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Effects of rarefaction on the characteristics of micro gas journal bearings*

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Abstract: Given the definition of the reference Knudsen number for micro gas journal bearings, the range in the number is related to the viscosity of air at different temperatures. A modified Reynolds equation for micro gas journal bearings based on Burgdorfer's first-order slip boundary condition is proposed that takes into account the gas rarefaction effect. The finite difference method (FDM) is adopted to solve the modified Reynolds equation to obtain the pressure profiles, load capacities and attitude angles for micro gas journal bearings at different reference Knudsen numbers, bearing numbers and journal eccentricity ratios. Numerical analysis shows that pressure profiles and non-dimensional load capacities decrease markedly as gas rarefaction increases. Attitude angles change conversely, and when the eccentricity ratio is less than 0.6, the attitude angles rise slightly and the influence of the reference Knudsen number is not marked. In addition, the effect of gas rarefaction on the non-dimensional load capacity and attitude angle decreases with smaller bearing numbers.

Key words: Reference Knudsen number, Rarefaction effect, Reynolds equation, Finite difference method (FDM)
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1 Introduction

The velocity slip phenomenon of rarefied gases has remained an intriguing problem in the fluid flow of micro-electro-mechanical systems (MEMS) devices, such as in the air lubrication of the head-disk interface of disk drives in the data storage industry (Gad-el-Hak, 1999; Myong, 2004; Karniadakis and Beskok, 2005). Similarly, because the rotor in a micro gas journal bearing-rotor system has an extremely high rotation velocity, the surface speed of the journal comes close to the speed of sound and the gap between the journal and the bearing is only ten or so microns, a gas lubrication film of even a few microns will become rarefied because of the large velocity gradient (Schaaf and Chambre, 1958).

According to the theory of rarefied gas dynamics (Kennard, 1938; Shen, 2003), when a gas becomes rarefied, velocity slip will occur at the surface of the journal and then the speed distribution of the gas lubrication film will change. This will obviously have an effect on the performance of micro gas bearings. Burgdorfer (1959) first proposed a Reynolds equation based on first-order velocity slip boundary condition to investigate the influence of rarefied gas on the performance of gas bearings. Hsia and Domoto (1983) later presented a second-order velocity slip model and the corresponding Reynolds equation by considering the second-order molecular slip effects. Mitsuya (1993) modified the coefficient of the second-order slip term in the second-order velocity slip model, to produce a 1.5-order velocity slip model. Beskok and Karniadakis (1999) proposed an empirical model by modifying the first-order slip coefficient, but the coefficients are unknown and can be obtained only by

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related theory or experiments. Sun *et al.* (2002) developed a new velocity slip model incorporating molecular dynamics to take into account the impact of molecular collisions, which play an important role in the interactions between molecules and the solid surface. Wu and Bogy (2003) modified the length scale of Taylor's expansion in the second-order velocity slip boundary condition in accordance with kinetic theory and obtained a new model of first-order, second-order velocity slip boundary condition. Shen *et al.* (2007) proposed a new model of first-order velocity slip boundary condition based on kinetic theory. Recently Wu (2008) derived a slip model for rarefied gas flows at an arbitrary Knudsen number using a somewhat more physical approach. Recent research showed that although the slip boundary can be classified into first-, second- and higher orders, only the first-order slip boundary model has actual physical interpretation. The others are based mainly on mathematical theory. The basic equation describing rarefied gas dynamics, the Boltzmann equation, is first-order in nature and thus the slip boundary should be first-order rather than higher orders. Moreover, the first-order velocity slip can avoid the ambiguity of determining the coefficients of the high order term in high order slip boundary condition.

Micro gas journal bearings have been investigated widely at home and abroad. Orr (2000) magnified the configuration of the micro gas journal bearing to study its performance using a macro gas journal bearings model. Piekos (1999; 2002) used numerical simulation to assess the performance of the micro gas journal using the orbit method. Liu *et al.* (2005) and Liu and Spakovszky (2007) proposed a new analytical model for axially fed gas journal bearings and used experimental testing of micro gas bearings to characterize and investigate their rotor dynamic behavior. Teo *et al.* (2006; 2008) investigated the unsteady flow effects in the bearing gap to quantify their impact on the bearing dynamic behavior. He developed a gas-bearing supported micro-air turbine to demonstrate repeatable, stable high-speed gas-bearing operation and verified the new micro-gas-bearing analytical models. None of the above studies considered the gas rarefaction effect. Lee *et al.* (2005) considered the gas rarefaction effect in an analysis of micro gas journal bearing performance in the first-order boundary condition. However, they ignored the influence of ambient pressure on the degree of gas rarefaction. Research on micro gas journal bearings started only recently in China. Based on the first-order, second-order slip boundary condition, Huang *et al.* (2006) investigated the steady characteristics of micro gas journal bearings at different bearing numbers and different eccentricity ratios. Zhou *et al.* (2007) analyzed the influence of gas temperature on the performance of micro gas journal bearings in the second-order slip boundary condition, based on the bearing number, gas temperature and the length/diameter ratio of the bearings. Huang *et al.* (2006) and Zhou *et al.* (2007) considered the gas rarefaction effect in the micro gas journal bearing, but ignored ambient pressure and temperature effects, or considered only the gas temperature.

When the bearing configuration is determined, the Knudsen number symbolizing the degree of gas rarefaction is proportional to the molecular mean free path, which is related to the gas viscosity, pressure, and temperature. Because gas viscosity is related mainly to temperature, the Knudsen number is influenced by two factors, pressure and temperature. This paper first gives the definition of the reference Knudsen number Kn_0 for micro gas journal bearings, and then presents reference Knudsen numbers at different temperatures for a range of viscosity values. A modified Reynolds equation for the micro gas journal bearings based on Burgdorfer's first-order slip boundary condition is then proposed. The finite difference method (FDM) is applied to solve the modified Reynolds equation to obtain the steady characteristics of micro gas journal bearings at different reference Knudsen numbers, bearing numbers and eccentricity ratios.

2 Variation in reference Knudsen number with temperature

The viscosity of gases, a physical property, can be obtained from experiments or from approximate calculations from theoretical models. The values of viscosity for air increase gradually with increasing temperature (Fig. 1).

In micro gas journal bearings, the reference Knudsen number Kn_0 is defined as: $Kn_0=\lambda/c$, where $\lambda = \mu\sqrt{\pi RT} / \sqrt{2} p_a$, the molecular mean free path, c

is the average radial clearance, μ is gas viscosity, R is a gas constant, T is the ambient temperature, and p_a is the ambient pressure. According to Irvine and Liley (1984), Fig. 2 is given to show the range of reference Knudsen numbers for a gas lubrication film of 10 μm average radial clearance at different ambient pressures and increasing temperature, where p_0 stands for a standard atm. The reference Knudsen number Kn_0 increases with increasing temperature. Thus, air becomes more rarefied at higher temperatures. As the ambient pressure increases, Kn_0 decreases. When the ambient pressure changes from one to ten standard atm, the values of Kn_0 are in the range between 10^{-3} and 10^{-1}. According to the kinetic theory of gases, the flow is located in the slip region. So the velocity boundary with the slip at the gas-solid interface should be taken into account.

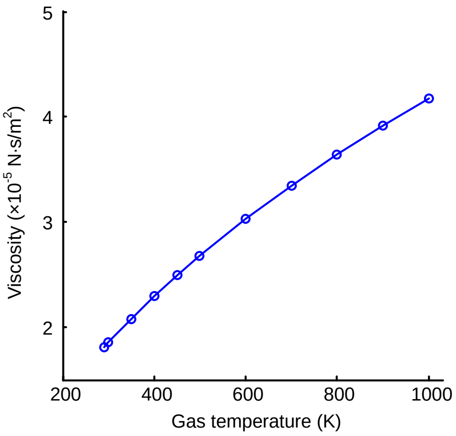

Fig. 1 Viscosity of air at different temperatures

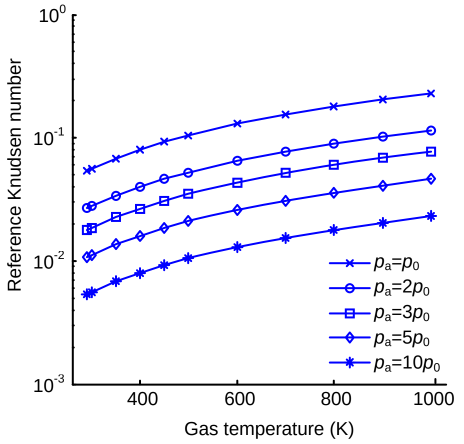

Fig. 2 Reference Knudsen numbers under different ambient pressures

3 Reynolds equation for micro gas journal bearings

In Fig. 3, the inner circle represents the journal, and the outer shadowed ring, the bearing. The journal rotates at a high angular velocity, ω, inside the stationary bearing. The radius of the journal is r, and of the bearing is $r+c$, where c is the average radial clearance. The distance e between O_b and O_j is the eccentricity, ϕ is the attitude angle, and L is the width of the bearing. For convenience, the flows in and out of the sides of the bearing are ignored, the flow is assumed to be isothermal and the time-dependent effect is not considered. Based on first-order slip boundary condition, the modified Reynolds equation is given as

$$\begin{aligned}&\frac{\partial}{\partial\theta}\left[ph^3\frac{\partial p}{\partial\theta}\left(1+6\frac{\lambda(p,T)}{h}\right)\right]\\&+r^2\frac{\partial}{\partial y}\left[ph^3\frac{\partial p}{\partial y}\left(1+6\frac{\lambda(p,T)}{h}\right)\right]\\&=6\mu(T)u_0r\frac{\partial}{\partial\theta}(ph),\end{aligned}\qquad(1)$$

where p is the gas film pressure, h the gas film thickness, λ the molecular mean free path, r the radius of the journal, $u_0=r\omega$, the surface velocity of the journal, Θ and y are the coordinate variables along the circumferential and axial directions respectively, and ζ is the dimensionless form for y.

Written in the dimensionless form, Eq.(1) becomes

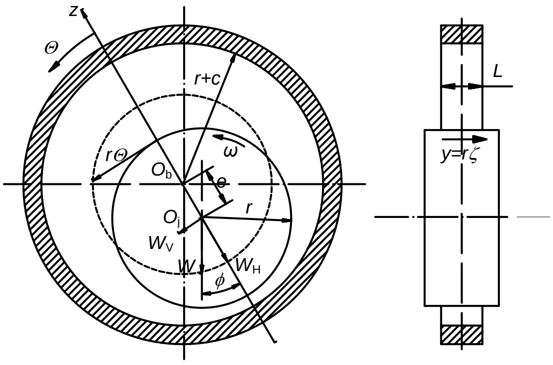

Fig. 3 Enlarged schematic sketch of micro gas journal bearings

$$\begin{aligned}&\frac{\partial}{\partial\theta}\left[\frac{\partial P}{\partial\theta}\left(PH^3+6Kn_0(p_\mathrm{a},T)H^2\right)\right]\\&+\frac{\partial}{\partial\zeta}\left[\frac{\partial P}{\partial\zeta}\left(PH^3+6Kn_0(p_\mathrm{a},T)H^2\right)\right]\\&=\Lambda\frac{\partial}{\partial\theta}(PH),\end{aligned}\qquad(2)$$

where $\zeta=y/r$, $P=p/p_\mathrm{a}$. The bearing number $\Lambda(p_\mathrm{a},T)=\frac{6\mu(T)\omega}{p_\mathrm{a}}\left(\frac{r}{c}\right)^2$, p_a is the pressure at a reference position and T is the temperature. $H=h/c=1+\varepsilon\cdot\cos\theta$ and ε is the eccentricity ratio, $\varepsilon=e/c$.

For convenience, the reference Knudsen number is defined as the ratio of the molecular free path λ to the average radial clearance c, i.e., $Kn_0(p_\mathrm{a}, T)=\lambda(p_\mathrm{a}, T)/c$, which indicates the degree of rarefaction of the gas lubrication flow and is determined by the ambient pressure and temperature of the gas flow.

The boundary condition for solving Eq. (2) is as follows:

(a) The gas pressure on both ends of the bearing equals the ambient pressure p_a, i.e., $P\left(\theta,\pm\frac{L}{2r}\right)=1.$

(b) The gas pressure P is an even function for ζ, i.e., $P(\theta, \zeta)=P(\theta, -\zeta)$.

(c) The gas pressure P is continuous at $\zeta=0$, i.e., $\left.\frac{\partial P}{\partial\zeta}\right|_{\zeta=0}=0.$

(d) The gas pressure P is a periodic function for Θ, i.e., $P(\theta, \zeta)=P(\theta+2\pi, \zeta)$

The load capacity of micro gas journal bearings consists of two components, one along the line of centers and the other normal to the line of centers (Fig. 3). The corresponding load capacities for micro gas journal bearings are given as

$$W_\mathrm{H}=2\int_0^{L/2}\int_0^{2\pi}pr\cos\theta\,\mathrm{d}\theta\,\mathrm{d}y,\qquad(3)$$

$$W_\mathrm{V}=2\int_0^{L/2}\int_0^{2\pi}pr\sin\theta\,\mathrm{d}\theta\,\mathrm{d}y.\qquad(4)$$

The dimensionless load capacity and attitude angle are given by

$$W=\sqrt{W_\mathrm{H}^2+W_\mathrm{V}^2}\Big/(p_\mathrm{a}2\pi rL),\qquad(5)$$

$$\phi=\arctan(W_\mathrm{V}/W_\mathrm{H}).\qquad(6)$$

4 Numerical analysis

In micro gas journal bearings, the modified Reynolds equation is a nonlinear partial differential equation. As an analytical solution for such an equation is difficult to obtain, a numerical method is applied for its solution. In this study, the central finite difference scheme is used to discretize the modified Reynolds equation in the θ and ζ directions. For simplicity, a uniform mesh size is employed. Therefore, Eq. (2) can be transformed into

$$\begin{aligned}&\left(P_{i,j}^{(k)}H_{i,j}^3+6Kn_0H_{i,j}^2\right)\frac{P_{i+1,j}^{(k+1)}-2P_{i,j}^{(k+1)}+P_{i-1,j}^{(k+1)}}{(\Delta\theta)^2}\\&+\left(P_{i,j}^{(k)}H_{i,j}^3+6Kn_0H_{i,j}^2\right)\frac{P_{i,j+1}^{(k+1)}-2P_{i,j}^{(k+1)}+P_{i,j-1}^{(k+1)}}{(\Delta\zeta)^2}\\&=\Lambda H_{i,j}\frac{P_{i+1,j}^{(k+1)}-P_{i-1,j}^{(k+1)}}{2\Delta\theta}+\Lambda\frac{\partial H_{i,j}}{\partial\theta}P_{i,j}^{(k)},\end{aligned}\qquad(7)$$

where $P_{i,j}^{(k+1)}, P_{i,j}^{(k)}$ are the current and last iteration values respectively, for the pressure of grid point (i, j). The successive over relaxation (SOR) method is used and the pressure distribution for the bearing can be obtained using an iterative calculation process.

The values of the main parameters for the bearing in the analysis are as follows: r=2.0 mm, L=0.4 mm, $p_\mathrm{a}=1.033\times10^5$ N/m^2, $\mu=1.88\times10^{-5}$ Pa·s. The circumferential dimensionless pressure profiles in the middle section of the micro gas journal bearings are given in Fig. 4, where the bearing number L is 2.4 and the reference Knudsen number is 0, 0.01, 0.1 or 0.5. The results show that the reference Knudsen number has a significant effect on the pressure profiles of micro gas journal bearings. As the reference Knudsen number Kn_0 increases (i.e., the degree of gas rarefaction increases), the dimensionless pressure decreases significantly. In particular, the pressure becomes quite low when Kn_0 is above 0.5. From the results, we conclude that a high degree of gas rarefaction would result directly in a low load capacity, which would impair the performance of micro gas journal bearings.

Fig. 5 shows the dimensionless load capacities W and Fig. 6 the attitude angles ϕ for micro gas journal bearings under different journal eccentricity ratios when the bearing number is 2.4.

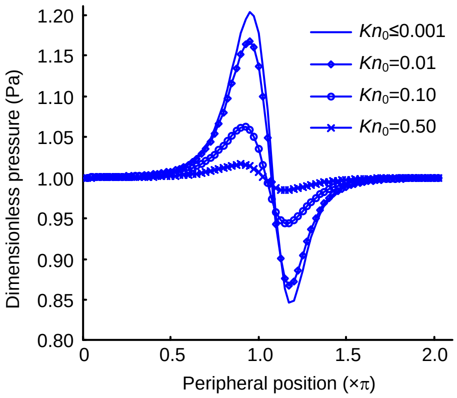

Fig. 4 Pressure profiles in the middle section of micro gas journal bearings (ζ=0)

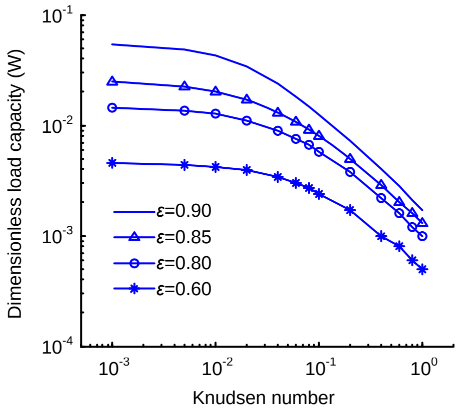

Fig. 5 Load capacities for micro gas journal bearings under different eccentricity ratios

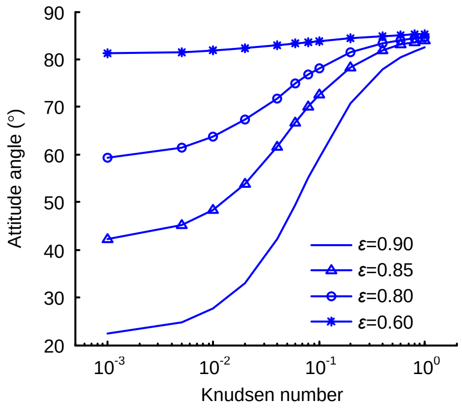

Fig. 6 Attitude angles for micro gas journal bearings under different eccentricity ratios

For a given eccentricity ratio, the dimensionless load capacity for micro gas journal bearings decreases as the Knudsen number Kn_0 increases. When the eccentricity ratio e is changed, the dimensionless load capacity shows a similar trend. Moreover, as the eccentricity ratio increases, the dimensionless load capacity increases.

In contrast, for a given eccentricity ratio, the attitude angles for micro gas journal bearings increases as the Knudsen number Kn_0 increases. The attitude angles show a similar trend at different eccentricity ratios, but when the eccentricity ratios increase, the change in the attitude angles increases. The influence of the Knudsen number on the attitude angle is not obvious when the eccentricity ratio is below 0.6, and the attitude angles are above 80°. According to fluid lubrication theory, the smaller are the attitude angles, the smaller is the vertical component of lubrication film force, and the more stable is the bearing. Therefore, the stability of micro gas journal bearings declines with larger reference Knudsen numbers.

The dimensionless load capacities and the attitude angles for micro gas journal bearings under different reference Knudsen numbers are given in Figs. 7 and 8, respectively. The dimensionless load capacities increase as the bearing number increases. With the same bearing numbers, the dimensionless load capacities decrease as the reference Knudsen number increases.

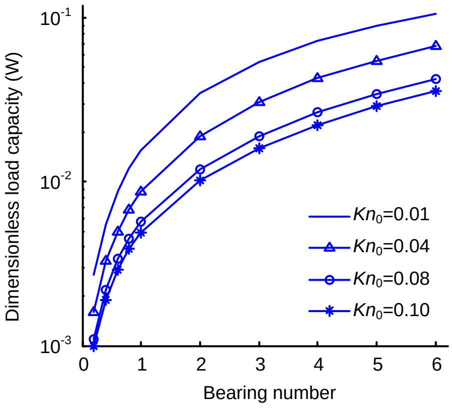

Fig. 7 Load capacities for micro gas journal bearings under different Kn_0

Unlike the dimensionless load capacities, the attitude angles decrease as the bearing number increases, and the change in attitude angles declines with increasing reference Knudsen numbers. However, when the bearing number is low (i.e., when the journal angular velocity is low), the effect of different reference Knudsen numbers on the dimensionless load capacity and the attitude angle is reduced.

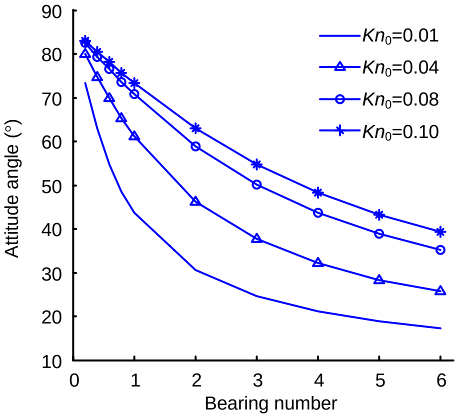

Fig. 8 Attitude angles for micro gas journal bearings under different Kn_0

From the above discussion, rarefied gas lubrication flow can have a significant effect on the characteristics of micro gas journal bearings. The unique performance of the bearings derives from the fact that they operate at a micro scale and rarefied gas changes the gas-solid boundary condition for the bearings and introduces velocity slip.

5 Conclusion

In MEMS devices, the characteristics of micro gas journal bearings differ from those of macro gas journal bearings. The conventional boundary condition is changed and a modified Reynolds equation is used. In this paper, the definition of the reference Knudsen number for micro gas journal bearings is given and the range of the reference Knudsen number is outlined according to the viscosity values of air at different temperatures. A modified Reynolds equation for micro gas journal bearings, based on Burgdorfer's first-order slip boundary condition, is given which takes account of the effects of gas rarefaction. The finite difference method is applied to solve the modified Reynolds equation to obtain the pressure profiles, load capacities and attitude angles for micro gas journal bearings at different reference Knudsen numbers, bearing numbers and eccentricity ratios. Numerical analysis confirmed that the reference Knudsen number has considerable influence on the performance of micro gas journal bearings. With larger reference Knudsen numbers, i.e., increased rarefaction, the pressure and non-dimensional load capacities decrease markedly and the attitude angles increase. With the same bearing numbers, the influence of reference Knudsen number on non-dimensional load capacity and the attitude angle is significant. Therefore, the effect of gas rarefaction should not be ignored in the analysis and design of micro gas bearings.

References

Beskok, A., Karniadakis, G.E., 1999. A model for flows in channels, pipes and ducts at micro and nano-scales. *Nanoscale and Microscale Thermophysical Engineering*, **3**(1):43-77. [doi:10.1080/108939599199864]

Burgdorfer, A., 1959. The influence of the molecular mean free path on the performance of hydrodynamic gas lubricated bearings. *Journal of Basic Engineering*, **81**(1):94-100.

Gad-el-Hak, M., 1999. The fluid mechanics of microdevices—the freeman scholar lecture. *Journal of Fluids Engineering*, **121**(1):5-33. [doi:10.1115/1.2822013]

Hsia, Y.T., Domoto, G.A., 1983. An experimental investigation of molecular rarefaction effects in gas lubricated bearings at ultra-low clearances. *Journal of Lubrication Technology*, **105**(1):120-130.

Huang, H., Meng, G.., Zhao, S., 2006. The effects of second-order slip flow on the steady performance of micro gas bearings. *Chinese Journal of Theoretical and Applied Mechanics*, **38**(5):68-673 (in Chinese).

Irvine, T.F., Liley, P.E., 1984. Steam and Gas Tables with Computer Equations. Academic Press, Orlando.

Karniadakis, G.E., Beskok, A., 2005. Micro Flow Fundamentals and Simulation. Springer-Verlag, New York, USA.

Kennard, E.H., 1938. Kinetic Theory of Gases. McGraw-Hill, New York, USA.

Lee, Y., Kwak, H., Kim, C., Lee, N., 2005. Numerical prediction of slip flow effect on gas-lubricated journal bearings for MEMS/MST-based micro-rotating machinery. *Tribology International*, **38**:89-96. [doi:10.1016/j.triboint.2004.01.003]

Liu, L.X., Teo, C.J., Epstein, A.H., Spakovszky, Z.S., 2005. Hydrostatic gas journal bearings for micro-turbomachinery. *Journal of Vibration and Acoustics*, **127**(2):157-164. [doi:10.1115/1.1897738]

Liu, L.X., Spakovszky, Z.S., 2007. Effects of bearing stiffness anisotropy on hydrostatic micro gas journal bearing dynamic behavior. *Journal of Engineering for Gas Turbines and Power*, **129**(1):177-184. [doi:10.1115/1.2180813]

Mitsuya, Y., 1993. Modified Reynolds equation for ultra-thin film gas lubrication using 1.5-order slip flow model and considering surface accommodation coefficient. *Journal of Tribology*, **115**:289-294. [doi:10.1115/1.2921004]

Myong, R.S., 2004. Gas slip models based on the Langmuir adsorption isotherm. *Physics of Fluids*, **16**(1):104-117. [doi:10.1063/1.1630799]

Orr, D.J., 2000. Macroscale Investigation of High Speed Gas

Bearing for MEMS Devices. PhD Thesis, Massachusettes Institute of Technology, USA.

Piekos, E.S., Breuer, K.S., 1999. Pseudospectral orbit simulation of nonideal gas-lubricated journal bearings for microfabricated turbomachines. *Journal of Tribology*, **121**(3):604-609. [doi:10.1115/1.2834110]

Piekos, E.S., Breuer, K.S., 2002. Manufacturing effects in microfabricated gas bearings: axially varying clearance. *Journal of Tribology*, **124**(4):815-821. [doi:10.1115/1.1430672]

Schaaf, S.A., Chambre, P.L., 1958. Flow of Rarefied Gas, Part H of Fundamentals of Gas Dynamics. Princeton University Press, USA.

Shen, Q., 2003. Rarefied Gas Dynamics. National Defense Industry Press, Beijing, China (in Chinese).

Shen, S., Chen, G., Crone, R.M., Anaya-Dufresne, M., 2007. A kinetic-theory based first order slip boundary condition for gas flow. *Physics of Fluids*, **19**:086101. [doi:10.1063/1.2754373]

Sun, Y.H., Chan, W.K., Liu, N., 2002. A slip model with molecular dynamics. *Journal of Micromechanics and Microengineering*, **12**(3):316-322. [doi:10.1088/0960-1317/12/3/318]

Teo, C.J., Liu, L.X., Li, H.Q., Ho, L.C., Jacobson, S.A., Ehrich, F.F., Epstein, A.H., Spakovszky, Z.S., 2006. High-speed Operation of a Gas-bearing Supported MEMS Air Turbine. Proceedings of the STLE/ASME International Joint Tribology Conference, San Antonio, Texas, USA, p.1303-1314. [doi:10.1115/IJTC2006-12173]

Teo, C.J., Spakovszky, Z.S., Jacobson, S.A., 2008. Unsteady flow and dynamic behavior of ultrashort Lomakin gas bearings. *Journal of Tribology*, **130**(1):011001. [doi:10.1115/1.2805403]

Wu, L., Bogy, D.B., 2003. New first and second order slip models for the compressible Reynolds equations. *Journal of Tribology*, **125**:558-561. [doi:10.1115/1.1538620]

Wu, L., 2008. A slip model for rarefied gas flows at arbitrary Knudsen number. *Applied Physics Letters*, **93**:253103. [doi:10.1063/1.3052923]

Zhou, J., Meng, G., Zhang, W., 2007. Characteristics of micro gas journal bearings. *Journal of Vibration and Shock*, **26**(9):30-33 (in Chinese).