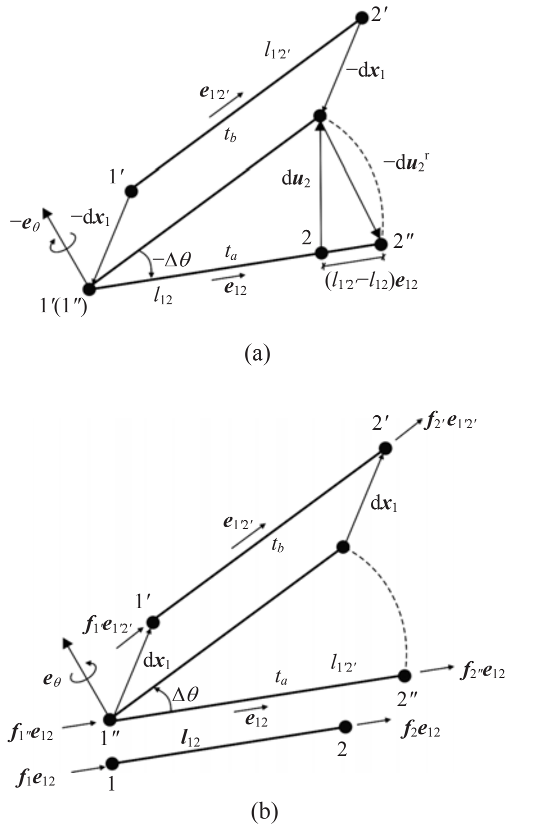

Fig.3 Illustration of particle internal force calculations. (a) Reversed rotation ($-\Delta\theta$) and translation ($-\mathrm{d}\boldsymbol{x}_1$); (b) Rotation ($\Delta\theta$) and translation ($\mathrm{d}\boldsymbol{x}_1$)

where $\boldsymbol{I}$ is a 3×3 unit matrix; $\boldsymbol{R}$ is the rotation matrix of $\Delta\theta$, $\Delta\theta$ is the angle between element 12 and 1′2′; $\mathrm{d}\boldsymbol{x}'$ is the position vector of node 2 in the local deformation coordinate at time t_b. $\mathrm{d}\boldsymbol{x}'=[l_{1'2'}\ 0\ 0]^{\mathrm{T}}$, where $l_{1'2'}$ is the length of element 12 at time t_b. Therefore, the relative displacement increment of deformation is

$$\mathrm{d}\boldsymbol{u}_2^{\mathrm{d}} = \mathrm{d}\boldsymbol{u}_2 + \mathrm{d}\boldsymbol{u}_2^{\mathrm{r}} = \mathrm{d}\boldsymbol{x}_2 - \mathrm{d}\boldsymbol{x}_1 - (\boldsymbol{R}^{\mathrm{T}} - \boldsymbol{I})\mathrm{d}\boldsymbol{x}'. \quad (8)$$

As a bar element, its deformation is only related to the variation of the bar length. Thus, instead of using Eq.(8), we can get the incremental deformation of a bar from the following equation:

$$\mathrm{d}\boldsymbol{u}_2^{\mathrm{d}} = (l_{1'2'} - l_{12})\boldsymbol{e}_{12}, \quad (9)$$

where l_{12} and $l_{1'2'}$ are the length of element 12 at time t_a and t_b, respectively. $\boldsymbol{e}_{12}$ is the directional vector of element 12 at time t_a.

After the fictitious rotation, element 1″2″ is parallel to element 12 (Fig.3b), which satisfied the basic assumptions of the material mechanics. The axial force of the element is

$$\boldsymbol{f}_{2''} = \boldsymbol{f}_a + \Delta\boldsymbol{f}_a = \left[\sigma_a A_a + \frac{E_a A_a}{l_{12}}(l_{1'2'} - l_{12})\right]\boldsymbol{e}_{12}, \quad (10)$$

where $\boldsymbol{f}_a$ is the axial force of element 12 at time t_a; $\Delta\boldsymbol{f}_a$ is the incremental axial force of element 12 at time t_b; σ_a is the axial stress at time t_a; E_a is Young's modulus; A_a is the section area of element 12. According to the static equilibrium condition, there is

$$\boldsymbol{f}_{1''} = -\boldsymbol{f}_{2''}. \quad (11)$$

As the axial force at the fictitious position is obtained, the element should be subjected a rotation $\Delta\theta$ and a translation $\mathrm{d}\boldsymbol{x}_1$ and gets back to its original position. In this process, only the direction of the element axial force is changed. Then we can get the real element axial force (Fig.3b):

$$\boldsymbol{f}_{2'} = -\boldsymbol{f}_{1'} = \left[\sigma_a A_a + \frac{E_a A_a}{l_{12}}(l_{1'2'} - l_{12})\right]\boldsymbol{e}_{1'2'}, \quad (12)$$

where $\boldsymbol{e}_{1'2'}$ is the directional vector of element 12 at time t_b. The axial force $\boldsymbol{f}_{1'}$ and $\boldsymbol{f}_{2'}$ is applied to the corresponding particle, respectively. The internal force of a particle can be identified by the summation of all axial forces of elements connected to the corresponding particle.

COMPUTATIONAL PROCEDURES FOR THE FINITE PARTICLE METHOD

The computational procedure of the FPM for bar assemblies is very simple. It can be summarized as follows.

(1) At time t=0, give all initial input information, such as the initial displacements, external force, constraint conditions. According to Eq.(6), $\boldsymbol{d}_{-1}$ is needed which can be obtained from the following equation:

$$\boldsymbol{d}_{-1} = \boldsymbol{d}_0 - \Delta t\dot{\boldsymbol{d}}_0 + \frac{1}{2}\Delta t^2 \boldsymbol{d}_0. \quad (13)$$

Substituting $\boldsymbol{d}_{-1}$, $\boldsymbol{d}_0$, $\boldsymbol{F}_0^{\mathrm{ext}}$ and $\boldsymbol{F}_0^{\mathrm{int}}$ into Eq.(6), we can get $\boldsymbol{d}_1$. Store $\boldsymbol{d}_0$ and $\boldsymbol{d}_1$, then move forward to the next step;

(2) At any step n, update positions of all nodes first;

(3) Determine the new axial force of each element at new position from Eqs.(9)~(12). Assemble them for each particle internal force $\boldsymbol{F}_n^{\mathrm{int}}$;

(4) If external forces are changed, update them;

(5) If mass values of particles are changed, update them;

(6) Substituting $\boldsymbol{d}_{n-1}$, $\boldsymbol{d}_n$, $\boldsymbol{F}_n^{\mathrm{ext}}$ and $\boldsymbol{F}_n^{\mathrm{int}}$ into Eq.(6), we can get $\boldsymbol{d}_{n+1}$. Store $\boldsymbol{d}_n$ and $\boldsymbol{d}_{n+1}$, then move forward to the next step;

(7) If time is less than the ending time, go to Step (2). Otherwise, stop.

NUMERICAL EXAMPLES

Three examples are presented in this section. The first one is dealt with a bar assembly with an infinitesimal mechanism. The other two refer to kinematically indeterminate structures with non-strain motion. These examples implied that the FPM is a very effective method in the analysis of kinematically indeterminate assemblies.

Example 1 A straight cable is under a load vertically acting at its center (Fig.4). Let load W=311.38 N, EA=564.92 N (E is Young's modulus of the cable material, A is the area of the cable section), the total length of the cable is 10160 mm, the initial tension is 4448.2 N. There is an infinitesimal mechanism existing in the structure. The traditional FEM could not be used in this example. The theoretical results are a vertical central displacement of 166.54 mm (Levy and Spillers, 1995). According to the FPM, the central deflection is 167.56 mm with an error less than 1%.

Example 2 A three-cable assembly, identical to that analyzed in Pellegrino (1990) with the force method (FM) is given in Fig.5. A pair of equal forces W is applied at nodes 1 and 2. The cable connected to the left fixed end is shortened by Δ=10, 20, 30 mm, respectively. We shall calculate the final shape of the assembly.

The results of nodal displacements from the FM and the FPM are given in Table 1, and the lengths of the cables are given in Table 2. They are all compared with the experimental results, parts of which (Δ=10 mm) were provided by Pellegrino (1990), whereas the rest were conducted by the authors. It is assumed that the experimental results are precise.

It has been found that when the initial shortening

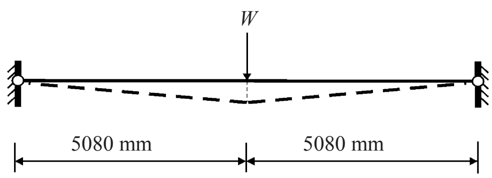

Fig.4 A straight pretension cable under a concentrated load at the center

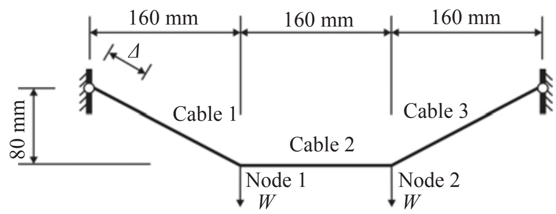

Fig.5 Suspension cable assembly

Table 1 Displacements for the cable assembly shown in Fig.5

Cable 1 shortening (mm)	Nodal displacement	FM result (mm)	FPM result (mm)	Experiment value (mm)	FM error (%)	FPM error (%)
10	d_{1x}	−5.160	−5.062	−5.0	3.20	−0.62
	d_{1y}	12.040	12.783	12.5	−3.68	2.26
	d_{2x}	−5.160	−5.068	−5.5	−6.18	−7.85
	d_{2y}	10.320	11.048	11.0	−6.18	−0.44
20	d_{1x}	−10.320	−9.796	−9.5	8.63	3.11
	d_{1y}	24.081	28.006	27.0	−10.81	3.73
	d_{2x}	−10.320	−9.847	−10.0	3.20	−1.53
	d_{2y}	20.641	23.914	24.0	−14.00	−0.36
30	d_{1x}	−15.480	−14.966	−15.0	3.20	−0.23
	d_{1y}	36.121	46.357	44.5	−18.83	4.17
	d_{2x}	−15.480	−14.986	−15.0	3.20	−0.09
	d_{2y}	30.961	42.740	41.5	−25.40	2.99

Table 2 Length of cables for the cable assembly shown in Fig.5

Cable 1 shortening (mm)	Cable number	FM result (mm)	FPM result (mm)	Theoretical value (mm)	FM error (%)	FPM error (%)
10	Cable 1	169.10	168.89	168.89	0.12	0.0
	Cable 2	169.01	160.00	160.00	5.63	0.0
	Cable 3	179.26	178.89	178.89	0.21	0.0
20	Cable 1	159.78	158.89	158.89	0.56	0.0
	Cable 2	160.04	160.00	160.00	0.02	0.0
	Cable 3	180.37	178.89	178.89	0.83	0.0
30	Cable 1	151.03	148.89	148.89	1.44	0.0
	Cable 2	160.08	160.00	160.00	0.05	0.0
	Cable 3	182.20	178.89	178.89	1.85	0.0

of the cable Δ is small, the displacements obtained from both the FM and the FPM are comparable, and close to the experimental results. The errors with the FM increase as Δ becomes large. When Δ=30 mm, the FM has up to 25.4% error in comparison with only 4.17% with the FPM. Furthermore, the FPM gives almost accurate cable length after the cables are shortened, while the FM has error up to 5.63%. Thus, the FPM can provide a very accurate analysis.

Example 3 A space assembly made of four pin-jointed bars is given in Fig.6. The coordinates of A, B, C, D are (−1, 0, 1), (0, 1, 0), (1, 0, 1) and (0, −1, 0), respectively. The gravity of the assembly is not considered here. All the members are non-physical and thus the units are omitted. A couple of force F and F' with the opposite direction and the same value 100 is applied on points A and C, respectively. The duration of F and F' are 50. The material properties of the four bars are the same to each other. Suppose Young's modulus E=1, mass density $\rho=1\times10^{-3}$. We set the section area with a relative large value $A=1\times10^{5}$. Thus there is no need to take the axial deformations into account in the analysis. A time step $\Delta t=1\times10^{-4}$ is used. Set the damping factor μ=0, 0.01, 0.1, and 1, respectively. The motions of the assembly are simulated with the FPM.

The motion trajectory of the assembly with μ=1 is shown in Fig.7. When the structure reaches the final static configuration, the four bars are on a straight line. The results of motion responses with different μ are given in Fig.8. It shows that the distance of bar AB is a constant, which implies that the assembly undergoes rigid body motions under external forces and that there is no deformation in it. When μ=0, the assembly always vibrates around the equilibrium configuration and cannot reach a static configuration (Fig.8a). As μ increases, the amplitude of the assembly reduces gradually (Figs.8b and 8c). And the assembly can be static in the equilibrium configuration at last. When the value of μ is large enough, the assembly will reach the final configuration in a very short time (Fig.8d). From the results of the analysis, it can be found that the FPM can simulate the motion behavior of the four-bar assembly effectively.

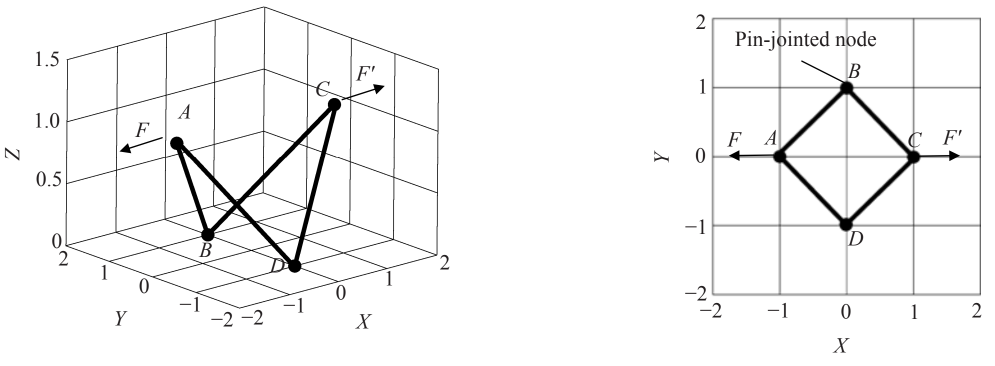

Fig.6 A space assembly made from four pin-jointed bars. (a) Axonometric view; (b) Plane view

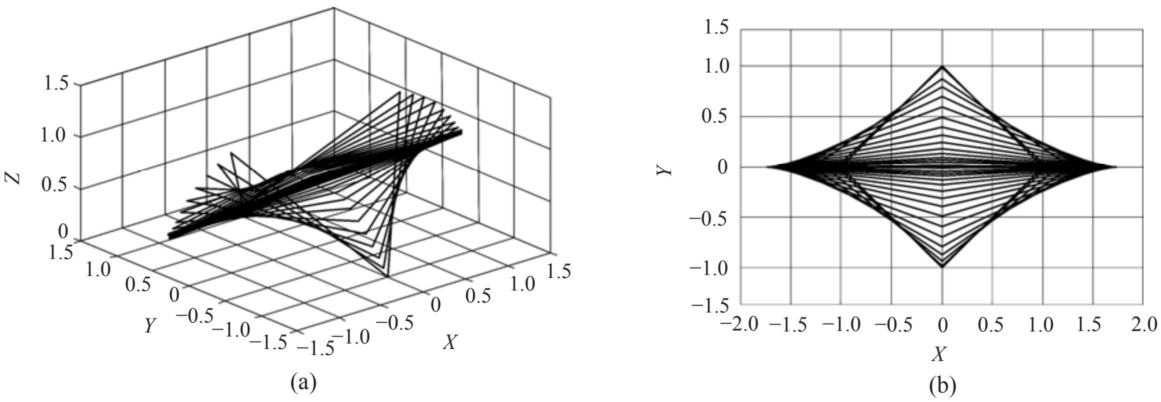

Fig.7 Motion trajectory of the four-bar assembly with μ=1. (a) Axonometric view; (b) Plane view

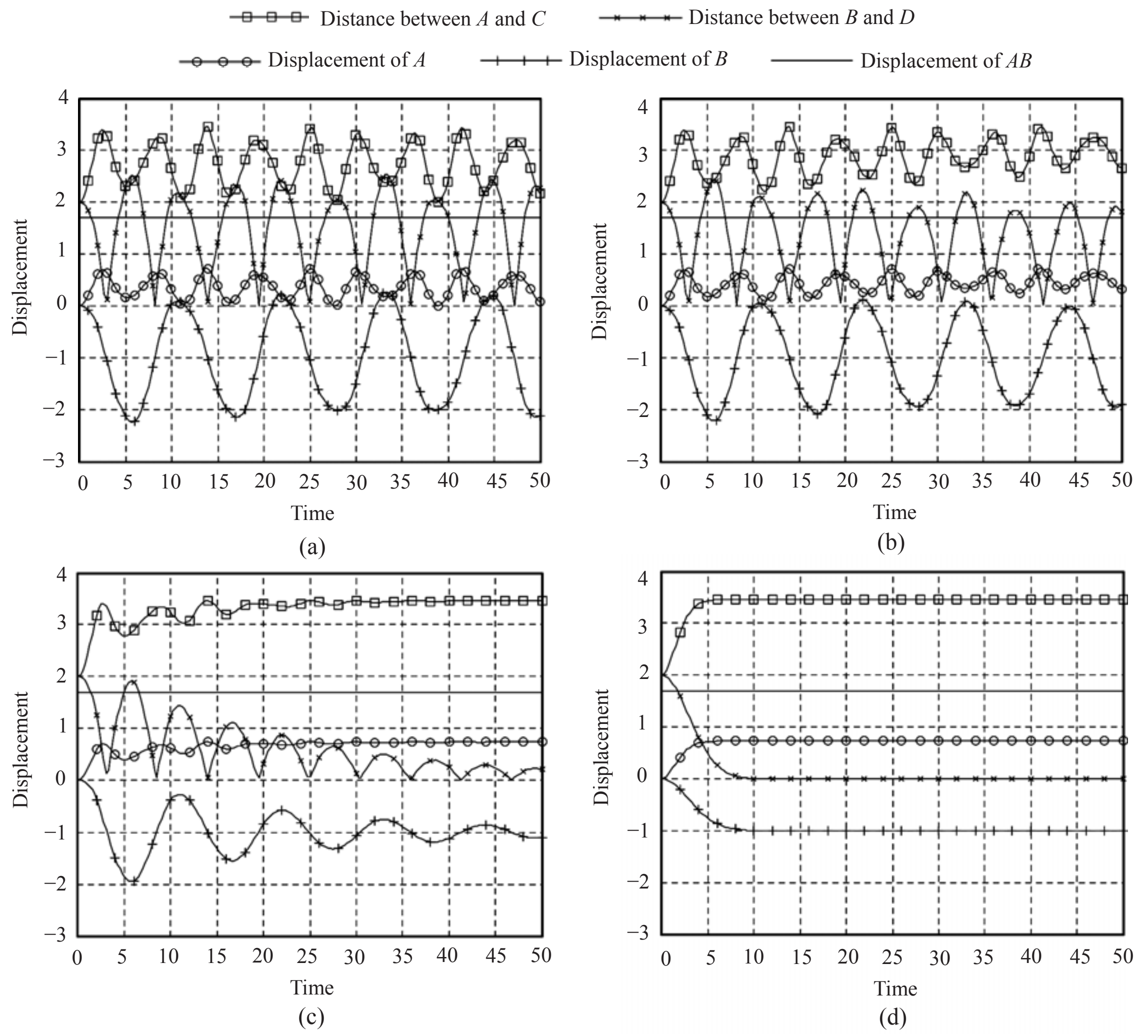

Fig.8 Variations of the motion response under external forces with different damping factor μ. (a) μ=0; (b) μ=0.01; (c) μ= 0.1; (d) μ=1

CONCLUSION

The FPM, a new algorithm for structure analysis, is presented in this study. This method is based on a new concept of mechanics. It is very easy to prescribe the force with the convected material frame and to calculate the displacement with the motion equation on each particle during the procedure of analysis. The formulations of space bar for kinematically indeterminate assemblies are derived and the basic computational procedures are also given.

By using the FPM, there is no need to solve nonlinear equations, or calculate the singular value decomposition of the equilibrium matrix. This method can simulate the motion behavior of the kinematically indeterminate structures, and produce a more accurate analysis result. Thus, it has the inherent advantage in the analysis of kinematically indeterminate structures.

However, the FPM has some problems on the computation time. A very small time step in the explicit time integrations has to be set. The analysis will be time-consuming as the number of particles increases. Thus, in the analysis of ordinary statically determinate or redundant assemblies, the FPM is not a replacement of traditional analysis method, such as the FEM. Instead, it is effective and accurate in the analysis of non-traditional structures, such as tension structures and structures with internal infinitesimal mechanisms or rigid body motions.

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