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Earth return path impedances of underground cable for three-layer earth

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Abstract: One of the factors that affect the parameters of an underground cable is earth return path impedance. Pollaczek developed a formula for the case of one-layer (homogenous) earth. But in practice the earth is composed of several layers. In this study we develop a new formula for earth return path impedance in the case of a three-layer earth. To check the accuracy of the obtained results, a comparison has been made with the finite element method (FEM). A comparison between the results of the Pollaczek formula and results of the obtained formula for a three-layer earth has been made, showing that the use of the Pollaczek formula instead of the actual formula can cause serious errors.

Key words: Underground cables, Earth return path impedance, Earth stratification, Finite element method (FEM)
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INTRODUCTION

Nowadays, there are such a variety of cable designs that developing a general formula for calculating the electrical parameters of all cables is difficult, if not impossible (Dommel, 1986). One of the most important factors that affect the impedance of the cable is the earth return path. The earth-return path self impedance of underground cables can reach up to 90% of the cable impedance, and the mutual impedance of underground cables, which in the case of underground cables is very significant, is contributed totally by the earth return path (Nguyen, 1998a).

A variety of studies have been made to obtain the earth return path contribution to transmission line impedances. The first attempts in this field were made in 1926. Carson (1926) obtained a correction term to consider the earth return path. But the Carson formula is true just for overhead transmission lines. Pollaczek (1926) developed a new formula for earth return path impedance, which is applicable for both overhead transmission lines and underground cables.

The Pollaczek formula is used only for homogenous earth (Pollaczek, 1926). But the earth is composed of several layers (Lagace *et al*., 1996; Zhang *et al*., 2005). Sunde (1968) extended a homogenous earth solution and proposed formulas in the case of a two-layer earth. Other formulas are proposed for two- and three-layer earth cases, but all of them are used for overhead lines (Ametani and Schinzinger, 1976; Nakagawa and Iwamoto, 1976; Papagiannis *et al*., 2005b). Tsiamitros *et al*.(2005) proposed a formula for earth return path impedance of underground cables in the case of a two-layer earth and developed a formula for the case of a three-layer earth.

All of the formulas developed for earth return path impedance include expressions with complex infinite integrals (Nguyen, 1998a). Some algebraic series were proposed for calculation of the Carson formula (Carson, 1926) and there are also some approximations for this formula (Alvarado and Betancourt, 1983; Davies *et al*., 1995; Noda, 2006). But integration of the Pollaczek formula is not so simple and there is no closed solution for Pollaczek's integral (Papagiannis *et al*., 2005a). Some closed-form

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approximations for calculation of Pollaczek's integral were proposed by (Wedepohl and Wilcox, 1973; Saad *et al*., 1996). But these formulas have their restrictions and they are applicable only for a limited range of frequencies, earth resistivity and cable configuration and geometry (Wedepohl and Wilcox, 1973; Saad *et al*., 1996).

Some numerical methods were proposed (Nguyen, 1998a; Papagiannis *et al*., 2005a) to solve Pollaczek's integral. Nguyen (1998b) also used a neural network method to evaluate earth return path impedance of homogenous earth. Some of the research used the finite element method (FEM) to obtain this impedance (Satsios *et al*., 1998; Papagiannis *et al*., 2000; 2003; Xu *et al*., 2002). Ametani (1980) proposed general models for the simulation of wave propagation, which are used to calculate the cable constants in the electromagnetic transients program (EMTP).

The Monte-Carlo method of integration is a nondeterministic method (Robert and Casella, 2004; Sadiku, 2004). Application of this method to calculate the integral of earth return path impedance leads to accurate results. FEM as a numerical method for the solution of electromagnetic problems can be used to compare the obtained results. This is because in this method the problems are analyzed regardless of their geometric complexities and the mathematical relations (Satsios *et al*., 1998; Papagiannis *et al*., 2000; 2003; Xu *et al*., 2002).

In this study we develop a formula for earth return path impedance of underground cables in the case of a three-layer earth. The results obtained with this new formula are justified by FEM results in the case of a three-layer earth. A comparison between the results of the Pollaczek formula and the results of our obtained formula for earth return path impedance of the three-layer earth is achieved.

EARTH RETURN PATH FOR THREE-LAYER EARTH

As mentioned above, the assumption of homogenous earth may cause serious errors. The earth is composed of several layers. In Fig.1, we assume two cables 1 and 2 buried in the first layer of a three-layer earth in depths h_1 and h_2. As is shown, the thickness of the first layer is d_1, the thickness of the second layer is d_2, and the third layer is just limited to the border with the second layer.

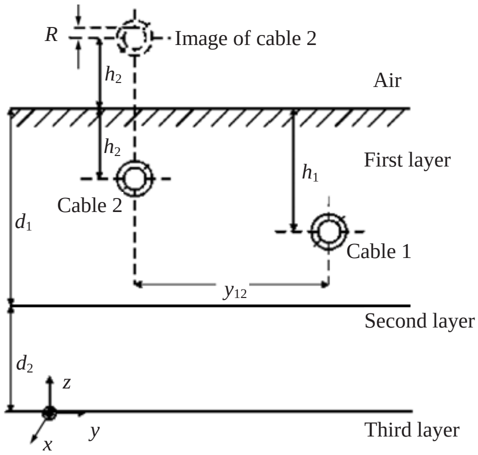

Fig.1 Configuration of cables 1 and 2

The permeability, permittivity and conductivity of the first layer are μ_1, ε_1 and σ_1, respectively. The same parameters for the second layer are μ_2, ε_2 and σ_2 and for the third layer these parameters are expressed as μ_3, ε_3 and σ_3, respectively.

Assume that there is a horizontal dipole in the place of cable 1 in Fig.1. In this research, as explained in (Sunde, 1968; Tsiamitros *et al*., 2005), the Hertzian vector, $\boldsymbol{\Pi}$, is used to obtain the earth return path impedance. The cables are long enough to be assumed to have infinite length. So it is sufficient to just consider the x component of $\boldsymbol{\Pi}$. These components in the air, the first, second and third layer are expressed as Π_{0x}, Π_{1x}, Π_{2x} and Π_{3x}, respectively.

These components are defined as follows:

In the air,

$$\Pi_{0x} = \int_0^\infty f_0(u)\,\mathrm{e}^{-\alpha_0 z}\, J_0(ru)\mathrm{d}u, \quad z \geq d_1 + d_2. \tag{1}$$

In the first layer,

$$\begin{cases} \Pi'_{1x} = \int_0^\infty [\, f_{1u}(u)\,\mathrm{e}^{-\alpha_1 z} + g_{1u}(u)\mathrm{e}^{\alpha_1 z}\,] J_0(ru)\mathrm{d}u, \\ \qquad d_1 \geq z \geq d_1 + d_2 - h_1, \\ \Pi_{1x} = \int_0^\infty [\, f_{1l}(u)\,\mathrm{e}^{-\alpha_1 z} + g_{1l}(u)\mathrm{e}^{\alpha_1 z}\,] J_0(ru)\mathrm{d}u, \\ \qquad d_2 \leq z \leq d_1 + d_2 - h_1. \end{cases} \tag{2}$$

In the second layer,

$$\Pi_{2x}=\int_0^\infty [f_2(u)\mathrm{e}^{-\alpha_2 z}+g_2(u)\mathrm{e}^{\alpha_2 z}]J_0(ru)\mathrm{d}u,\quad 0\le z\le d_2. \tag{3}$$

In the third layer,

$$\Pi_{3x}=\int_0^\infty f_3(u)\mathrm{e}^{-\alpha_3 z}J_0(ru)\mathrm{d}u,\quad z\le 0. \tag{4}$$

J_0 is the Bessel function of the first kind and zero order, $r=(x^2+y^2)^{1/2}$, $\alpha_i=(u^2+\gamma_i^2)^{1/2}$, $\gamma_i^2=\mathrm{j}\omega\mu_i(\sigma_i+\mathrm{j}\omega\varepsilon_i)$, j is the imaginary unit, and i is the layer number (i=0, 1, 2 and 3 for the air, the first, second and third layer, respectively), and $\omega=2\pi f$ is the angular velocity.

To obtain the Π function, firstly we should find f and g in Eqs.(1)~(4). But in the above equations it is not possible to find these functions directly. So we use an auxiliary configuration as shown in Fig.2. We assume a cable of the height of h_3 above the first layer.

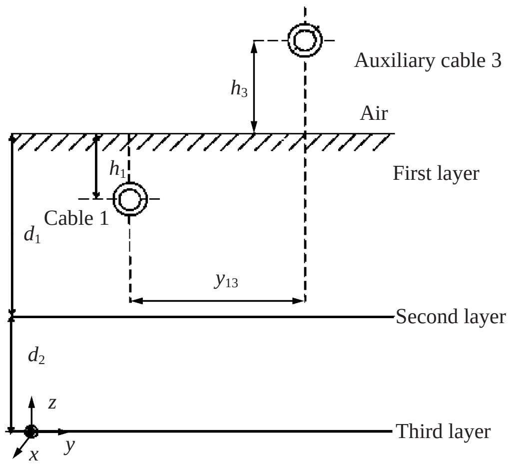

Fig.2 Auxiliary configuration of cables

With this assumption, the new Hertzian vectors, with dipole in the place of cable 3, as given in (Sunde, 1968; Tsiamitros *et al*., 2005), are defined as

$$\Pi'_{0x}=\int_0^\infty\left(C\frac{u}{\alpha_0}\mathrm{e}^{-\alpha_0|z-(d_1+d_2+h_3)|}+g_0(u)\mathrm{e}^{-\alpha_0 z}\right)J_0(ru)\mathrm{d}u, \tag{5}$$

$$\Pi'_{1x}=\int_0^\infty [f_1(u)\mathrm{e}^{\alpha_1 z}+g_1(u)\mathrm{e}^{-\alpha_1 z}]J_0(ru)\mathrm{d}u, \tag{6}$$

$$\Pi'_{2x}=\int_0^\infty [f_2(u)\mathrm{e}^{\alpha_2 z}+g_2(u)\mathrm{e}^{-\alpha_2 z}]J_0(ru)\mathrm{d}u, \tag{7}$$

$$\Pi'_{3x}=\int_0^\infty f'_3(u)\mathrm{e}^{\alpha_3 z}J_0(ru)\mathrm{d}u, \tag{8}$$

where $C=\mathrm{j}\omega\mu_0 l_\mathrm{d}I/(4\pi\gamma_0^2)$, l_d is the dipole length, $l_\mathrm{d}I$ is the moment of the dipole, and the other parameters are defined as above.

The boundary conditions in different media are as follows:

For $z=d_1+d_2$,

$$\gamma_0^2\Pi'_{0x}=\gamma_1^2\Pi_{1x},\ \frac{1}{\mu_0}\gamma_0^2\frac{\partial\Pi'_{0x}}{\partial z}=\frac{1}{\mu_1}\gamma_1^2\frac{\partial\Pi'_{1x}}{\partial z}. \tag{9}$$

For $z=d_1$,

$$\gamma_1^2\Pi'_{1x}=\gamma_2^2\Pi'_{2x},\ \frac{1}{\mu_1}\gamma_1^2\frac{\partial\Pi'_{1x}}{\partial z}=\frac{1}{\mu_2}\gamma_2^2\frac{\partial\Pi'_{2x}}{\partial z}. \tag{10}$$

For $z=0$,

$$\gamma_2^2\Pi'_{2x}=\gamma_3^2\Pi'_{3x},\ \frac{1}{\mu_2}\gamma_2^2\frac{\partial\Pi'_{2x}}{\partial z}=\frac{1}{\mu_3}\gamma_3^2\frac{\partial\Pi'_{3x}}{\partial z}. \tag{11}$$

By solving the above equations, we obtain

$$f_1=\frac{2Cu\mu_1\gamma_0^2\mathrm{e}^{-\alpha_0 h_3}K_3}{\gamma_1^2[K_3(\alpha_0\mu_1+\alpha_1\mu_2)\mathrm{e}^{\alpha_1(d_1+d_2)}+K_2\mathrm{e}^{\alpha_1(d_2-d_1)}(\alpha_0\mu_1-\alpha_1\mu_2)]}, \tag{12}$$

$$g_1=\frac{2Cu\mu_1\gamma_0^2\mathrm{e}^{-\alpha_0 h_3}\mathrm{e}^{2\alpha_1 d_2}K_2}{\gamma_1^2[K_3(\alpha_0\mu_1+\alpha_1\mu_2)\mathrm{e}^{\alpha_1(d_1+d_2)}+K_2\mathrm{e}^{\alpha_1(d_2-d_1)}(\alpha_0\mu_1-\alpha_1\mu_2)]}. \tag{13}$$

In the above equations,

$$K_2=\alpha_1\mu_2-(\mathrm{e}^{\alpha_2 d_2}-K_1\mathrm{e}^{-\alpha_2 d_2})\frac{\alpha_2\mu_1}{\mathrm{e}^{\alpha_2 d_2}+K_1\mathrm{e}^{-\alpha_2 d_2}},$$

$$K_3=\alpha_1\mu_2+(\mathrm{e}^{\alpha_2 d_2}-K_1\mathrm{e}^{-\alpha_2 d_2})\frac{\alpha_2\mu_1}{\mathrm{e}^{\alpha_2 d_2}+K_1\mathrm{e}^{-\alpha_2 d_2}},$$

where

$$K_1=\frac{\alpha_2\mu_3-\alpha_3\mu_2}{\alpha_2\mu_3+\alpha_3\mu_2}.$$

The mutual impedance between two parallel conductors 1 and 3 (Fig.2) is obtained as follows (Sunde, 1968; Tsiamitros *et al*., 2005):

$$Z'_{13}=\int_{-\infty}^{+\infty}\frac{\gamma_1^2\Pi'_{1x(z=d_1+d_2-h_1,\,y=y_{13})}}{l_\mathrm{d}I}\mathrm{d}x. \tag{14}$$

With setting f_1 and g_1 in Π'_{1x},

$$Z'_{13}=\gamma_1^2\int_0^\infty\frac{f_1(u)\mathrm{e}^{\alpha_1(d_1+d_2-h_1)}+g_1(u)\mathrm{e}^{-\alpha_1(d_1+d_2-h_1)}}{l_\mathrm{d}I}\cdot\left(\int_{-\infty}^{+\infty}J_0(u\sqrt{x^2+y_{13}^2})\mathrm{d}x\right)\mathrm{d}u. \tag{15}$$

The expression $\int_{-\infty}^{+\infty} J_0(u\sqrt{x^2+y_{13}^2})\mathrm{d}x$ can be replaced by $2\cos(uy_{13})/u$ as in (Sunde, 1968; Tsiamitros *et al*., 2005). At last Z'_{13} is obtained as

$$Z'_{13}=\frac{\mathrm{j}\omega\mu_1\mu_0}{\pi}\cdot\int_0^{\infty}\frac{\mathrm{e}^{-\alpha_0 h_3}[K_3\mathrm{e}^{\alpha_1(d_1+d_2-h_1)}+K_2\mathrm{e}^{2\alpha_1 d_2}\mathrm{e}^{-\alpha_1(d_1+d_2-h_1)}]\cos(uy_{13})}{K_3(\alpha_0\mu_1+\alpha_1\mu_2)\mathrm{e}^{\alpha_1(d_1+d_2)}+K_2\mathrm{e}^{\alpha_1(d_2-d_1)}(\alpha_0\mu_1-\alpha_1\mu_2)}\mathrm{d}u. \tag{16}$$

Eq.(16) for mutual impedance between conductors 1 and 3 has been obtained with the assumption that the dipole is in the place of conductor 3. With the assumption of the dipole in the place of cable 1, the same relation has to be obtained for the mutual impedance between conductors 1 and 3. This is true because of the reciprocity theory. This theory states that in a linear, bilateral, single circuit network, the ratio of excitation to response is constant, when the position of excitation and response is interchanged (Cheng, 2001; Tsiamitros *et al*., 2005). We obtained the formula in one case. Now we can obtain this formula in another way. With the assumption of the dipole in the place of cable 1,

$$Z'_{13}=\int_{-\infty}^{+\infty}\frac{\gamma_0^2\Pi_{0x(z=d_1+d_2+h_3,\,y=-y_{13})}}{l_\mathrm{d}I}\mathrm{d}x. \tag{17}$$

With replacement and some simplification,

$$Z'_{13}=\gamma_0^2\int_0^{\infty}\frac{2f_0(u)\mathrm{e}^{-\alpha_0(d_1+d_2+h_3)}\cos(uy_{13})}{l_\mathrm{d}I}\mathrm{d}u. \tag{18}$$

So with the theory of reciprocity and comparison between two expressions for Z'_{13}, f_0 is obtained as

$$f_0(u)=\frac{l_\mathrm{d}I\,u\mathrm{j}\omega\mu_1\mu_0}{2\pi\gamma_1^2}\cdot\frac{\mathrm{e}^{\alpha_0(d_1+d_2)}[K_3\mathrm{e}^{\alpha_1(d_1+d_2-h_1)}+K_3\mathrm{e}^{2\alpha_1 d_2}\mathrm{e}^{-\alpha_1(d_1+d_2-h_1)}]}{K_3(\alpha_0\mu_1+\alpha_1\mu_2)\mathrm{e}^{\alpha_1(d_1+d_2)}+K_2\mathrm{e}^{\alpha_1(d_2-d_1)}(\alpha_0\mu_1-\alpha_1\mu_2)}. \tag{19}$$

Now Π_{0x} is defined. With boundary conditions for $\boldsymbol{\Pi}$, f_{1u} and g_{1u} are defined as follows:

$$f_{1u}(u)=\frac{C_1 u}{\alpha_1}(\alpha_1\mu_0+\alpha_0\mu_1)L_1(u)\mathrm{e}^{\alpha_1(d_1+d_2)}, \tag{20}$$

$$g_{1u}(u)=\frac{C_1 u}{\alpha_1}(\alpha_1\mu_0-\alpha_0\mu_1)L_1(u)\mathrm{e}^{-\alpha_1(d_1+d_2)}, \tag{21}$$

where

$$L_1(u)=\frac{K_3\mathrm{e}^{\alpha_1(d_1+d_2-h_1)}+K_3\mathrm{e}^{2\alpha_1 d_2}\mathrm{e}^{-\alpha_1(d_1+d_2-h_1)}}{K_3(\alpha_0\mu_1+\alpha_1\mu_2)\mathrm{e}^{\alpha_1(d_1+d_2)}+K_2\mathrm{e}^{\alpha_1(d_2-d_1)}(\alpha_0\mu_1-\alpha_1\mu_2)}, \tag{22}$$

$$C_1=\mathrm{j}\omega\mu_1 l_\mathrm{d}I/(4\pi\gamma_1^2).$$

So Π_{1x} is formed as follows:

$$\Pi_{1x}=\int_0^{\infty}\left[\frac{C_1 u}{\alpha_1}(\alpha_1\mu_0+\alpha_0\mu_1)L_1(u)\mathrm{e}^{\alpha_1(d_1+d_2)}\mathrm{e}^{-\alpha_1 z}+\frac{C_1 u}{\alpha_1}(\alpha_1\mu_0-\alpha_0\mu_1)L_1(u)\mathrm{e}^{-\alpha_1(d_1+d_2)}\mathrm{e}^{\alpha_1 z}\right]J_0(ru)\mathrm{d}u. \tag{23}$$

From (Sunde, 1968; Tsiamitros *et al*., 2005),

$$Z'_{12}=\int_{-\infty}^{+\infty}\frac{\gamma_1^2\Pi_{1x(z=d_1+d_2-h_2,\,y=y_{12})}}{l_\mathrm{d}I}\mathrm{d}x. \tag{24}$$

At last, the expression for mutual impedance between cables 1 and 2 in Fig.1 is obtained as

$$Z'_{12}=\frac{\mathrm{j}\omega\mu_1}{2\pi}\int_0^{\infty}\frac{\cos(uy_{12})}{\alpha_1}\left[\frac{p_{10}p_{12}\mathrm{e}^{-\alpha_1|h_1-h_2|}+p_{10}q_{12}\mathrm{e}^{-\alpha_1(2d_1-h_1-h_2)}}{p_{12}p_{10}-q_{12}q_{10}\mathrm{e}^{-2\alpha_1 d_1}}+\frac{q_{10}p_{12}\mathrm{e}^{-\alpha_1(h_2+h_1)}+q_{10}q_{12}\mathrm{e}^{-\alpha_1(2d_1-|h_1-h_2|)}}{p_{12}p_{10}-q_{12}q_{10}\mathrm{e}^{-2\alpha_1 d_1}}\right]\mathrm{d}u, \tag{25}$$

where $p_{10}=\mu_0\alpha_1+\mu_1\alpha_0$, $q_{10}=\mu_0\alpha_1-\mu_1\alpha_0$, $p_{12}=K_3$, $q_{12}=K_2$.

To apply this formula for the self earth return path impedance, we should replace y_{12} with the outer radius of cable and make both h_1 and h_2 equal to the depth of the said cable.

In the above expression, by setting the parameters of the third layer (μ_3, ε_3, σ_3) equal to the parameters of the second layer (μ_2, ε_2, σ_2), the expression for the earth return path impedance of a two-layer earth [which is introduced in (Tsiamitros *et al*., 2005) with Eq.(8)] has been obtained.

If in the above equation set the electromagnetic properties of the third layer (μ_3, ε_3, σ_3) and of the second layer (μ_2, ε_2, σ_2) equal to the similar parameters of the first layer (μ_1, ε_1, σ_1), the well-known Pollaczek formula for earth return path impedance in the case of homogenous earth will be obtained.

NUMERICAL INTEGRATION METHOD

As mentioned above, the integral of earth return path impedance is complicated. Therefore, finding a suitable method for solving this integral is important.

Unlike deterministic numerical methods, Monte Carlo methods are non-deterministic (probabilistic or stochastic) numerical methods employed in solving mathematical and physical problems (Robert and Casella, 2004). Monte Carlo methods are applied in simulation and sampling (Sadiku, 2004).

For one-dimensional (1D) integration several quadrature formulas exist. The numbers of such formulas are relatively few for multidimensional integration. It is for such multidimensional integrals that the Monte Carlo technique becomes valuable. The quadrature formulas become very complex for multiple integrals, while the Monte Carlo method remains almost unchanged (Sadiku, 2004). Because of the good convergency of Monte Carlo integration procedures, the crude Monte Carlo method will be used. The integrals of earth return path impedance relations have an oscillatory characteristic, so convergence is very important in the integration of these integrals. The proposed method shows good convergence. The method is not complex and is very easy to program in the computer.

The algorithm and formulation of this method are introduced in the Appendix.

COMPARISON WITH FEM

Now to modify the obtained relation Eq.(25), we compare the results of the relation Eq.(25) for earth return path impedance with the results of FEM in the case of a three-layer earth. The comparison has been done for a range of frequencies between 50 and 10^6 Hz. We assumed a system of cables. The cables were buried in the first layer of the three-layer earth at a depth of 1 m. The permeability of all media was set to 1 and the relative permittivity was equal to 0. The separation between the two considered cables was 35 cm. The radius of the core was 1.9 cm and the outer radius was 4.2 cm. Two different cases obtained by practical tests in (Lagace *et al*., 1996) were considered. The conditions of these two cases are shown in Table 1.

Table 1 Parameters of three-layer earth

Case	ρ_1 (Ω·m)	ρ_2 (Ω·m)	ρ_3 (Ω·m)	d_1 (m)	d_2 (m)
1	30	9.4	500	3.4	25.5
2	128	1930	520	3.1	15.0

Comparisons have been done by the formula

$$Difference\ (\%) = \frac{|Element_{\text{FEM}} - Element_{\text{formula}}|}{|Element_{\text{FEM}}|} \times 100. \tag{26}$$

In this relation $Element_{\text{FEM}}$ is the real or imaginary part of self or mutual impedance obtained by FEM and $Element_{\text{formula}}$ is the real or imaginary part of self or mutual earth return path impedance obtained by the three-layer earth formula. Results are illustrated in Figs.3a~3d.

COMPARISON WITH THE POLLACZEK FORMULA

The Pollaczek formula was used to calculate earth return path impedance of underground cables buried in homogenous earth. In this section, a comparison between the results of the Pollaczek formula and the proposed formula for the three-layer earth is made. All of the conditions and parameters were as in the previous section. The conductivity of the first layer was used in the Pollaczek formula.

$$Difference\ (\%) = \frac{|Element_{\text{Pollaczek}} - Element_{\text{formula}}|}{|Element_{\text{Pollaczek}}|} \times 100, \tag{27}$$

in which $Element_{\text{Pollaczek}}$ is the real or imaginary part of the self or mutual earth return path impedance obtained by the Pollaczek formula and $Element_{\text{formula}}$ is previously defined. The results of this comparison are illustrated in Figs.4a~4d.

DISCUSSION

It can be seen in Figs.3a~3d that the difference between the results obtained by relation Eq.(25) and by FEM for earth return path impedance in all parts is less than 5% (for a wide range of frequencies). Therefore we can judge that the obtained relation for earth return path impedance in the case of a

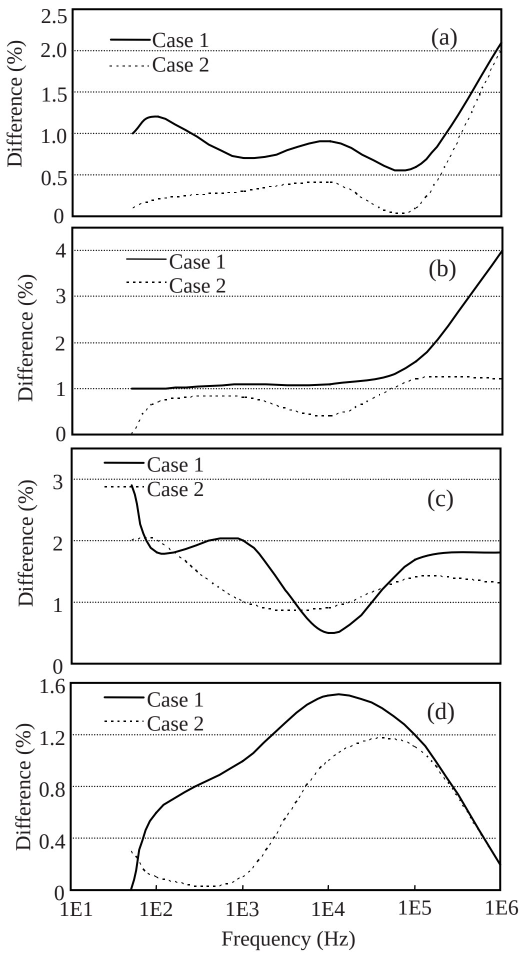

Fig.3 Difference between real parts of self impedance (a), imaginary parts of self impedance (b), real parts of mutual impedance (c), and imaginary parts of mutual impedance (d) obtained by relation Eq.(25) and by FEM for a three-layer earth

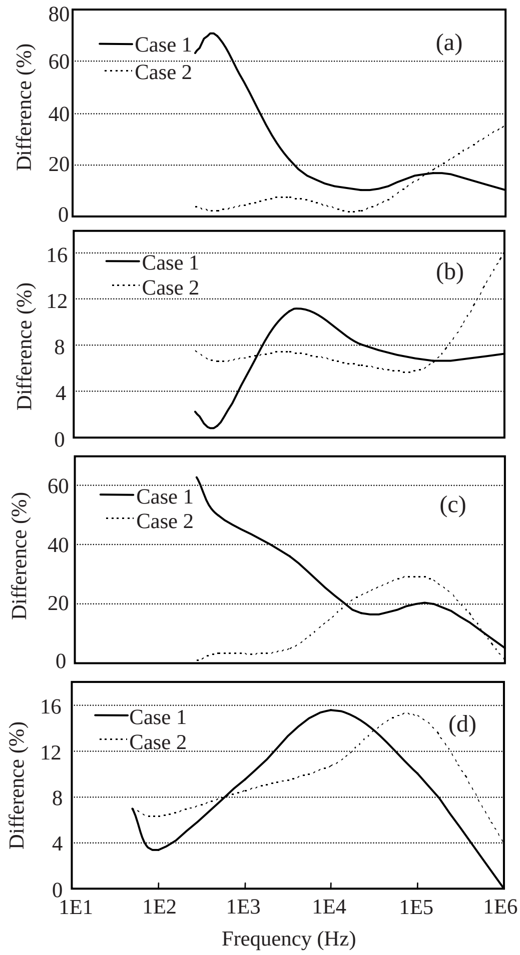

Fig.4 Difference between real parts of self impedance (a), imaginary parts of self impedance (b), real parts of mutual impedance (c), and imaginary parts of mutual impedance obtained by relation Eq.(25) and by the Pollaczeck formula for a three-layer earth

three-layer earth is so accurate that it can be used in power system analysis.

The comparison between the values obtained by relation Eq.(25) and the Pollaczek formula for earth return path impedance shows that the use of the Pollaczek formula instead of the three-layer earth formula causes serious errors in the case of a three-layer earth. This difference can reach more than 60% for the real part and about 20% for the imaginary part.

CONCLUSION

(1) A new formula for obtaining the earth return path impedance of underground cables in the case of a three-layer earth is developed.

(2) As has been said, the earth is composed of several layers and neglecting this fact may cause serious errors.

(3) The results of the obtained formula (obtained by aid of the Monte Carlo integration method) with the results obtained by FEM for self and mutual earth return path impedance are compared and the comparison is shown to be in good agreement and it confirms the accuracy of the obtained relation.

(4) Finally, a comparison between the results of the Pollaczek formula and the obtained formula was made.

The results show that the use of the Pollaczek formula instead of the actual formula can cause serious errors.

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APPENDIX

Crude Monte Carlo integration

Suppose we wish to evaluate the integral

$$I=\int_R f(X)\mathrm{d}X\,, \tag{A1}$$

where R is an n-dimensional space. Let $X=(X^1, X^2, \ldots, X^n)$ be a random variable that is uniformly distributed in R (Sadiku, 2004). Then $f(X)$ is a random variable with a mean value

$$\frac{1}{|R|}\int_R f(X)\mathrm{d}X=\frac{I}{|R|} \tag{A2}$$

and a variance

$$\frac{1}{|R|}\int_R (f(X))^2 \mathrm{d}X - \left(\frac{1}{|R|}\int_R f(X)\mathrm{d}X\right)^2, \qquad \text{(A3)}$$

where

$$|R| = \int_R \mathrm{d}X. \qquad \text{(A4)}$$

If we take N independent samples of X, i.e., X_1, X_2, ..., X_N, all having the same distribution as X, we might expect the average $\sum_{i=1}^{N} f(X_i)/N$ to be close to the mean of $f(X)$. Thus, from Eq.(A2), we have

$$I = \frac{|R|}{N}\sum_{i=1}^{N} f(X_i). \qquad \text{(A5)}$$

This Monte Carlo formula applies to any integration over a finite region R. For a simple illustration, consider the 1D integral:

$$I = \int_a^b f(X)\mathrm{d}X. \qquad \text{(A6)}$$

Applying Eq.(A5) to 1D integral yields

$$I = \frac{b-a}{N}\sum_{i=1}^{N} f(X_i), \qquad \text{(A7)}$$

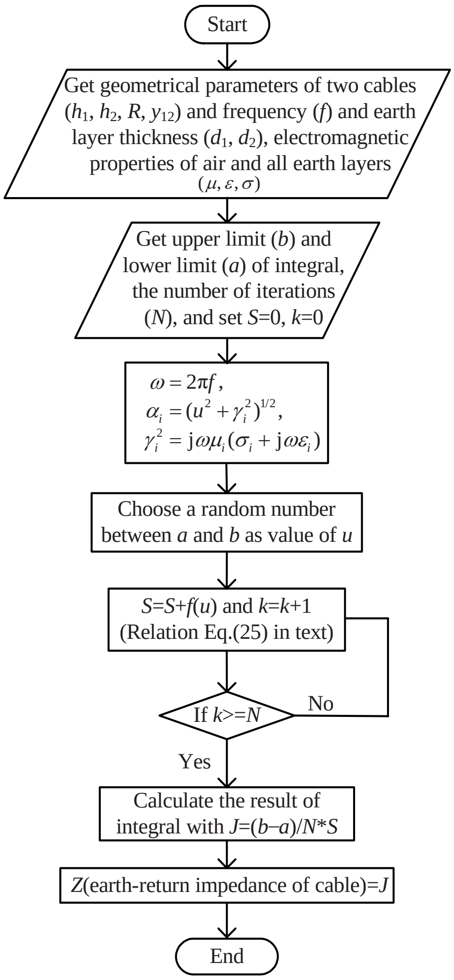
where X_i is a random number in the interval (a, b); i.e.,

$$X_i = a + (b-a)U, \qquad \text{(A8)}$$

where U is a random number between 0 and 1 (Sadiku, 2004).

To apply to the case of the integral of earth return path impedance of a three-layer earth, set a=0, b=10, and the number of iterations is $N=10^7/(2\ln f)$.

Computation flowchart



h_1, h_2: depths of cables 1 and 2
R: outer radius of cables
y_{12}: separation of cables
f: frequency
d_1, d_2: thicknesses of the first and second layer
i=0, 1, 2, 3 denote the air, the first, second and third layer
μ_i: permeability of medium i
ε_i: permittivity of medium i
σ_i: conductivity of medium i
a, b: lower and upper limits of earth return path integral
N: number of iterations
ω: angular velocity
j: imaginary unit