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Finite element reliability analysis of slope stability*

Xiao-hui TAN†1, Jian-guo WANG[2]

([1]School of Resource and Environmental Engineering, Hefei University of Technology, Hefei 230009, China)

([2]School of Civil Engineering, Hefei University of Technology, Hefei 230009, China)

†E-mail: tantan9666@126.com

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Abstract: The method of nonlinear finite element reliability analysis (FERA) of slope stability using the technique of slip surface stress analysis (SSA) is studied. The limit state function that can consider the direction of slip surface is given, and the formulations of FERA based on incremental tangent stiffness method and modified Aitken accelerating algorithm are developed. The limited step length iteration method (LSLIM) is adopted to calculate the reliability index. The nonlinear FERA code using the SSA technique is developed and the main flow chart is illustrated. Numerical examples are used to demonstrate the efficiency and robustness of this method. It is found that the accelerating convergence algorithm proposed in this study proves to be very efficient for it can reduce the iteration number greatly, and LSLIM is also efficient for it can assure the convergence of the iteration of the reliability index.

Key words: Slope stability, Finite element method, Reliability analysis, Limited step length iteration method (LSLIM), Accelerating convergence

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INTRODUCTION

Due to uncertainties of soil parameters and load conditions, it is very important to carry out reliability analysis of slopes. There are several reliability analysis methods in structural engineering, such as the first order reliability analysis method (FORM), Monte-Carlo simulation method and response surface method, among which FORM is the most popular and it is used in this study. The most frequently used slope stability analysis method of reliability analysis is the limit equilibrium method (Duncan, 1996; Auvinet and Gonzalez, 2000; Bhattacharya *et al*., 2003). With the development of numerical computation methods, some researchers begin to use the finite element method to carry out reliability analysis of slopes. In finite element analysis (FEA), soil mass is usually considered as elasto-plastic material, so the procedure of reliability analysis of slope stability belongs to the category of nonlinear finite element reliability analysis (FERA). There are mainly two kinds of finite element methods of slope stability, one is based on the technique of slip surface stress analysis (SSA) (Kim and Li, 1997), and the other is based on the technique of the strength reduction method (Griffiths and Lane, 1999; Swan and Seo, 1999; Manzari and Nour, 2000; Chugh, 2003; Zheng and Liu, 2005). Mellah *et al*.(2000) analyzed the stability of a slope and computed the contour of the reliability index using the first method. In that article, soil parameters are considered as stochastic variables. Griffiths and Fenton (2004) calculated the failure probability of a slope using the second method, and soil parameters are considered as random fields. In this study, the SSA technique is adopted to carry out FERA of slope stability since it can calculate both the overall reliability index and the position of the corresponding slip surface.

An essential step in FORM is the determination of the so-called design point $\boldsymbol{x}^*$ for a limit state

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function $Z=g(\boldsymbol{X})$. This design point is the point on the surface $g(\boldsymbol{X})=0$ that has the maximum probability density in a transformed standard normal space. This point is used to construct first order approximations of the limit state function. In order to get the value of the design point, repeated evaluations of $g(\boldsymbol{X})$ and its gradient $\nabla g(\boldsymbol{X})$ at a sequence of trial points are required (Koduru and Haukaas, 2006).

In the nonlinear FERA using the SSA technique, there are several methods for computing $\nabla g(\boldsymbol{X})$, e.g., the perturbation method, the Neumann expansion method, the partial differentiation method (Yamazaki and Shinozuka, 1988; Bhattacharyya and Chakraborty, 2002; Imai and Frangopol, 2000). Among these methods, the partial differentiation method is very efficient and accurate, but it is still time-consuming because it is composed of multiple iterative loops for the searching of the design point. Therefore, it is necessary to improve the solution algorithm of the FERA of slope stability.

Since the limit state function of slope stability analysis is a complicate implicit function of soil parameters, it may lead to non-convergence in the searching of a design point. Thus, new algorithms that can assure the convergence should be studied.

In order to solve the problems mentioned above, an accelerating convergence algorithm of the nonlinear FERA that can speed up the running of nonlinear FERA code is presented, and another iterative algorithm of FORM—limited step length iteration method (LSLIM) is proposed to perform the reliability analysis of slope stability. During the FEA, the soil mass is assumed to be an elastic-perfectly plastic material and the Mohr-Coulomb failure criterion is adopted. The limit state function is proposed and its gradients with respect to basic variables are developed.

LIMIT STATE FUNCTION AND ITS GRADIENTS WITH RESPECT TO BASIC VARIABLES

Limit state function based on the SSA technique

Assuming the soil mass is an elastic-perfectly plastic material and meets the Mohr-Coulomb failure criterion, the limit state function that can consider the slip direction of an arbitrary slip surface can be defined as follows (Tan, 2007):

$$Z = g(\boldsymbol{X}) = \sum_{i=1}^{n_e} Z_i \Delta l_i, \tag{1}$$

where

$$\begin{aligned} Z_i = g_i(\boldsymbol{X}) = & -[(\sigma_x + \sigma_y)/2 - \cos(2\theta)\cdot(\sigma_x - \sigma_y)/2 \\ & + \tau_{xy}\sin(2\theta)]_i \tan\varphi_i + c_i - [\sin(2\theta)\cdot(\sigma_x - \sigma_y)/2 \\ & + \tau_{xy}\cos(2\theta)]_i, \end{aligned} \tag{2}$$

where $Z_i=g_i(\boldsymbol{X})$ and Δl_i represent the limit state function and the length of the slip surface of the ith finite element, respectively. $\boldsymbol{X}=(X_1, X_2, \ldots, X_n)$ represents the vector of basic variables, which consists of n soil variables, e.g., c, φ, ψ, γ, E, μ (where c, φ, ψ, γ, E, μ represent cohesion, internal friction angle, expansive angle, unit weight, elastic module, Poisson's ratio, respectively); n_e represents the number of finite elements on the slip surface; σ_x, σ_y, τ_{xy} and θ are the stress components and the angle of the slip surface to the horizontal plane of the ith element, respectively.

Gradients of limit state function with respect to basic variables

Defining stress vector $\boldsymbol{\sigma}=(\sigma_x, \sigma_y, \tau_{xy})^{\mathrm{T}}$, the gradients of limit state function with respect to the basic variables can be given as follows from Eqs.(1)~(2):

$$\nabla g(\boldsymbol{X}) = \sum_{i=1}^{n_e} \nabla_i g(\boldsymbol{X}) \Delta l_i, \tag{3}$$

where

$$\nabla_i g(\boldsymbol{X}) = \frac{\partial g_i(\boldsymbol{X},\boldsymbol{\sigma})}{\partial \boldsymbol{X}} + \frac{\partial g_i(\boldsymbol{X},\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{X}}, \tag{4}$$

Therefore, the main effort in evaluating $\nabla g(\boldsymbol{X})$ is devoted to computing the matrix $\dfrac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{X}}$.

Gradients of displacement increment with respect to basic variables

For the computation of $\dfrac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{X}}$, we must compute the displacement increment, stress increment, and their gradients with respect to basic variables.

There are several procedures to calculate the displacement increment and stress increment in the deterministic finite element method, e.g., the incremental tangent stiffness method, the incremental initial stress method, the incremental initial strain method (Wang and Shao, 1988). In this study the incremental tangent stiffness method is used and modified for the calculation of the displacement increment, stress increment, and their gradients with respect to basic variables.

The iterative equilibrium equation of the incremental tangent stiffness method is:

$$(\boldsymbol{K}_0)^{k+1}\Delta\boldsymbol{d}_{i+1}^{k+1}=\boldsymbol{F}^{k+1}+\boldsymbol{R}_i^{k+1}, \tag{5}$$

then

$$\Delta\boldsymbol{d}_{i+1}^{k+1}=[(\boldsymbol{K}_0)^{k+1}]^{-1}(\boldsymbol{F}^{k+1}+\boldsymbol{R}_i^{k+1}), \tag{6}$$

$$\left(\frac{\partial\Delta\boldsymbol{d}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}=[(\boldsymbol{K}_0)^{k+1}]^{-1}\left[\left(\frac{\partial\boldsymbol{F}}{\partial\boldsymbol{X}}\right)^{k+1}+\left(\frac{\partial\boldsymbol{R}}{\partial\boldsymbol{X}}\right)_i^{k+1}-\left(\frac{\partial\boldsymbol{K}_0}{\partial\boldsymbol{X}}\right)^{k+1}\Delta\boldsymbol{d}_{i+1}^{k+1}\right], \tag{7}$$

where k (k=0, 1, …, N−1; N is the number of total load steps) and i (i=0, 1, …) represent the number of load increment and the number of iteration of nonlinear equations, respectively; $\boldsymbol{K}_0$ is the initial tangent stiffness matrix; $\Delta\boldsymbol{d}$ is the displacement increment; $\boldsymbol{R}$ is the vector of internal force.

Assume a_i represent the modified accelerating factor, the updated displacement of the modified Aitken accelerating convergence algorithm is given as follows:

$$\Delta\tilde{\boldsymbol{d}}_{i+1}^{k+1}=a_i\Delta\boldsymbol{d}_{i+1}^{k+1}, \tag{8}$$

then

$$(\boldsymbol{K}_0)^{k+1}\Delta\tilde{\boldsymbol{d}}_{i+1}^{k+1}=a_i(\boldsymbol{F}^{k+1}+\boldsymbol{R}_i^{k+1}), \tag{9}$$

therefore

$$\left(\frac{\partial\Delta\tilde{\boldsymbol{d}}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}=a_i(\boldsymbol{K}_0^{k+1})^{-1}\left[\left(\frac{\partial\boldsymbol{F}}{\partial\boldsymbol{X}}\right)^{k+1}+\left(\frac{\partial\boldsymbol{R}}{\partial\boldsymbol{X}}\right)_i^{k+1}-\left(\frac{\partial\boldsymbol{K}_0}{\partial\boldsymbol{X}}\right)^{k+1}\Delta\boldsymbol{d}_{i+1}^{k+1}\right]$$
$$=a_i\left(\frac{\partial\Delta\boldsymbol{d}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}. \tag{10}$$

From Eqs.(8) and (10), we can find that the accelerating method of $\Delta\boldsymbol{d}_{i+1}^{k+1}$ is the same as that of $\left(\frac{\partial\Delta\boldsymbol{d}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}$. Therefore, the accelerating procedure of $\Delta\boldsymbol{d}_{i+1}^{k+1}$ and $\left(\frac{\partial\Delta\boldsymbol{d}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}$ can be carried out synchronously.

Gradients of stress increment with respect to basic variables

Based on the relationship of stress increment $\Delta\boldsymbol{\sigma}$ and displacement increment $\Delta\boldsymbol{d}$:

$$\Delta\boldsymbol{\sigma}=\boldsymbol{D}_{\text{ep}}\boldsymbol{B}\Delta\boldsymbol{d}, \tag{11}$$

and the general expression of $\boldsymbol{D}_{\text{ep}}$:

$$\boldsymbol{D}_{\text{ep}}=\boldsymbol{D}_{\text{e}}-(1-m)\boldsymbol{D}_{\text{p}}, \tag{12}$$

we can get

$$(\Delta\boldsymbol{\sigma})_{i+1}^{k+1}=\boldsymbol{D}_{\text{e}}\boldsymbol{B}(\Delta\boldsymbol{d})_{i+1}^{k+1}-(1-m)(\boldsymbol{D}_{\text{p}})_{i+1}^{k+1}\boldsymbol{B}(\Delta\boldsymbol{d})_{i+1}^{k+1}, \tag{13}$$

where $\boldsymbol{B}$ is the strain matrix; $\boldsymbol{D}_{\text{ep}}$ is the elasto-plastic matrix; $\boldsymbol{D}_{\text{e}}$ and $\boldsymbol{D}_{\text{p}}$ are the elastic and plastic matrixes, respectively; m is a factor of proportionality.

By differentiation, we can get

$$\left(\frac{\partial\Delta\boldsymbol{\sigma}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}=\frac{\partial\boldsymbol{D}_{\text{e}}}{\partial\boldsymbol{X}}\boldsymbol{B}\Delta\boldsymbol{d}_{i+1}^{k+1}+(1-m)\left(\frac{\partial\boldsymbol{D}_{\text{p}}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}\boldsymbol{B}\Delta\boldsymbol{d}_{i+1}^{k+1}$$
$$-(1-m)(\boldsymbol{D}_{\text{p}})_{i+1}^{k+1}\boldsymbol{B}\left(\frac{\partial\Delta\boldsymbol{d}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1}+\boldsymbol{D}_{\text{e}}\boldsymbol{B}\left(\frac{\partial\Delta\boldsymbol{d}}{\partial\boldsymbol{X}}\right)_{i+1}^{k+1} \tag{14}$$
$$+\frac{\partial m}{\partial\boldsymbol{X}}(\boldsymbol{D}_{\text{p}})_{i+1}^{k+1}\boldsymbol{B}\Delta\boldsymbol{d}_{i+1}^{k+1}.$$

Since $\boldsymbol{D}_{\text{p}}$ is the function of $\boldsymbol{X}$, m and $\boldsymbol{\sigma}$, the gradient of $\boldsymbol{D}_{\text{p}}$ with respect to $\boldsymbol{X}$ will take the form of Eq.(15) by use of the chain rule of differentiation:

$$\frac{\partial\boldsymbol{D}_{\text{p}}}{\partial\boldsymbol{X}}=\frac{\partial\boldsymbol{D}_{\text{p}}(\boldsymbol{X},\boldsymbol{\sigma})}{\partial\boldsymbol{X}}+\frac{\partial\boldsymbol{D}_{\text{p}}(\boldsymbol{X},\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}}\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{X}}. \tag{15}$$

The detailed expressions of m, $\boldsymbol{D}_{\text{p}}$, $\frac{\partial\boldsymbol{D}_{\text{p}}(\boldsymbol{X},\boldsymbol{\sigma})}{\partial\boldsymbol{X}}$ and $\frac{\partial\boldsymbol{D}_{\text{p}}(\boldsymbol{X},\boldsymbol{\sigma})}{\partial\boldsymbol{\sigma}}$ were deduced by Tan (2007).

METHODS OF RELIABILITY ANALYSIS

In structural reliability analysis using the FORM, the most popular iterative searching algorithm of the design point is perhaps the so-called first order second moment method (FOSM), which was originally developed by Hasofer and Lind (1974), and later extended to non-normal random variables by Rackwitz and Fiessler (1978). However, it has been demonstrated that if the limit state function is highly nonlinear, the iterative computation of the design point and reliability index will probably be divergent in the FOSM algorithm. Since limit state function of the FERA of slope stability is usually a highly nonlinear implicit function of soil parameters, it is necessary to study some new iterative algorithms to assure the successful running of the nonlinear FERA code. In this study, a new iterative algorithm—LSLIM, is presented to carry out the reliability analysis.

LSLIM is another searching algorithm of FORM. The iterative procedure in standard normal space has been listed by Gong (2003). Since the FERA for the calculation of stress increment is carried out in the original space, the iteration of design point and reliability index should also be performed in the original space. When basic variables are independent normally distributed variables, the iterative formulas are derived as follows:

$$\alpha_{X_i}^{(k+1)}=\frac{\dfrac{x_i^{(k)}-\mu_{X_i}}{\sigma_{X_i}}-\lambda^{(k)}\dfrac{\partial g(\boldsymbol{x}^{(k)})}{\partial x_i}\sigma_{X_i}}{\sqrt{\sum_{j=1}^{n}\left(\dfrac{x_j^{(k)}-\mu_{X_j}}{\sigma_{X_j}}-\lambda^{(k)}\dfrac{\partial g(\boldsymbol{x}^{(k)})}{\partial x_j}\sigma_{X_j}\right)^2}},\quad(16a)$$

$$\beta^{(k+1)}=-\frac{g_X(x_1^{(k)},\ \dots,\ x_n^{(k)})+\sum_{i=1}^{n}\dfrac{\partial g(\boldsymbol{x}^{(k)})}{\partial x_i}(\mu_{X_i}-x_i^{(k)})}{\sum_{i=1}^{n}\alpha_{X_i}^{(k+1)}\dfrac{\partial g(\boldsymbol{x}^{(k)})}{\partial x_i}\sigma_{X_i}},\quad(16b)$$

$$X_i^{(k+1)}=\mu_{X_i}+\alpha_{X_i}^{(k+1)}\sigma_{X_i}\beta^{(k+1)},\quad(16c)$$

$$\lambda^{(k+1)}=\lambda^{(k)}/f,\quad(16d)$$

where k is the iteration step, $\lambda^{(k)}$ is the step length of the kth iteration, f is the step length adjusting coefficient.

COMPUTATION OF THE OVERALL ELIABILITY INDEX OF SLOPE STABILITY

Based on the theoretical deduction above, we compiled the code for computing the overall reliability index of slope stability using FERA based on the SSA technique. The code is written in VISUAL FORTRAN 6.5, and it is composed of three main loops: (1) The iterative loop of design point and reliability index. This is the outmost loop of the program. The flow chart of the main program using the LSLIM algorithm is illustrated in Fig.1, where k means the number of iterations and ε is the tolerance error; (2) The iterative loop of nonlinear FERA using the incremental tangent stiffness method and accelerating convergence algorithm (it consists of several other loops, such as the loop of load increments, the loop of each element, and the loop of each Gauss integral point). The task of this loop is to calculate the vector of stress $\boldsymbol{\sigma}$ and its gradient $\dfrac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{X}}$ at each trial point $\boldsymbol{x}^*$. This loop contains nonlinear finite element computation and it is the most time-consuming part of the whole program; (3) The searching loop of the minimum reliability index of all geometrically possible slip surfaces.

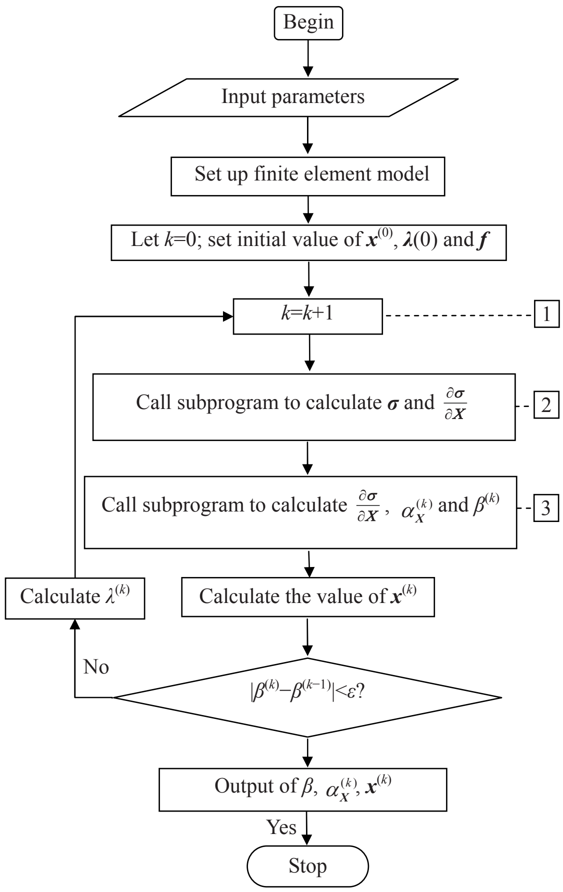

Fig.1 Flow chart of main program using LSLIM

EXAMPLES

To verify the efficiency and robustness of the proposed method and the corresponding nonlinear FERA code developed by the authors, two examples were presented.

Example 1 There is a three-layer slope. The cross section of this slope is illustrated in Fig.2 and soil parameters are listed in Table 1. Consider φ_1, c_2, φ_2, c_3, φ_3 are normally distributed random variables whose statistical properties are listed in Table 2. The coefficients of correlation are $\rho(c_2, \varphi_2)$=0.1 and $\rho(c_3, \varphi_3)$= 0.3. For the special slip surface shown in Fig.2, the reliability index (β) is 4.963 according to Chen (2003).

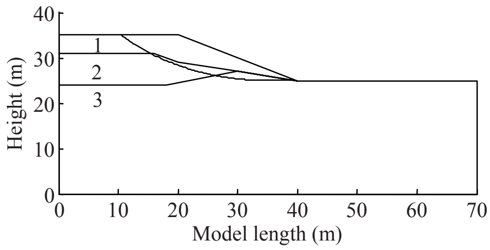

Fig.2 Cross section and slip surface of the slope for Example 1. 1, 2, 3: soil layer

Table 1 Soil parameters for Example 1

Soil layer	c (kPa)	φ (°)	ψ (°)	γ (kN/m^3)	E (kPa)	μ
1	0.0	38.0	0.0	19.5	1.0E+4	0.25
2	5.3	23.0	0.0	19.5	1.0E+4	0.25
3	7.2	20.0	0.0	19.5	1.0E+4	0.25

Table 2 Statistical properties of basic variables for Example 1

Basic variable	φ_1 (°)	c_2 (kPa)	φ_2 (°)	c_3 (kPa)	φ_3 (°)
Mean value (μ_X)	38.0	5.3	23.0	7.2	20.0
Coefficient of variation (δ_X)	0.13	0.13	0.12	0.03	0.14

For comparison, we also compute the reliability index of this specific slip surface. In the FERA, the type of finite element is an eight-node quadrangle element, and there are altogether 1037 nodes and 320 elements. The boundary conditions are as follows: the bottom of the mesh is fixed, and the horizontal displacements on both sides of the mesh are fixed. Gravity loading is applied at single step. Under the same condition as Chen (2003), we get β=4.997 using the method of FERA based on the SSA technique proposed in this study. We can find that this value is very similar to the result of Chen (2003).

For further comparison, we also adopt another normally used slope stability analysis method—Bishop's simplified method (Bishop, 1955)—for the calculation of reliability indices, and some other cases of coefficient of variation, e.g., $\boldsymbol{\delta}_X$=[0.1, 0.1, 0.1, 0.1, 0.1]$^\text{T}$, $\boldsymbol{\delta}_X$=[0.2, 0.2, 0.2, 0.2, 0.2]$^\text{T}$, $\boldsymbol{\delta}_X$=[0.3, 0.3, 0.3, 0.3, 0.3]$^\text{T}$ are assumed. The results are shown in Fig.3, where "SSA" means the results of nonlinear FERA using the SSA technique proposed in this study, and "BSP" means the results of reliability analysis using the Bishop's simplified method. The iterative algorithms of the reliability index for SSA and BSP are both LSLIM. From Fig.3, we can find that the results of these two methods are very similar, which demonstrates that the method proposed in this study is feasible and accurate.

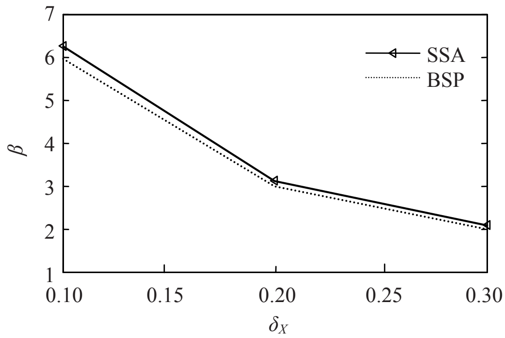

Fig.3 Reliability index β vs coefficient of variation δ_X for Example 1

Example 2 Fig.4 shows a two-layer slope whose parameters are listed in Table 3 (Griffiths and Fenton, 2004). In the FERA, there are altogether 831 nodes and 250 eight-node quadrangle elements. The boundary conditions are the same as those for Example 1. Gravity loading is applied at single step. Parameters c_1 and c_2 are considered as independent normally distributed random variables. The mean

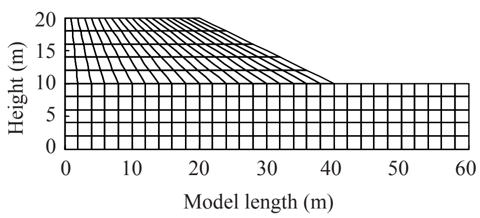

Fig.4 Meshing of the slope section for Example 2

Table 3 Soils parameters for Example 2

Soil layer	c (kPa)	φ (°)	ψ (°)	γ (kN/m^3)	E (kPa)	μ
1 (upper layer)	50.0	0.0	0.0	20.0	1.0E+5	0.3
2 (bottom layer)	73.1	0.0	0.0	20.0	1.0E+5	0.3

values of c_1 and c_2 are the same as those listed in Table 3, and their coefficients of variation are denoted as δ_{c_1} and δ_{c_2}, respectively.

1. Analysis of iteration numbers and iterative procedures

In order to study the characteristics of the method of nonlinear FERA using the SSA technique with the Aitken accelerating convergence algorithm based on incremental tangent stiffness method and LSLIM, we suppose δ_{c_1} and δ_{c_2} vary from 0.1 to 0.3, respectively, and both algorithms of FOSM and LSLIM are studied. For simplicity, we use β to represent $\beta_{\min}$ in the following text.

Tables 4 and 5 are the comparative analyses of iteration numbers of several cases.

Table 4 Contrast of iteration numbers (FOSM)

δ_{c_1}	δ_{c_2}	N_β^*	$N_{\rm FERA}^*$	N_β^{**}	$N_{\rm FERA}^{**}$
0.2	0.3	4	404	4	173
0.3	0.3	Divergent	−	Divergent	−

$\delta_{c_1}, \delta_{c_2}$: coefficient of variation; N_β: iteration number of β that represents the iteration number of the reliability index of the outmost loop in Fig.1; $N_{\rm FERA}$: iteration number of FERA that represents the summation of iterative numbers for solving nonlinear equilibrium equations at each trial point; * without acceleration; ** with acceleration

Table 5 Contrast of iteration numbers (LSLIM)

δ_{c_1}	δ_{c_2}	$\lambda^{(0)}$	f	N_β^*	$N_{\rm FERA}^*$	N_β^{**}	$N_{\rm FERA}^{**}$
0.2	0.3	1.0	1.5	4	404	4	173
0.3	0.3	0.01	1.5	5	505	6	241

$\delta_{c_1}, \delta_{c_2}$: coefficient of variation; N_β: iteration number of β that represents the iteration number of the reliability index of the outmost loop in Fig.1; $N_{\rm FERA}$: iteration number of FERA that represents the summation of iterative numbers for solving nonlinear equilibrium equations at each trial point; * without acceleration; ** with acceleration

From Tables 4 and 5, we can find that although the iteration numbers of reliability indices will perhaps increase a little after the adoption of the accelerating algorithm proposed in this study, the total iteration number of FERA will decrease greatly both for FOSM and LSLIM.

Comparing Table 4 and Table 5, we can also find that in some cases (e.g., δ_{c_1}=0.2, δ_{c_2}=0.3), the iteration numbers are equal both for FOSM and LSLIM. However, in some other cases (e.g., δ_{c_1}=0.3, δ_{c_2}=0.3), the iterative procedure of FOSM is divergent, but the iterative procedure of LSLIM is convergent if the values of $\lambda^{(0)}$ and f is properly selected in LSLIM. Therefore, LSLIM are very useful for it can assure the successful iteration of the reliability index.

In order to compare the iterative procedures of FOSM and LSLIM, Fig.5a and Fig.5b are plotted for FOSM and LSLIM, respectively. In both figures, $\delta_{c_1} = \delta_{c_2} = 0.1$ and an accelerating algorithm is adopted. It is clearly shown that the iterative procedure of FOSM is not convergent but the iterative procedure of LSLIM is convergent. Therefore, the convergence of LSLIM is better than that of FOSM for the specific example.

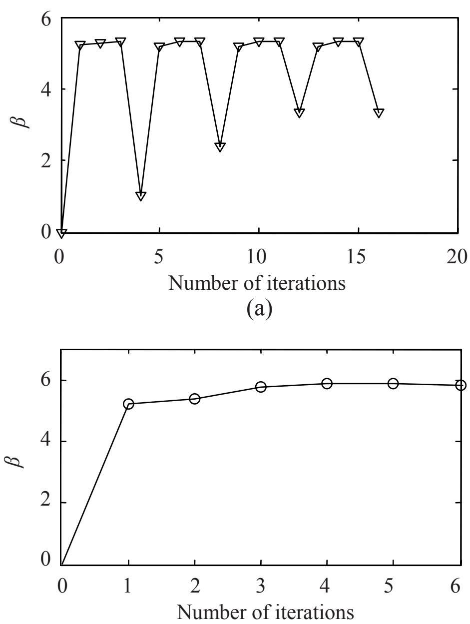

Fig.5 Reliability index β vs number of iterations for Example 2. (a) FOSM; (b) LSLIM

2. Analysis of reliability index and position of slip surface

The important results of nonlinear FERA using the SSA technique of slope stability are the reliability index and the position of the corresponding slip surface.

The relationship between the reliability index (β) and coefficient of variation (δ) is plotted in Fig.6, where β is calculated using LSLIM. The results of the reliability index using BSP are also plotted in this figure for comparison. It shows that the two methods have nearly the same value of β for several different cases of coefficient of variation, which demonstrates that the method proposed in this study is feasible and accurate.

Through many calculations, we find that although the values of reliability indices are different for several different cases of the coefficient of variation, the positions of slip surfaces corresponding to the minimum reliability index (β_{min}) obtained from nonlinear FERA using the SSA technique are nearly the same, which is illustrated in Fig.7. For comparison, the position of the minimum factor of safety (Fs_{min}) obtained from the deterministic FEA is also plotted in Fig.7. We can find that the position of the slip surface corresponding to β_{min} and Fs_{min} is different. Thus, it is necessary to search for the value of β_{min} and its corresponding position of the slip surface. Otherwise, if we take the slip surface of Fs_{min} for the critical slip surface and calculate its reliability index, we will get a larger value of the reliability index, which is unsafe for slope stability analysis.

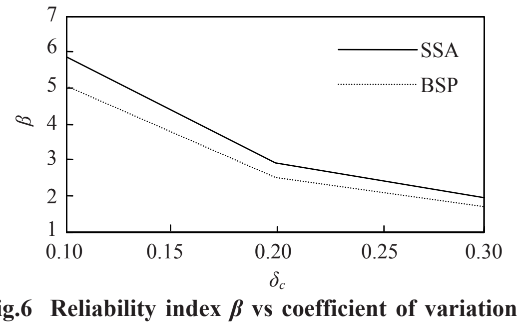

Fig.6 Reliability index β vs coefficient of variation for Example 2 (LSLIM, $\delta_{c_1}=\delta_{c_2}=\delta_c$)

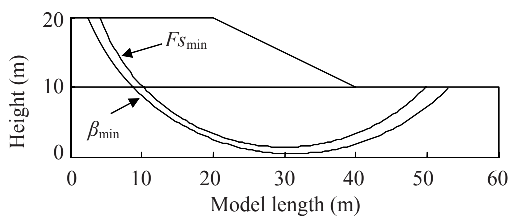

Fig.7 Positions of slip surfaces for Example 2

CONCLUSION

Nonlinear FERA of slope stability is studied in this paper. The reliability index of the slope is computed using the SSA technique and the solution method of nonlinear equilibrium equations is the incremental tangent stiffness method. In FEA, the soil mass is assumed to be elastic-perfectly plastic material and Mohr-Coulomb failure criterion is adopted.

In order to speed up the running of nonlinear stochastic finite element code, the formulae of the Aitken accelerating convergence algorithm based on the incremental tangent stiffness method are deduced, which aims to reduce the iteration number of the solution of finite element equilibrium equations and their partial derivative equations. At the same time, another iterative algorithm of the first order reliability method—LSLIM is proposed to perform the reliability analysis of slope stability. The flow chart of nonlinear FERA of slope stability is proposed and the code is compiled using VISUAL FORTRAN 6.5.

Through the analyses of two examples, the method and code of nonlinear FERA using the SSA technique are confirmed. It is found that the accelerating convergence algorithm proposed in this study proves to be very efficient for it can reduce the iteration number greatly, and LSLIM is also efficient for it can assure the convergence of the iteration of the reliability index. From the results of nonlinear FERA of these examples, we can find that the relationships between the reliability index and variation coefficient are the same both for SSA and BSP, and the difference between these two methods is small. Therefore, the method of nonlinear FERA using the SSA technique proposed in this study is accurate. Another benefit of this method is that it can calculate the position of the slip surface of β_{min}, which is different from that of Fs_{min}.

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