



Reliability assessment of networks-on-chip based on analytical models

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Abstract: As technology scales down, the reliability issues are becoming more crucial, especially for networks-on-chip (NoCs) that provide the communication requirements of multi-processor systems-on-chip. Reliability evaluation based on analytical models is a precise method for dependability analysis before and after designing the fault-tolerant systems. In this paper, we accurately formulate the inherent reliability and vulnerability of some popular NoC architectures against permanent faults, also depending on the employed routing algorithm and traffic model. Based on this analysis, effects of failures in the links, switches and network interfaces on the packet delivery of NoCs are determined. Besides, some extensions to evaluate a fault-tolerant method and some routing algorithms are described. The analyses are validated through appropriate simulations. The results thus obtained are exactly the same as or very close to the analytical ones.

Key words: Networks-on-chip (NoCs), Traffic model, Routing algorithm, Reliability assessment, Permanent fault
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INTRODUCTION

As CMOS technology scales down into the nano-technology domain and the complexity of evolving integrated circuits design increases, VLSI systems become more and more vulnerable to permanent faults in addition of transient faults. Furthermore, relaxing the requirement of 100% correctness in the operation of various components and on-chip interconnections increases the yield and intensely reduces the manufacturing cost, but necessitates some system-level fault-tolerance (Dumitras and Marculescu, 2003). Thus, tolerance is required against manufacturing faults and defects as well as the permanent faults caused by accelerated aging effects such as electro-migration, strikes of high-energy particles and voltage or thermal variations. Designing the reliable systems that tolerate permanent faults has been extensively studied, although in parallel processing networks research has mostly focused on large scale and multi-computer systems and their interconnections.

The network-on-chip (NoC) design paradigm (Dally and Towles, 2001) has been proposed as the best scalable communication infrastructure for multi-processor system-on-chip (MP-SoC) designs. Among the different topologies, the 2D-mesh structure has been widely used because of its simple structure and implementation. The main problem with the mesh topology is its large diameter that has negative effect on communication latency. Torus topology was proposed to reduce the latency of mesh through connecting the switches on the edges to the switches on the opposite edges with wrap-around channels. However, the long wrap-around connections may result in excessive delay, but this problem can be avoided by folding the torus, as illustrated in Fig.1a (Dally and Seitz, 1986). In this paper we will investigate the folded torus in addition to mesh topology.

It is essential to design reliable NoCs considering all aspects such as system-level design, topology, routing algorithms and router design. On the other hand, the reliability assessment is a key method for dependability evaluation which can be used to make

decisions in the design of reliable systems. The reliability assessment can be done in two ways, analytically or by simulation. In analytical assessment methods, the evaluation is performed by finding the exact or approximate relations between the system reliability and the reliabilities of system components in different conditions as well as finding the probability of correct operation in erroneous situations. But, the reliability assessment through simulation is performed by considering the system behavior under the known failure rates of components or under fault injection.

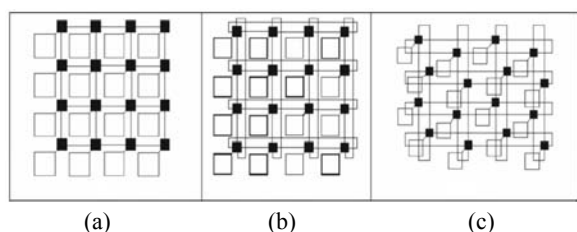


Fig.1 Different topologies: (a) 4×4 mesh; (b) 4×4 torus; (c) 4×4 folded torus

In this paper, some probabilistic and analytical models are presented to evaluate the intrinsic reliability and vulnerability of mesh- and torus-connected NoCs based on the computation of average path length for all packets traversing the network with a specific routing algorithm and traffic model. The main investigated routing algorithms are XY and $XY-YX$ because of their simplicity. But, this analysis is extensible to other topologies and adaptive algorithms. Besides, to obtain more realistic results, the analytical formulations are presented for different traffic models including the hot-spot and transpose traffics (Kim *et al.*, 2006; Schonwald *et al.*, 2007) in addition to the uniform traffic model. This analysis is performed for permanent faults that lead to permanent failures in the switches, links and network interfaces. To verify the analytical models, extensive simulations are done and it is observed that the simulation results match very well with the analytical results.

RELATED WORKS

While performance analysis for NoCs has been reported extensively, reliability assessment for NoCs

has not received much attention. The reliability evaluation has been widely used for multi-computer systems with hypercube, mesh and torus interconnections. Kim *et al.*(1989) investigated an analytical model for reliability evaluation of hypercube multi-computers based on a task-based model. Mohapatra and Das (1995) and Chang and Mohapatra (1998) derived the task-based analytical models termed as ‘sub-mesh dependability’ for mesh-connected processors. A simulation-based reliability analysis was introduced in Abachi and Walker (1997) to compare the different topologies of message passing architectures. Wang *et al.*(2003) presented a probability model to predict the reliability and availability of mesh-connected multi-computer systems based on the connectivity probability. In most of these works only the node failure has been addressed in which a node is supposed the same as a core, whereas in our analysis a node is composed of a switch, a network interface and a core, and the link failures are considered as well.

In NoC domain, Mondal *et al.*(2006) provided a model for determining the probability that a NoC link fails due to manufacturing variation in addition to measuring the impact of a link failure on the number of cycles needed for the communication. Lehtonen *et al.*(2007) presented a fault-tolerant analysis of different mesh NoC architectures including the topology, the router structure and the number of network interfaces. Dalirsani *et al.*(2007) proposed an analytical model to assess the reliability of a NoC based system-on-chip, but this model was designed for analysis of transient faults’ effects on switches and routing algorithms.

In this work, the general aspects of NoCs (which have no built-in fault-tolerance mechanism) affecting the system reliability, such as topology, network size, routing algorithms and traffic patterns are analytically investigated. Therefore, our work is different from the previous works since those have been performed either for large scale systems with different applications from NoCs or for a specific aspect of NoCs. Thus, to the best of our knowledge, this is the first system-level and general analytical reliability assessment of mesh- and torus-based NoCs with respect to different routing algorithms, traffic models and network sizes.

PRELIMINARIES

Networks-on-chip

An $M \times N$ 2D-mesh network consists of a set of nodes $V = \{(x, y) : 0 \leq x \leq N-1, 0 \leq y \leq M-1\}$, where each node (x, y) is connected to its neighbors $(x \pm 1, y)$ and $(x, y \pm 1)$ if they exist, and a set of $2MN - (M+N)$ bidirectional links E .

An $M \times N$ 2D-torus network is the same as a 2D-mesh but with $(M+N)$ additional links in which N links connect the lower nodes $(x, 0)$ to the upper nodes $(x, M-1)$ and M links connect the left most nodes $(0, y)$ to the right most nodes $(N-1, y)$. Hereafter, we mean ‘folded torus’ whenever we use ‘torus’.

A NoC can be defined as a set of switches, network interfaces and point-to-point links interconnecting the cores of an MP-SoC. In this architecture each node consists of a switch or router, a core and a network interface where the latter can be assumed as a portion of the core. In NoC we use two unidirectional links instead of each bidirectional link. Thus, hereafter we mean ‘unidirectional link’ whenever we use ‘link’. A 4×4 -mesh NoC and some simple paths between some source and destination nodes are depicted in Fig.2.

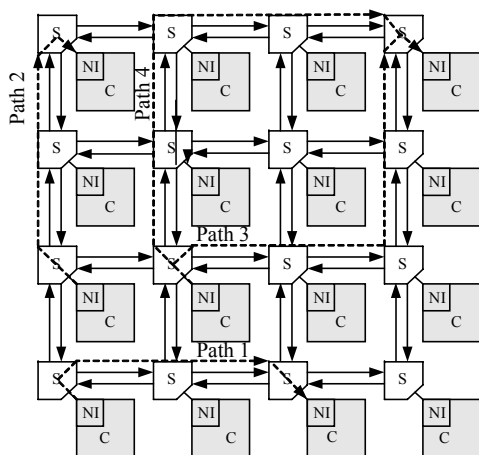


Fig.2 A 4×4 -mesh NoC with some simple paths
S: switch; C: core; NI: network interface

In XY routing algorithms, there is one path between a source and a destination. A packet is first routed in the X direction and then in the Y direction to reach the destination. But in XY - YX routing algorithms there are two paths for most of source-destination pairs. In this way, if a path fails, the

packets are sent to the destination via another path, if it exists. To compare the NoCs with different sizes, routing algorithms and traffic models, we define the following parameters (metrics):

- Path length (PL): It is defined as the number of hop-by-hop links that exist on a path between a source and a destination node.
- Path reliability (PR): It is the reliability of a path between a source and a destination core traversed by a packet, and its failure rate is a function of the failure rates of the network components in the path.
- Packet completion probability (PCP): It is defined as the number of intact received packets divided by the total number of injected packets into the network.
- Packet drop probability (PDP): It is defined as the complement of PCP and equals $1 - \text{PCP}$.

Fault model

The employed fault model in the proposed analytical models is based on the following assumptions:

1. A link failure, switch failure or network interface failure does not affect the operation of any other component.
2. The failure rates of the links are the same. This applies to the failure rates of the switches and the failure rates of the network interfaces.
3. Any link, switch or network interface can fail and the faulty components are unusable (as a common case in the manufacturing faults). This means that data will not be correctly transmitted over the faulty components.
4. No new fault occurs during a routing process.

In the third assumption, we consider the faults that cause the whole component to become faulty and unusable. Therefore, although not all stuck at 0 or 1 faults lead to the whole switch failure, these faults on the links and network interfaces cause a packet either not to reach its destination or with high probability to be received incorrectly. Thus, we assume that the packets are either not sent or received over a faulty component but overwritten in the output ports connected to the faulty component, or received incorrectly. In addition, for fault detection we assume that there exists a mechanism such as built-in-self-test that detects the permanent faults while the system is being used.

ANALYTICAL RELIABILITY MODELS

To perform analytical reliability assessment for mesh and torus NoCs and design the appropriate models, we use the average values of parameters mentioned in the previous section. Therefore, these parameters can be computed if, for a specific traffic pattern and topology, we have computed the ‘average path length’ of all the packets traversing the network:

Average path length (APL) is defined as the sum of minimal (or shortest) path lengths between any source and destination nodes according to the traffic pattern, divided by the total number of these minimal paths. It should be noted that for any source and destination pairs only one minimal path is considered and APL is independent of the routing algorithms but depends on the traffic pattern and topology. Thus, APL differs from the average distance that depends only on the topology.

We present two analytical models to assess the reliability of NoCs. In the first model, the reliability functions are used. This model is useful for the permanent faults caused by voltage or thermal variations, accelerated aging effects and even low quality manufacturing process while the system is being used (Koren and Krishna, 2007). In the second model, the reliability assessment is performed with the assumption that one or more permanent faults have occurred. These faults can be manufacturing faults or faults caused by accelerated aging effects or strikes of high-energy particles, and so on.

For the first model assuming the failure rates of the links, switches and network interfaces to be λ_L , λ_S and λ_{NI} , respectively, then the reliability functions of these components can be stated as $R_L(t)$, $R_S(t)$ and $R_{NI}(t)$ in which $R_i(t) = e^{-\lambda_i t}$. As stated in the previous section, the network interface can be assumed as a portion of the core. In this case, we can use $R_C(t)$ instead of $R_{NI}(t)$ in which $R_C(t)$ is the reliability function of a core including the network interface.

Definition 1 (Path reliability, PR) For all packets traversing the network, PR is computed by the following general equation in which PL is the path length and $R_L(t)$, $R_S(t)$ and $R_{NI}(t)$ are the reliability functions of the links, switches and network interfaces, respectively:

$$PR(t) = R_L^{PL}(t) \cdot R_S^{PL+1}(t) \cdot R_{NI}^2(t). \quad (1)$$

To correctly send a packet to the destination, the network interfaces and switches in the source and destination nodes must be healthy, in addition to the middle links and switches. The number of links in a path is PL and the number of switches in a path including the source and destination ones is (PL+1). Thus, Eq.(1) is valid. In Eq.(1), it is assumed that the reliability functions are constant in the period of time in which a packet is transmitted from a source to a destination. Eq.(1) can be used in the first model to compute the average path reliabilities for different routing algorithms.

In the following, based on the different traffic patterns we present the reliability models for mesh and torus NoCs that use XY and XY-YX routing algorithms.

Mesh topology

1. Uniform traffic

In this traffic pattern, a core sends the packets randomly to any other cores in the network with equal probability. This results in the fact that for a high number of packets each core sends/receives an equal number of packets to/from any other cores. In our models, the packet generation mechanism is not important. The APL parameter for this traffic model is computed using Eq.(2):

$$APL_{uni} = \frac{1}{MN(MN-1)} \sum_{i=1}^{MN(MN-1)} L(Path_i) = \frac{M+N}{3}, \quad (2)$$

where M and N are the number of rows and columns in the mesh, respectively, and $L(Path_i)$ is the length of the i th path. The term $MN(MN-1)$ is the total number of paths in this mesh under the uniform traffic model. Since according to APL definition we consider only one minimal path between any source-destination pair, $MN(MN-1)$ showing the number of all unique source-destination pairs also equals the total number of paths.

In the first model, to compute the average path reliability for XY routing we should replace PL in Eq.(1) by APL that equals APL_{uni} for this traffic model. Thus, by removing the time notation in Eq.(1) for brevity and replacing PL by APL_{uni} from Eq.(2), the average path reliability for XY routing is computed by

$$APR_{XY} = R_L^{(M+N)/3} \cdot R_S^{(M+N)/3+1} \cdot R_{NI}^2 \quad (3)$$

In XY - YX routing, if the source and destination are in the same row or column, there is one path between them called '1-way path' (Path 1 and Path 2 in Fig.2); otherwise, there are two paths called '2-way path' (Path 3 and Path 4 in Fig.2). Thus, to compute the average path reliability for this routing we should combine the average path reliabilities for two types of available paths based on Eq.(4):

$$APR_{XY-YX} = \frac{M+N-2}{MN-1} \cdot APR_{1-way} + \frac{(M-1)(N-1)}{MN-1} \cdot APR_{2-way} \quad (4)$$

where APR_{1-way} and APR_{2-way} are the average path reliabilities for 1- and 2-way paths in XY - YX routing, respectively. In addition, the factors of the first and second terms are fractions of the total paths that are 1- and 2-way paths, respectively. Hereafter, we consider only $N \times N$ meshes to obtain simpler equations. Thus, since 1-way paths are common between XY and XY - YX , APR_{1-way} in Eq.(4) is obtained similar to APR_{XY} by replacing PL by APL_{1-way} in which APL_{1-way} is the average path length of 1-way paths in the mesh under the uniform traffic and is computed by

$$APL_{1-way} = \frac{1}{2N^2(N-1)} \sum_{i=1}^{2N^2(N-1)} L(\text{Path}_{1-way i}) = \frac{1}{3}(N+1), \quad (5)$$

where the upper index of summation is the total number of 1-way paths in $N \times N$ meshes.

APR_{2-way} is computed by

$$APR_{2-way} = R_S^2 \cdot R_{NI}^2 [1 - (1 - R_S^{APL_{2-way}-1} \cdot R_L^{APL_{2-way}})^2], \quad (6)$$

where

$$APL_{2-way} = \frac{N+1}{N-1} \cdot APL_{uni} - \frac{2}{N-1} \cdot APL_{1-way} = \frac{2}{3}(N+1). \quad (7)$$

The long term in Eq.(6) is obtained based on the reliability of two same parallel ways in 2-way paths. In addition, Eq.(7) is based on the fact that the total number of unique paths equals the sum of 1-way and

2-way paths, and thus the factors are obtained from the ones used in Eq.(4) for M equal to N . Therefore, the average path reliability for XY - YX routing in $N \times N$ mesh can be obtained using Eqs.(4)~(7).

APR_{XY-YX} is greater than APR_{XY} based on Eqs.(3) and (4). As a result, having more permissible paths between the source and destination nodes in a routing algorithm, more average path reliability is reached. In addition, since $R_i(t)$ is not greater than 1, the APR parameters decrease when the mesh size increases. In other words, when the mesh size increases the PCP is decreased.

In the second model, we analyze the NoC reliability for different routing strategies and traffic models from another perspective. To do so, we compute the PCP or PDP parameters (which are the average values) when one or more permanent failures occur in the NoC architecture (the computation of PDP is simpler).

(1) One component failure. When a unidirectional link fails in an $N \times N$ mesh, the average probability that a packet does not reach the destination or reaches with incorrect data using XY routing under the uniform traffic is $PDP_{XY,1-L} = APL_{uni}/[4N(N-1)]$ in which the denominator is the number of total unidirectional links. Similarly, we have $PDP_{XY,1-S} = (APL_{uni}+1)/N^2$ for a switch failure and $PDP_{XY,1-NI} = 2/N^2$ for a network interface failure, because the total number of switches is N^2 , and the numbers of network interfaces and switches on a path are 2 and $(APL_{uni}+1)$, respectively. For XY - YX routing the following equations are used:

$$PDP_{XY-YX,1-L} = \frac{2}{N+1} \cdot \frac{APL_{1-way}}{4N(N-1)} = \frac{1}{6N(N-1)}, \quad (8)$$

$$PDP_{XY-YX,1-S} = \frac{2}{N+1} \cdot \frac{APL_{1-way}+1}{N^2} + \frac{N-1}{N+1} \cdot \frac{2}{N^2} = \frac{2(4N+1)}{3N^2(N+1)}, \quad (9)$$

where the factors $2/(N+1)$ and $(N-1)/(N+1)$ are fractions of the total paths that are 1- and 2-way paths, respectively. Eq.(8) is obtained through the fact that in XY - YX routing, one link failure can stop only the packets traversing 1-way paths, whereas for one switch failure (Eq.(9)) this can occur in both the source and destination nodes of 2-way paths (second

term) in addition to the 1-way paths (first term). Thus, Eq.(9) is obtained by applying the factors used in Eq.(4) for the case $M=N$. For one network interface failure, PDP for XY - YX routing equals $PDP_{XY,1-NI}$, which has been stated before.

(2) Two component failures. For two link failures and two switch failures, using the probability computations, we estimate the PDP parameters as follows:

$$PDP_{XY,2-L} = 2PDP_{XY,1-L} - PDP_{XY,1-L}^2, \tag{10}$$

$$PDP_{XY,2-S} = 2PDP_{XY,1-S} - PDP_{XY,1-S}^2, \tag{11}$$

$$PDP_{XY-YX,2-L} = \frac{2}{N+1}(2q_1 - q_1^2) + \frac{N-1}{N+1} \cdot 2q_2^2 + 2 \cdot \frac{2}{N+1} \cdot 2q_1 \cdot \frac{N-1}{N+1} \cdot 2q_2, \tag{12}$$

$$PDP_{XY-YX,2-S} = \frac{2}{N+1}(2q_3 - q_3^2) + \frac{N-1}{N+1}(2q_4^2 + q_5) + 2 \cdot \frac{2}{N+1} \cdot q_3 \cdot \frac{N-1}{N+1} \cdot q_4. \tag{13}$$

Eqs.(10) and (11) are based on the fact that two link or switch failures are independent of each other. In Eq.(12), q_1 equals $APL_{1-way}/[4N(N-1)]$ and q_2 equals $APL_{2-way}/[4N(N-1)]$ or $2q_1$. The q_1 stands for the failure probability of 1-way paths because of one link failure and q_2 stands for the failure probability of one of the two ways in 2-way paths when one link fails. The third term counts for the estimation of the common effect of two link failures on both 1- and 2-way paths. In this term the double of q_1 and q_2 are used since there are two unidirectional links between any adjacent nodes. In Eq.(13), q_3 and q_4 are equal to $(APL_{1-way}+1)/N^2$ and $(APL_{2-way}-1)/N^2$, respectively. Here, q_3 stands for the failure probability of 1-way paths because of one switch failure and q_4 stands for the failure probability of one of the two ways in 2-way paths (excluding the source and destination switches) when one switch fails. In addition, q_5 stands for the failure probability of the source or destination switches in a 2-way path and equals $4/N^2-1/N^4$.

For two network interface failures, PDPs for XY and XY - YX are the same and equal $4/N^2-1/N^4$.

2. Hot-spot traffic

In the hot-spot traffic pattern, one or more cores called hot-spot nodes receive more traffic in addition to the uniform traffic. Suppose that each core re-

ceives one packet and the hot-spot core receives E extra packets. Then, the fraction of all packets that the hot-spot node receives besides the packets from the uniform traffic is $h=E/(N^2+E)$. The derived formulae for the uniform traffic model are also applicable to the hot-spot traffic model, but APL used here is different from the parameter used in the uniform traffic. When one hot-spot node exists, APL is computed using Eq.(14) in terms of E and h :

$$APL_{hs} = \frac{E \cdot APL_{rel} + N^2 \cdot APL_{uni}}{N^2 + E} = h \cdot APL_{rel} + (1-h) \cdot APL_{uni}, \tag{14}$$

where APL_{rel} is the average path length of all the extra packets that enter the hot-spot node, which is computable based on the hot-spot location. For example, if anyone of the four adjacent nodes of the mesh center is a hot-spot, then APL_{rel} for that node is $N^3/[2(N^2-1)]$ and $(N^3+N)/[2(N^2-1)]$ for even and odd mesh sizes, respectively. If APL_{rel} is greater than APL_{uni} , then APL_{hs} will be greater than APL_{uni} and as a result, the PDP parameters for the hot-spot traffic will be greater than the ones for the uniform traffic.

In $N \times N$ mesh networks, the average path length of the packets entering the central nodes is smaller than the one for the corner nodes. Thus, if the hot-spot nodes are near the center, PDP will be lower.

When M hot-spot nodes exist in the mesh, the APL_{hs} parameter is computed as follows:

$$APL_{hs} = \frac{\sum_{i=1}^M E_i \cdot APL_{reli} + N^2 \cdot APL_{uni}}{N^2 + \sum_{i=1}^M E_i} = \sum_{i=1}^M h_i \cdot APL_{reli} + \left(1 - \sum_{i=1}^M h_i\right) \cdot APL_{uni}, \tag{15}$$

where $0 \leq \sum_{i=1}^M h_i \leq 1$ and $h_i = E_i / \left(N^2 + \sum_{i=1}^M E_i\right)$.

To compute PDP for XY routing, APL_{hs} is used; for XY - YX routing, we need to compute $APL_{2-way,hs}$ and $APL_{1-way,hs}$. The former can be computed using Eq.(7) when APL_{hs} and $APL_{1-way,hs}$ were computed, and the latter is computed similar to APL_{hs} in Eq.(15) according to

$$\begin{aligned} \text{APL}_{1\text{-way,hs}} &= (A + B) \left/ \left[2(N-1) \left(N^2 + \sum_{i=1}^M E_i \right) \right] \right. \\ &= \sum_{i=1}^M h_i \cdot \text{APL}_{1\text{-way,rel}i} + \left(1 - \sum_{i=1}^M h_i \right) \cdot \text{APL}_{1\text{-way}}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} A &= 2(N-1) \sum_{i=1}^M E_i \cdot \text{APL}_{1\text{-way,rel}i}, \\ B &= 2N^2(N-1) \cdot \text{APL}_{1\text{-way}}. \end{aligned}$$

3. Transpose traffic

In the first transpose traffic pattern, a core (i, j) sends the packets only to core $(N-1-j, N-1-i)$ in $N \times N$ mesh networks. But, with the second traffic pattern, a core (i, j) sends the packets only to core (j, i) . The APL parameter for the first transpose traffic model is computed using Eqs.(17) and (18):

$$\begin{aligned} \sum_{i=1}^{N^2-N} L(\text{Path}_i) &= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (|N-1-j-i| + |N-1-i-j|) \\ &= \frac{2}{3} N(N^2-1), \end{aligned} \quad (17)$$

$$\text{APL}_{\text{tp}1} = \frac{1}{N^2-N} \sum_{i=1}^{N^2-N} L(\text{Path}_i) = \frac{2}{3}(N+1), \quad (18)$$

where $L(\text{Path}_i)$ shows the length of the i th path, and the upper index of summation shows the number of paths in the first transpose traffic model. As shown in Eq.(18), $\text{APL}_{\text{tp}1}$ equals $\text{APL}_{2\text{-way}}$ in the uniform traffic model. Due to the symmetric relation between the first and second transpose patterns, $\text{APL}_{\text{tp}2}$ equals $\text{APL}_{\text{tp}1}$ in addition to the other reliability parameters. Thus, we use the notation ‘tp’ for these two transpose traffic models.

Similar to the uniform traffic, the PR parameters for transpose traffic models are computed with Eq.(1). In this way, we have the following equations:

$$\text{APR}_{XY,\text{tp}} = R_L^{\text{APL}_{\text{tp}}} \cdot R_S^{\text{APL}_{\text{tp}}+1} \cdot R_{\text{NI}}^2, \quad (19)$$

$$\text{APR}_{XY-YX,\text{tp}} = R_S^2 \cdot R_{\text{NI}}^2 \cdot \left[1 - \left(1 - R_S^{\text{APL}_{\text{tp}}-1} R_L^{\text{APL}_{\text{tp}}} \right)^2 \right]. \quad (20)$$

Eq.(20) is based on the fact that only 2-way paths exist in these traffic models.

(1) One component failure. The principal approach to analyze the network when some components fail, is the same for different traffic models. Therefore, when a link fails in an $N \times N$ mesh, the average probability that a packet does not reach the destination or reaches with incorrect data using XY routing under the transpose traffic equals $\text{APL}_{\text{tp}}/[4N(N-1)]$. Similarly, for a switch failure PDP equals $(\text{APL}_{\text{tp}}+1)/N^2$ and for a network interface failure equals $2/N^2$. For XY - YX routing, PDP for one link failure is zero (PCP is one) since there is no 1-way path in transpose traffic models. For one switch or one network interface failure, PDP equals $2/N^2$.

(2) Two component failures. The PDP parameters for XY routing in the case of two link failures and two switch failures are computed similar to Eqs.(10) and (11) but with the transpose traffic parameters.

For other traffic patterns such as complement and bit-reversal, similar formulations can be derived. For example, APL for the complement traffic in mesh NoCs equals N and based on this parameter PDPs are computable.

Torus topology

The formulations for different traffic patterns in torus topology are obtained similar to mesh topology. But the basic parameters such as APL and the number of links are different. In addition, the minimum network size N is 3.

1. Uniform traffic

The APL parameter for this traffic model in $N \times N$ torus networks is computed by

$$\text{APL}_{\text{uni}} = \begin{cases} N/2, & N = 2k-1, \\ N/2 + N/[2(N^2-1)], & N = 2k. \end{cases} \quad (21)$$

The average path reliabilities for XY and XY - YX are obtained similar to that of mesh topology but with the proper APL parameters. To compute APR_{XY-YX} in torus networks we first need to compute $\text{APL}_{1\text{-way}}$ and $\text{APL}_{2\text{-way}}$. $\text{APL}_{1\text{-way}}$ for $N \times N$ torus networks is computed using Eq.(22), but $\text{APL}_{2\text{-way}}$ is computed using Eq.(7) after computing APL_{uni} and $\text{APL}_{1\text{-way}}$ for $N \times N$ torus networks.

$$\text{APL}_{1\text{-way}} = \begin{cases} (N+1)/4, & N = 2k-1, \\ (N+1)/4 + 1/[4(N-1)], & N = 2k. \end{cases} \quad (22)$$

(1) One component failure. When a link fails in an $N \times N$ torus, the average probability that a packet does not reach the destination or reaches with incorrect data using XY routing is $PDP_{XY,1-L} = APL_{uni} / (4N^2)$ in which the denominator is the number of total uni-directional links in an $N \times N$ torus. This PDP parameter is smaller than that of mesh topology since the number of links in torus is greater. Similarly, we have $PDP_{XY,1-S} = (APL_{uni} + 1) / N^2$ for a switch failure and $PDP_{XY,1-NI} = 2 / N^2$ for a network interface failure. For XY - YX routing and a link failure, Eq.(23) is used:

$$PDP_{XY-YX,1-L} = \frac{2}{N+1} \cdot \frac{APL_{1-way}}{4N^2} = \begin{cases} 1 / (8N^2), & N = 2k - 1, \\ 1 / [8(N^2 - 1)], & N = 2k. \end{cases} \quad (23)$$

But for a switch failure PDP is computed using Eq.(24) obtained by replacing APL_{1-way} for mesh in Eq.(9) by APL_{1-way} for torus:

$$PDP_{XY-YX,1-S} = \begin{cases} \frac{5N+1}{2N^2(N+1)}, & N = 2k - 1, \\ \frac{5N-4}{2N(N^2-1)}, & N = 2k. \end{cases} \quad (24)$$

(2) Two component failures. Similar to mesh networks, Eqs.(10)~(13) are used for $N \times N$ torus networks but with the APL and PDP parameters obtained in this section and the number of total links equal to $4N^2$ instead of $4N(N-1)$.

2. Hot-spot traffic

Similar to the mesh topology, Eqs.(14)~(16) are applicable for torus topology; however, because of the structure of a torus and its extra links, APL_{rel} for all of nodes in a torus is the same and equals APL_{uni} . Thus, according to Eqs.(14)~(16) APL_{hs} equals APL_{uni} in torus networks. In addition, $APL_{1-way,hs}$ equals $APL_{1-way,uni}$ and $APL_{2-way,hs}$ equals $APL_{2-way,uni}$. This property leads to an important result:

In torus networks, the number and place of hot-spot nodes and the number of extra packets that enter these nodes have no effect on PDPs relative to the uniform traffic.

3. Transpose traffic

The APL parameter for this traffic model in $N \times N$ torus networks is computed by

$$APL_{tp} = \begin{cases} (N+1) / 2, & N = 2k - 1, \\ (N+1) / 2 + 1 / [2(N-1)], & N = 2k. \end{cases} \quad (25)$$

The average path reliabilities for XY and XY - YX are obtained similar to that of mesh topology using Eqs.(19) and (20) but with the new APL parameter.

One component failure. When a link fails in an $N \times N$ torus, the average probability that a packet does not reach its destination or reaches with incorrect data using XY routing equals $APL_{tp} / (4N^2)$. Similarly, for a switch failure PDP equals $(APL_{tp} + 1) / N^2$ and for a network interface failure equals $2 / N^2$. For XY - YX routing, similar to mesh PDP for one link failure in torus is zero. Also, for one switch or one network interface failure PDP equals $2 / N^2$.

We can derive similar formulations for other traffic patterns. For example, APL for the complement traffic in the torus NoCs equals $N/2$.

ANALYTICAL MODELS EXTENSIONS

Extensions to other routing algorithms

We can extend the reliability analysis to the partially or fully adaptive routing algorithms. Based on the analyses in previous sections, PDPs for XY - YX routing are smaller than that of XY routing. This is due to the existence of 2-way paths in XY - YX routing. Thus, having more adaptivity more reliability is achieved.

Even if a routing algorithm is fully adaptive and minimal, it is still vulnerable to a single permanent failure, since in the case of one link or switch failure in a path, the packets are blocked if the source and destination nodes belong to the same row or column. In addition, a network interface or a switch failure can at least prohibit the packets from entering or leaving from the related core.

Glass and Ni (1994) introduced three partially adaptive routing algorithms: west-first, north-last, and negative-first. All of these algorithms use the fully adaptive paths almost in half of the situations in average and use only one path (similar to XY) in the remaining situations. Since all the minimal routing algorithms suffer from the unreliability of 1-way paths, we can compute the minimum PDP for these algorithms based on PDP_{XY} and PDP_{XY-YX} . For example, in west-first routing if the X coordinate of the

destination is not less than the X coordinate of the source, then the algorithm can use one of the total shortest paths between the source and destination nodes (almost in half of the situations). Otherwise, it can use only one shortest path between them.

Thus, the minimum PDP in the west-first routing algorithm (also usable for north-last routing) for one link or switch failure is derived from Eq.(26) when the uniform traffic or other traffic patterns are used:

$$PDP_{\min} = U \left(\frac{1}{2} - \frac{1}{2(N+1)} \right) + V \left(\frac{1}{2} + \frac{1}{2(N+1)} \right). \quad (26)$$

For negative-first routing the following equation can be used:

$$PDP_{\min} = U \left(\frac{1}{2} - \frac{1}{N+1} \right) + V \left(\frac{1}{2} + \frac{1}{N+1} \right). \quad (27)$$

In Eqs.(26) and (27), U stands for PDP_{XY} for one link or switch failure and V stands for PDP_{XY-YX} for one link or switch failure in mesh or torus NoCs with respect to the used traffic pattern. In addition, for fully adaptive routing algorithms $PDP_{\min}=V$ or PDP_{XY-YX} for one link or switch failure. In these equations the coefficients are the average fractions of nodes in each half. The results are the minimum amount of PDPs since even for non 1-way paths in the fully adaptive half, it is probable that a packet be stopped when only one failure occurs. This case occurs when a packet reaches the node that is placed on the same row or column as the destination node and a failure occurs on the straight path between this node and the destination node. However, for more failures these partially adaptive routing algorithms have smaller PDPs compared with $XY-YX$ routing. The extensions for the other routing algorithms with different traffic models can similarly be derived.

Since the minimal fully adaptive routing algorithms are vulnerable even against one failure, the design of non-minimal adaptive routing algorithms is required to obtain more fault-tolerant routings. In the non-minimal and fault-tolerant routing algorithms many failures cannot lead to a packet drop, but they can increase the latency at least for the packets traversing the 1-way paths (this case is true for the source and destination nodes with the same row or column).

The latency is defined as the total time needed to convey a packet from a source to a destination and is the sum of the time needed to transmit a packet over the links, the time to send a packet from the core to the switch in the source node and from the switch to the core in the destination node, and the average waiting time in each switch multiplied by the number of switches in the path. The average waiting time (AWT) in the switches depends on the passing traffic rate and the structure of switches and routers. The average numbers of switches and links in a path equal $(APL+1)$ and APL , respectively. If the packet length is more than one flit, then the packet length in flits (Pkt_len) minus one is added to the total latency. Therefore, if we suppose sending a packet from the core to the switch and vice versa each takes one clock cycle and if we use the worm-hole switching in which one clock cycle is needed to transfer a flit over one link, then we can compute the average latency for a packet in cycles by

$$\text{Latency} = \text{AWT}(APL + 1) + APL + Pkt_len + 1. \quad (28)$$

On the other hand, in the non-minimal and fault-tolerant routing algorithms used in mesh NoCs each link or switch failure in a 1-way path increases both the path length and the number of nodes in the path at least by two (Fig.3). Thus, the minimum latency overhead due to a single failure equals $(2AWT+2)$ cycles for the 1-way paths. For the whole NoC it is smaller since a failure leads to an increase in latency for a fraction of paths.

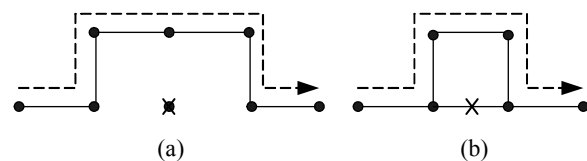


Fig.3 Possible effect of (a) a switch and (b) a link failure on the 1-way paths using a non-minimal and fault-tolerant routing algorithm

Analysis of an improvement method

Since a switch failure has more impact on the NoCs compared to a link or a network interface failure, we will analyze a method in which the permanent faults in the switches are partially tolerated. Kim *et al.*(2006) introduced a decoupled router architecture with a row-column switch in which for some

permanent faults, the switch bypasses the incoming packets in the same direction. A similar work was presented in Greenfield *et al.*(2007) in which the bypass is performed in a mode, called ‘through traffic mode’ (or bypass mode), using a wrapper.

In XY routing when a switch acts in the bypass mode, probably the local core (depending on the architecture) but definitely the cores that are accessible by turning or changing the direction in this switch will be unreachable. Assuming XY routing under the uniform traffic, the maximum amount of PDP when a faulty switch acts in the bypass mode is computed by

$$PDP_{XY} = \frac{(N-1)^2 L_1 + 2(N^2-1)L_2}{N^2(N^2-1) \cdot APL_{uni}} = \frac{3N+1}{N^2(N+1)}, \quad (29)$$

where the first term in the numerator, $(N-1)^2 L_1$, represents the number of paths that should turn in the faulty switch multiplied by their average path length, and the second term, $2(N^2-1)L_2$, represents the number of paths in which the local core connected to the faulty switch acts as a source or destination, multiplied by their average path length. In average, L_1 and L_2 are equal to APL_{uni} , which leads to the last result in Eq.(29). If only the turns are impossible (which means the local core is accessible in this switch), then

$$PDP_{XY} = \frac{(N-1)^2 L_1}{N^2(N^2-1) \cdot APL_{uni}} = \frac{N-1}{N^2(N+1)}. \quad (30)$$

For XY - YX routing, PDP simply equals $2/N^2$ since the packets are dropped only when the local core is the source or destination. Simulation results show noticeable improvement when a failure occurs in a switch and it acts in bypass mode. Similar analysis can be done for other traffic patterns.

EVALUATION AND DISCUSSION

Simulation setup

In this section we illustrate the analytical results and compare them with the simulation results. To verify the correctness of the analytical assessments, appropriate simulations were conducted using a modified version of Noxim (Fazino *et al.*, 2008), a

cycle-accurate open source SystemC simulator of a mesh-based NoC. We modified this simulator to support fault injection and torus topology. Since the obtained analytical results are independent of packet length, buffer size and packet injection rate, in all these experiments, the packet length and buffer size were set to four flits and the packet injection rate was set to 0.01 packet/cycle/node to be far enough from the saturation point for different NoC sizes. The experiments were carried out on $N \times N$ NoCs with N from 3 to 10.

Each simulation was initially run for 10000 cycles to allow transient effects to stabilize and afterwards, it was executed for 40000 cycles in each iteration that includes some injected faults. To obtain the simulation results for one link or switch failure in each scenario, 200 to 500 iterations each including a random fault were executed. In this manner, the minimum number of injected packets into $N \times N$ NoCs is about 7×10^5 for $N=3$ to 8×10^6 for $N=10$. In addition, for more link or switch failures in each scenario, at least 1000 iterations each including some random faults were executed. So the minimum number of injected packets into $N \times N$ NoCs is about 3.6×10^6 for $N=3$ to 4×10^7 for $N=10$.

Experimental results

Before showing the results obtained from both analytical models and simulations, we present some simulation results to give a perception about how different numbers of failures can affect NoC reliability. Fig.4 presents the packet drop probabilities as a function of different numbers of link failures for XY and XY - YX routing algorithms when the mesh dimension N varies from 4 to 6 under the uniform traffic pattern. PDPs increase almost linearly with the number of failures. Fig.5 illustrates the effects of the same defect ratios on PDPs for different mesh sizes and routing algorithms under the uniform traffic pattern. We selected 4.2% and 11.1% defect ratios for link and switch failures, respectively, because a 3×3 mesh with one link or switch failure has these defect ratios. In Fig.5 PDPs vary almost linearly for N equal to 3 to 7, but actually the slope is reduced when N is increased. Based on Figs.4 and 5 and the fact that the number of nodes in NoCs is increasing, it is clear that it is important to analyze and design the permanent fault-tolerant NoCs.

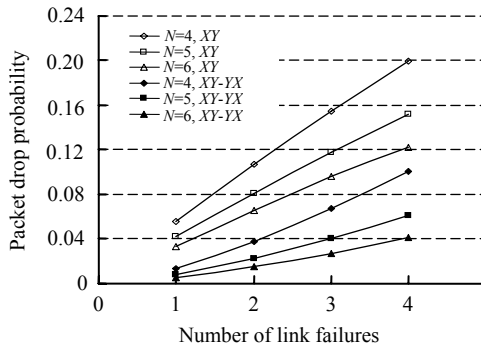


Fig.4 Unreliability as a function of different numbers of link failures and mesh sizes

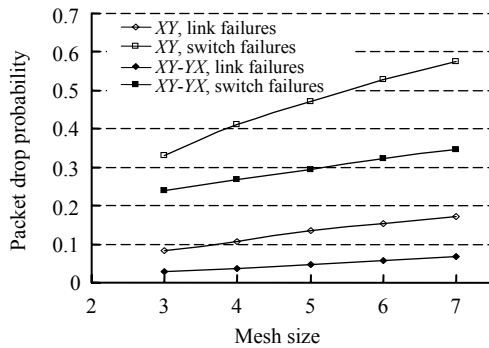


Fig.5 Effect of mesh size and the same percentage of failures on unreliability

The next figures that show PDPs for at most two failures make sense when we want to find the intrinsic reliability of different NoCs against the permanent run-time faults or manufacturing faults. Since APL increases linearly with N and the number of links or switches in $N \times N$ NoCs increases proportionally to the square of N , PDP decreases with the reverse of N ($O(1/N)$). Figs.6a and 6b present the packet drop probabilities when one or two link or switch failures occur in $N \times N$ meshes under the uniform traffic pattern. Fig.6a presents the results assuming XY and $XY-YX$ routing algorithms for link failures and Fig.6b presents the results for switch failures. Figs.6c and 6d illustrate the similar results under the hot-spot traffic pattern in which four adjacent nodes of the mesh center are all hot-spot nodes with $h=0.10$. In Figs.6c and 6d, for XY routing the analytical and simulation results are shown, but for $XY-YX$ only the simulation results are shown, which are reasonable and in conformance with the results for this routing in Figs.6a and 6b. Since the hot-spot nodes are placed near the mesh center, their relative APLs are smaller than the

overall APL under the uniform traffic. Thus, based on Eq.(15) APL under the hot-spot traffic is smaller than APL under the uniform traffic and as a result PDPs in Figs.6c and 6d are smaller than the ones in Figs.6a and 6b. In general, the effect of hot-spot nodes on PDPs is not great. As shown in Fig.6, PDPs for $XY-YX$

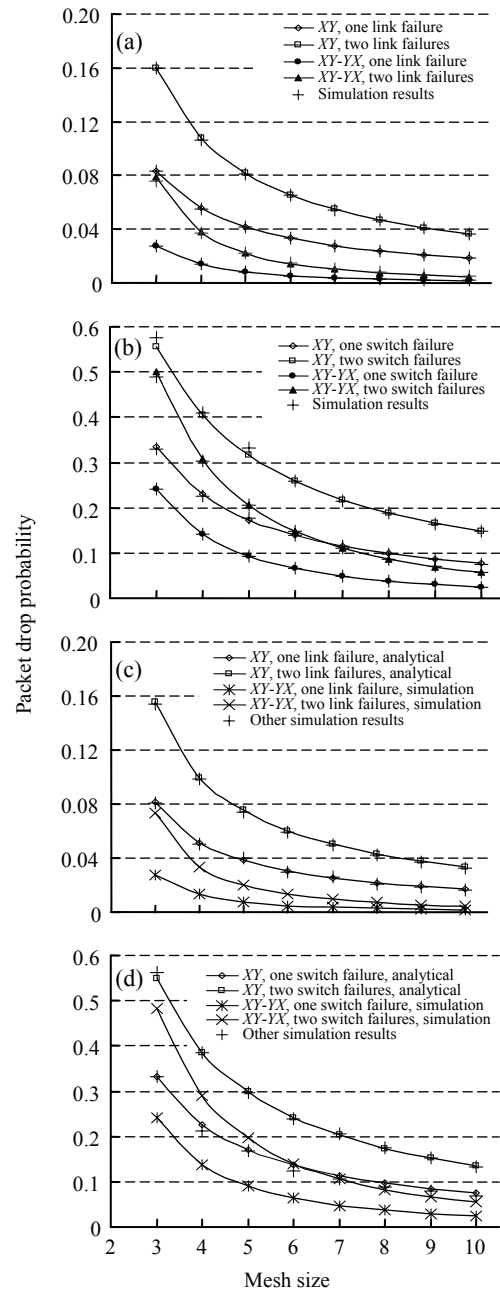


Fig.6 Unreliability as a function of mesh size (a) under the uniform traffic pattern for link failures, (b) under the uniform traffic pattern for switch failures, (c) under the hot-spot traffic pattern for link failures, and (d) under the hot-spot traffic pattern for switch failures

are smaller than the ones for XY with equal conditions. Also, PDPs for two link failures using XY - YX are always smaller than the ones for one link failure using XY routing. But for switch failures this is true for N greater than 6.

Fig.7a presents the packet drop probabilities when one link or switch failure occurs in $N \times N$ meshes under the transpose traffic pattern. In this figure, for XY - YX routing only the analytical results are shown since the simulation results are identical to the analytical ones as their equations are simple. In addition, PDPs for XY routing under the transpose traffic are greater than those under the uniform traffic in Figs.6a and 6b, whereas for XY - YX the opposite is true.

Figs.7b and 7c show the packet drop probabilities for link and switch failures in $N \times N$ torus NoCs under the uniform and transpose traffic patterns, respectively. As seen in these figures, PDPs for torus NoCs are smaller than the ones for mesh NoCs (Figs.6a, 6b and 7a) with equal conditions since torus networks have shorter APLs and more links relative to mesh networks. Fig.8 illustrates some simulation results for $N \times N$ torus NoCs under the hot-spot traffic pattern. In two hot-spots scenario the hot-spot nodes are placed in the corners with $h=0.06$, but in four hot-spots scenario the hot-spot nodes are placed around the center with $h=0.10$. This figure confirms the analytical results stated before, as different numbers and places of hot-spot nodes and the amount of extra packets that enter these nodes have no effect on PDPs, and all are equal to the ones under the uniform traffic.

In the illustrated figures the analytical results are identical or very close to the simulation results. For example, in Figs.6a and 6b the difference between the analytical and simulation results varies from 0 to 3%. Besides, it must be noted that in all conditions PDPs will be rapidly increased when the number of failures increases.

Fig.9 shows the effect of h and different numbers of hot-spots around the mesh center on PCP when the network size N is 6. As shown in this figure, PCPs are slowly increased while h is increased. In addition, in one switch failure scenario the number of hot-spots has more effect on PCP relative to one link failure scenario.

To validate the improvement method stated before, Fig.10 presents the results for XY routing under

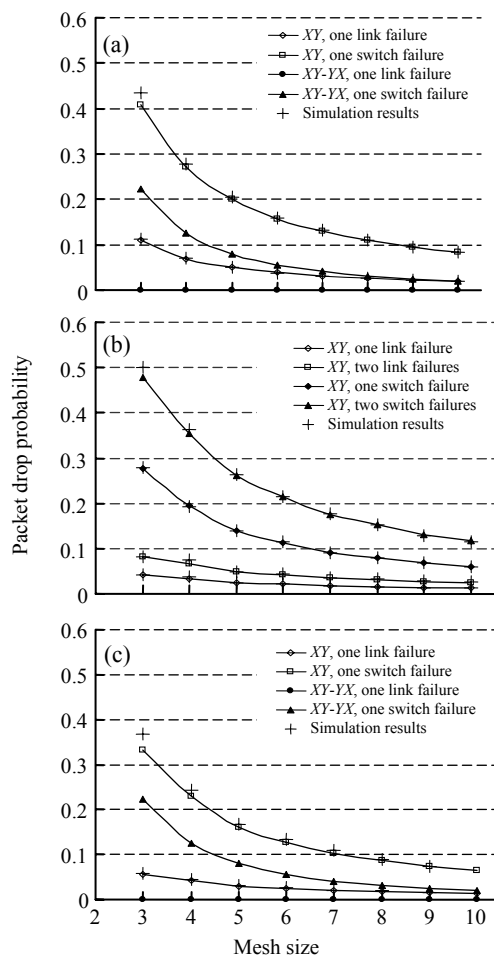


Fig.7 Unreliability as a function of (a) mesh size under the transpose traffic pattern, (b) torus size under the uniform traffic pattern, and (c) torus size under the transpose traffic pattern

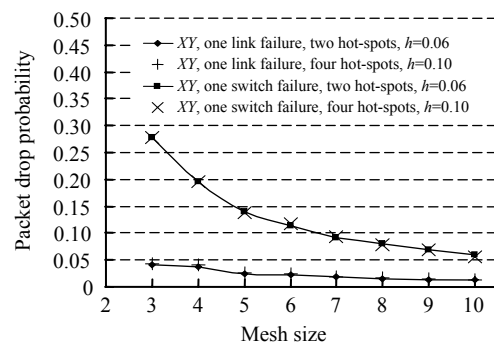


Fig.8 Effect of different hot-spot scenarios on unreliability in torus NoCs

the uniform traffic when a switch fails. In this figure, the analytical and simulation results are identical, which confirms Eqs.(29) and (30). In the normal

mode the whole switch fails where its PCP is the complement of a curve in Fig.6b. But in the bypass mode, there exist two different situations depending on the accessibility to the local core (Eqs.(29) and (30)). As shown in Fig.10, using the bypass switches can greatly enhance the system reliability.

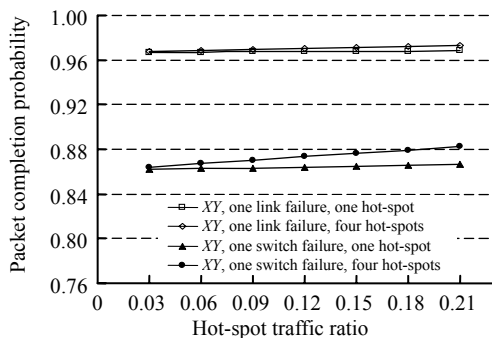


Fig.9 Effect of h and the number of hot-spots on reliability

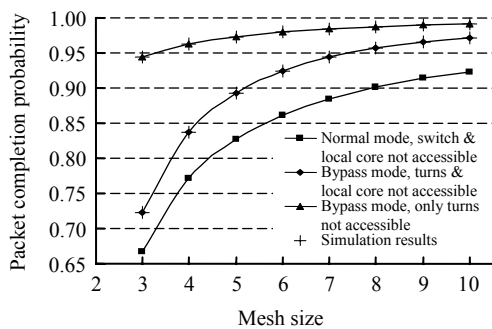


Fig.10 Effect of a bypass switch on reliability

In Fig.11 the effect of some link and switch failures on the average latency of the packets traversing the 1-way paths in a 5×5 mesh NoC is presented according to Eq.(28) when a non-minimal and fault-tolerant routing algorithm is used based on Fig.3. The results are analytical and obtained with the assumption that at most one failure occurs in the vicinity of a switch on a 1-way path. As stated before, the minimum latency overhead for both one link and one switch failure equals $(2AWT+2)$ cycles for the 1-way paths where the failure occurs. However, since switch failures impact more fractions of 1-way paths relative to link failures, their overall latency is higher (Fig.11).

In addition to the design of fault-tolerant routing algorithms, design of fault-tolerant components can be very beneficial. In general, the failure probabilities of cores, mainly processor elements and memory blocks in MP-SoCs, are greater than those for NoC

elements due to their larger areas. Thus, designing the fault-tolerant cores can highly increase the overall reliability. However, a switch failure can cause its eight adjacent links, one network interface, and consequently the local core to become unusable, which means the impact of one switch failure is higher than the sum of impacts of one link and one network interface (or core) failure. As a result, the design of permanent fault-tolerant switches with either bypass mode or another method is required, and it is more effective than the design of fault-tolerant links or network interfaces.

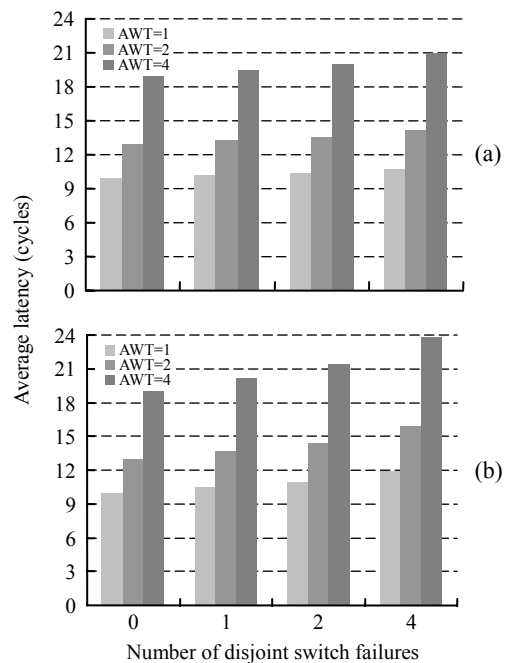


Fig.11 The latency due to (a) effect of link failures and AWT and (b) effect of switch failures and AWT

CONCLUSION AND FUTURE WORK

In this paper using the probability models and defining some beneficial parameters, the inherent reliabilities of mesh- and torus-connected NoCs are analytically evaluated under different routing algorithms and traffic patterns. The analytical results are identical or very close to the simulation results, confirming the correctness of the models. This analysis can be groundwork for reliability assessment of more realistic NoCs with more practical traffics to find the inherent reliability and design of more reliable ones.

This may be performed by also considering characteristics such as performance (including throughput and latency) and power consumption to obtain more precise analytical reliability models. Besides, it would be beneficial to investigate the effects of permanent faults in different parts of a switch or router on the whole NoC. In addition, based on the presented reliability models it is feasible to assess the reliability improvement methods and fault-tolerant designs to validate and quantify their enhancements. These methods may be incorporated in either the architectures or routing algorithms.

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