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Improved response surface method for anti-slide reliability analysis of gravity dam based on weighted regression*

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Abstract: The aim of this study was to design and construct an improved response surface method (RSM) based on weighted regression for the anti-slide reliability analysis of concrete gravity dam. The limitation and lacuna of the traditional RSM were briefly analyzed. Firstly, based on small experimental points, research was devoted to an improved RSM with singular value decomposition techniques. Then, the method was used on the basis of weighted regression and deviation coefficient correction to reduce iteration times and experimental points and improve the calculation method of checking point. Finally, a test example was given to verify this method. Compared with other conventional algorithms, this method has some strong advantages: this algorithm not only saves the arithmetic operations but also greatly enhances the calculation efficiency and the storage efficiency.

Key words: Response surface method (RSM), Reliability, Gravity dam, Singular value decomposition, Weighted regression, Deviation coefficient

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1 Introduction

The basic purpose of structural reliability analysis is to obtain the probabilistic responses of structural systems with uncertain design parameters, such as loadings, material parameters (strength, elastic modulus, Poisson's ratio, etc.), and shape dimensions. Among the methods available for these problems, the response surface method (RSM) is a powerful tool (Liu and Moses, 1994). The theory and methods of RSM have been developed significantly during the last twenty years. Although from a theoretical viewpoint, the field has reached a stage where the developed methodologies are becoming widespread, the RSM used to analyze large structures is still a complex and difficult task. To solve this problem, a rigorous series of tests has been carried out. Moore and Ping (1999) constructed confidence intervals about the difference in mean responses at the stationary point and alternate points based on the proposed delta method and the *F*-projection method and compared coverage probabilities and interval widths. Zheng and Das (2000) proposed an improved RSM and applied it to the reliability analysis of a stiffened plate structure. Guan and Melchers (2001) evaluated the effect of response surface parameter variation on structural reliability. Youn and Choi (2004) proposed the hybrid mean value (HMV) method for highly efficient and stable reliability based the design optimization (RBDO) by evaluating the probabilistic constraint effectively. Gupta and Manohar (2004) used the RSM to study the extremes of von Mises stress in nonlinear structures under Gaussian excitations. Gomes and Awruch (2004) compared the RSM and the artificial neural network (ANN) techniques. Kaymaz and McMahon (2005) suggested a new response surface called

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ADAPRES, in which a weighted regression method was applied in place of normal regression. Wong *et al.* (2005) proposed an adaptive design approach to overcome the problem that the solution of the reliability analysis initially diverged when the loading was applied in sequence in the non-linear finite element (NLFE) analysis, and made several suggestions to improve the robustness of RSM. Jiang *et al.* (2006) improved the method to fit the indeterminate coefficients of response surface. Jin and Yuan (2007) presented an RSM based on the least-squares support vector machines (LS-SLM) aiming at the reliability analysis problems with implicit performance function. Chebbah (2007) dealt with the optimization of tube hydro forming parameters to reduce defects which might occur at the end of forming process such as necking and wrinkling by the RSM. Cheng *et al.* (2008) presented a new ANN based the RSM in conjunction with the uniform design method for predicting failure probability of structures. Gavin and Yau (2008) used the higher order polynomials to approximate the true limit state more accurately in contrast to recently proposed algorithms which focused on the positions of sample points to improve the accuracy of the stochastic response surface method (SRSM). Zou *et al.* (2008) presented an accurate and efficient Monte Carlo simulation method for limit-state-based reliability analysis at both component and system levels, using a response surface approximation of the failure indicator function. Nguyen *et al.* (2009) proposed an adaptive construction of the numerical design, in which the response surface was fitted by the weighted regression technique, which allowed the fitting points to be weighted according to their distance from the true failure surface and their distance from the estimated design point. To date, however, most reliability methods, such as the first-order reliability method (FORM) (Hong *et al*., 1999), the second-order reliability method (SORM) (Koyluoglu and Nielsen, 1994; Der Kiureghian and Dakessian, 1998), the weighted regression method (WRM) (Qiu and Orazem, 2004; Triantafyllopoulos, 2006), and the space reduced weighted regression method (SRWRM) (Zhao and Lu, 2006), cannot be used to analyze large structures. These traditional reliability methods have two aspects of deficiencies. On one hand, limited state function is usually implicit when we use a finite element method (FEM) to analyze structure. It leads to difficulty in obtaining implicit limited state function's partial derivatives for basic random variables. On the other hand, to overcome the above defects, some reliability methods use polynomial response surface function to fit implicit limited state function, but the number of basic random variables is very large when we analyze large structures. And these reliability methods need more experimental points to confirm the indeterminate coefficients of these basic random variables. It follows that, during the process, the calculation efficiency and the storage efficiency of these methods are very low. Even in some large structures, it is impossible to obtain enough experimental points. Therefore, most of reliability methods can be used only to analyze small structures.

In this paper, we established an improved RSM based on the weighted regression from the aspects of the regression of sample points, the selection of experimental points, the determined method of weight matrix, and the calculation method of checking point. And we demonstrated that the improved RSM can be used to analyze large structures. Then we used this method to analyze the anti-slide reliability of a concrete gravity dam. Finally, we gave a test example to verify and analyze the convergence and stability of the proposed method.

2 Establishment of the improved response surface method based on weighted regression

In this section, we improve the RSM based on weighted regression and make this method applicable in large structures such as gravity dams. The algorithm is shown to have good convergence and stability and greatly enhances calculation efficiency and the storage efficiency compared with other conventional algorithms.

2.1 Establishment of the response surface function

Using a second-order polynomial response surface function $\tilde{y}(\boldsymbol{x})$ to fit the implicit limited state function $g(\boldsymbol{x})$, we have

$$\tilde{y}(\boldsymbol{x}) = b_0 + \sum_{j=1}^{n} b_j x_j + \sum_{j=1}^{n} c_j x_j^2, \qquad (1)$$

where x_j and n are basic random variables (in this study, the basic random variables are random elastic modulus of elements of gravity dam model) and the number of basic random variables, respectively; b_0, b_j, and c_j are indeterminate coefficients.

However, the number of basic random variables is very large when we analyze large structures. It is impossible to obtain the indeterminate coefficients by the traditional RSM because we can obtain only m sample points, which cannot reach the number of $2n+1$ to fit the second-order polynomial response surface function $\tilde{y}(\boldsymbol{x})$. Thus, we try to use the second-order polynomial response surface function $\tilde{y}(\boldsymbol{x})$ to best approximate the implicit limited state function $g(\boldsymbol{x})$ by m sample points.

We select m ($m<2n+1$) experimental points $\boldsymbol{x}_i$ (i=1, 2, …, m), and calculate the implicit limit state function value $g(\boldsymbol{x}_i)$ which corresponds to the experimental points $\boldsymbol{x}_i=[x_{i1}, x_{i2}, \ldots, x_{in}]^{\mathrm{T}}$, and then obtain the sample vector $\boldsymbol{y}=[g(\boldsymbol{x}_1), g(\boldsymbol{x}_2), \ldots, g(\boldsymbol{x}_m)]^{\mathrm{T}}$.

Set $\boldsymbol{b}=[b_0, b_1, \ldots, b_n, c_1, c_2, \ldots, c_n]^{\mathrm{T}}$ as the solution vector which is to be determined, and use m experimental points $\boldsymbol{x}_i$ to compose the experimental matrix $\boldsymbol{A}$ as

$$\boldsymbol{A}=\begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1n} & x_{11}^2 & x_{12}^2 & \cdots & x_{1n}^2 \\ 1 & x_{21} & x_{22} & \cdots & x_{2n} & x_{21}^2 & x_{22}^2 & \cdots & x_{2n}^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{m1} & x_{m2} & \cdots & x_{mn} & x_{m1}^2 & x_{m2}^2 & \cdots & x_{mn}^2 \end{bmatrix}. \tag{2}$$

By singular value decomposition of the experimental matrix $\boldsymbol{A}$, we have

$$\boldsymbol{A}=\boldsymbol{U}\begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{V}^{\mathrm{T}}, \tag{3}$$

where $\boldsymbol{\Sigma}$ is an $m\times m$ diagonal matrix, and $\boldsymbol{U}$ and $\boldsymbol{V}$ are the m-order and $(2n+1)$-order unitary matrices, respectively.

The solution vector $\boldsymbol{b}$ is given as

$$\boldsymbol{b}=\boldsymbol{V}\begin{bmatrix} \boldsymbol{\Sigma}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{y}. \tag{4}$$

Set weight matrix $\boldsymbol{M}$ as an $m\times m$ diagonal matrix which gives m experimental points $\boldsymbol{x}_i$ weight value.

$$\boldsymbol{M}=\begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_m \end{bmatrix}. \tag{5}$$

Rewrite the solution vector $\boldsymbol{b}$ as

$$\boldsymbol{b}=\boldsymbol{V}\begin{bmatrix} (\boldsymbol{M\Sigma})^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}[\boldsymbol{MU}]^{\mathrm{T}}\boldsymbol{y}. \tag{6}$$

2.2 Establishment of the weight matrix M

The most important part of using the weighted regression is to determine suitable weight factors. We want to achieve two goals when using the second-order polynomial response surface function $\tilde{y}(\boldsymbol{x})$ to approximate the implicit limited state function $g(\boldsymbol{x})$. The first goal is to have the value of the implicit limited state function $g(\boldsymbol{x})$ equal to zero. The second goal is to approximate the implicit limited state function $g(\boldsymbol{x})$ around checking the point $\boldsymbol{x}_0$. They are achieved in this study as follows.

Among the responses from the implicit limited state function corresponding to the design matrix, the best design is selected based on closeness to a zero value, which indicates that the experimental point is close to the limit state. Thus, to achieve the first goal, we establish the first optimization object function as

$$g_{\text{best}}^1=\min_{i=1}^{m}\left|g(\boldsymbol{x}_i)\right|. \tag{7}$$

Moreover, the center point $\boldsymbol{x}_1$ is close to the checking point $\boldsymbol{x}_0$ in the iteration process, which indicates that the experimental points closed to the center point $\boldsymbol{x}_1$ should obtain greater weight. Thus, to achieve the second goal, we establish the second optimization object function as

$$g_{\text{best}}^2=\max_{i=1}^{m}\left\|\boldsymbol{x}_i-\boldsymbol{x}_1\right\|_2. \tag{8}$$

Then, through the advantages of the above two optimization object functions, we find the following expression to be suitable to obtain the weight for each experimental point:

$$w_i=\alpha\frac{g_{\text{best}}^1}{|g(\boldsymbol{x}_i)|}+\beta\frac{g_{\text{best}}^2-\|\boldsymbol{x}_i-\boldsymbol{x}_1\|_2}{g_{\text{best}}^2},\ \alpha+\beta=1. \quad (9)$$

Thus, we establish the weight matrix $\boldsymbol{M}$ as

$$\boldsymbol{M}=\text{diag}\{w_i\}. \quad (10)$$

2.3 Selection of m experimental points x_i in the initial iterative step

In the initial iterative step, we select m experimental points $\boldsymbol{x}_i$ as

$$\boldsymbol{x}_1^0=[u_1,u_2,\cdots,u_n]^{\text{T}}, \quad (11)$$

$$\boldsymbol{x}_i^0=[u_1+r_{i1}\sigma_1,u_2+r_{i2}\sigma_2,\cdots,u_j+r_{ij}\sigma_j,\cdots,u_n+r_{in}\sigma_n]^{\text{T}},$$
$$i=2,3,\cdots,m,\quad j=1,2,\cdots,n, \quad (12)$$

where u_j and σ_j are the expected value and mean square deviation of basic random variables x_j, respectively; r_{ij} is random number in the interval [$-v$, v], where v is deviation factor.

2.4 Deviation factor adjusting

Deviation factor has an influence on the convergence speed in the iterative procedure. Thus, to improve the convergence speed, we adjust the deviation factor in each iterative step as

$$v^k=v^{k-1}\left\|\boldsymbol{x}_1^k-\boldsymbol{x}_0^k\right\|_2\Big/\left\|\boldsymbol{x}_1^{k-1}-\boldsymbol{x}_0^{k-1}\right\|_2, \quad (13)$$

where k is the iterative step number.

2.5 Calculation method of checking point x_0

Derivation calculus to the second-order polynomial response surface function $\tilde{y}(\boldsymbol{x})$ is complex. Thus, we adopt an improved method based on the Lagrange multiplier rule as follows.

We can obtain the design checking point $\boldsymbol{x}_0$ through solving the constrained optimization problem Eq. (14) as

$$\begin{aligned}&\min\beta=f(\boldsymbol{x}_0)\\&=\sqrt{\left(\frac{x_{01}-u_1}{\sigma_1}\right)^2+\left(\frac{x_{02}-u_2}{\sigma_2}\right)^2+\cdots+\left(\frac{x_{0n}-u_n}{\sigma_n}\right)^2},\quad (14)\\&\qquad\text{s.t.}\quad g(\boldsymbol{x}_0)=0,\end{aligned}$$

where β is the reliability index.

Substituting Eq. (1) into the constrained optimization problem Eq. (14), the constrained optimization problem is rewrite as

$$\begin{aligned}&\min f(\boldsymbol{x}_0)=\left(\frac{x_{01}-u_1}{\sigma_1}\right)^2+\left(\frac{x_{02}-u_2}{\sigma_2}\right)^2+\cdots+\left(\frac{x_{0n}-u_n}{\sigma_n}\right)^2,\\&\qquad\text{s.t.}\quad \tilde{y}(\boldsymbol{x}_0)=b_0+\sum_{j=1}^{n}b_jx_{0j}+\sum_{j=1}^{n}c_jx_{0j}^2=0.\end{aligned} \quad (15)$$

Based on the Lagrange multiplier rule, we can rewrite the constrained optimization problem Eq. (15) as

$$\begin{cases}f_{x_{01}}(\boldsymbol{x}_0)+\lambda\bar{g}_{x_{01}}(\boldsymbol{x}_0)=0,\\ \qquad\vdots\\ f_{x_{0n}}(\boldsymbol{x}_0)+\lambda\bar{g}_{x_{0n}}(\boldsymbol{x}_0)=0,\\ \tilde{y}(\boldsymbol{x}_0)=0.\end{cases} \quad (16)$$

Unfolding Eq. (16) as

$$\begin{cases}\begin{bmatrix}2/\sigma_1+2c_1\lambda & & & \\ & 2/\sigma_2+2c_2\lambda & & \\ & & \ddots & \\ & & & 2/\sigma_n+2c_n\lambda\end{bmatrix}\begin{bmatrix}x_{01}\\x_{02}\\\vdots\\x_{0n}\end{bmatrix}\\ \quad=\begin{bmatrix}-\lambda b_1+2\sigma_1u_1\\-\lambda b_2+2\sigma_2u_2\\\vdots\\-\lambda b_n+2\sigma_nu_n\end{bmatrix},\\ \tilde{y}(\boldsymbol{x}_0)=0.\end{cases} \quad (17)$$

Then solve Eq. (17), and we can obtain

$$x_{0j}=\frac{1}{2/(\lambda\sigma_j)+2c_j}\left(-b_j+\frac{2\sigma_ju_j}{\lambda}\right),\quad j=1,2,\cdots,n. \quad (18)$$

From Eqs. (17) and (18), we find that the second-order polynomial $\tilde{y}(\boldsymbol{x}_0)$ is the single-variable λ function. Thus, we have

$$\tilde{y}(\boldsymbol{x}_0)=\tilde{y}(\lambda)=0. \quad (19)$$

By the Binary Search, we can solve Eq. (19) as follows:

Step 1: Taking two values λ_1, λ_2 to fit the conditions $\tilde{y}(\lambda_1)\times\tilde{y}(\lambda_2)<0$, and making $\lambda=(\lambda_1+\lambda_2)/2$.

Step 2: When $\tilde{y}(\lambda_1)\times\tilde{y}(\lambda)<0$, making $\lambda_2=\lambda$ and $\lambda=(\lambda_1+\lambda_2)/2$.

Step 3: When $\tilde{y}(\lambda_2)\times\tilde{y}(\lambda)<0$, making $\lambda_1=\lambda$ and $\lambda=(\lambda_1+\lambda_2)/2$.

Through the above iterative process, we can obtain the value of the variable λ. Substituting λ into Eq. (18), we can obtain the value of the design checking point $\boldsymbol{x}_0$.

2.6 Basic steps of the improved response surface method based on weighted regression

The basic steps of the improved RSM based on weighted regression are as follows:

Step 1: In k iterative step, we obtain m experimental points $\boldsymbol{x}_i$ through Eqs. (11) and (12). We calculate the implicit limit state function value $g(\boldsymbol{x}_i)$ which corresponds to the experimental points $\boldsymbol{x}_i=[x_{i1}, x_{i2}, \ldots, x_{in}]^{\mathrm{T}}$, and then obtain the sample vector $\boldsymbol{y}=[g(\boldsymbol{x}_1), g(\boldsymbol{x}_2), \ldots, g(\boldsymbol{x}_m)]^{\mathrm{T}}$. We obtain weight matrix $\boldsymbol{M}$ by Eqs. (7)–(10) and then get the solution vector $\boldsymbol{b}$ by Eq. (6).

Step 2: We obtain checking point $\boldsymbol{x}_0^k$ by the improved method based on the Lagrange multiplier rule, and then we calculate center point $\boldsymbol{x}_1^{k+1}$ at the next iterative step as

$$\boldsymbol{x}_1^{k+1}=\boldsymbol{x}_1^k+(\boldsymbol{x}_0^k-\boldsymbol{x}_1^k)\frac{g(\boldsymbol{x}_0^k)}{g(\boldsymbol{x}_0^k)-g(\boldsymbol{x}_1^k)}. \quad (20)$$

Step 3: We obtain deviation factor v^{k+1} by Eq. (13) and obtain m experimental points $\boldsymbol{x}_i$ through Eqs. (11) and (12). If the reliability index $||\beta_k-\beta_{k-1}||\le\varepsilon$, stop the iterative procedure. If the reliability index $||\beta_k-\beta_{k-1}||>\varepsilon$, return to Step 1.

2.7 Numerical example

We provide a numerical example to verify and analyze the convergence and stability of this method.

Set the implicit limited state function $g(x,y)=\exp(0.2x+6.2)-\exp(0.47y+5.0)$, where basic random variables x and y obey the standard normal distribution and $\alpha=0.7$, $\beta=0.3$. We use two experimental points in this method compared with 3–5 experimental points used in other conventional algorithms. We obtain the comparison result when using the same initial deviation factor v^0 as shown in Table 1. We obtain the iterative process when initial deviation factor $v^0=3$ as shown in Table 2.

Table 1 Final results of the example

Method	v^0	k	β	e (%)
FORM			2.3493	0.00
TLM	2	4	2.0944	10.85
	3	6	1.8421	21.59
	10	60	0.3939	83.23
WRM	2	4	2.3494	0.00
	3	6	2.3508	0.06
	10	8	2.4279	3.35
SRWRM	2	4	2.3496	0.01
	3	6	2.3504	0.05
	10	5	2.4270	3.31
Proposed method	2	4	2.3492	0.00
	3	6	2.3502	0.04
	10	8	2.3557	0.27

FORM: first-order reliability method; TLM: traditional linear method; WRM: weighted regression method; SRWRM: space reduced weighted regression method; v^0: initial deviation factor; k: iterative step number; β: reliability index; e: relative error of reliability index

Table 2 Iterative procedure of the example (initial deviation factor=3) in the proposed method

v^k	k	β	e (%)
1.8371E−01	1	2.9446	25.33946
3.3100E−02	2	2.3664	0.72788
1.8370E−03	3	2.3504	0.04682
2.5969E−04	4	2.3503	0.04257
2.6841E−05	5	2.3503	0.04257
8.2334E−06	6	2.3502	0.03831
8.2334E−06	7	2.3499	0.02554
8.2334E−06	8	2.3493	0.00000

v^0: initial deviation factor; k: iterative step number; β: reliability index; e: relative error of reliability index

3 Numerical analysis of the gravity dam model

The gravity dam is 160 m high, the normal pool level (NPL) is 155 m deep, and the level of the back of the dam is 10 m deep. The elevations of upstream and downstream broken-line sloping surface relative to foundation plane are 80 m and 140 m, respectively. The concrete strength of the gravity dam is C20. The finite element model of the gravity dam is divided into 2432 elements. The model consisted of 8-node iso-parametric plane elements for the dam and foundation. The density of the dam is 2450 kg/m^3, and the Poisson's ratio λ=0.18. The initial elastic modulus of dam E=3.50 GPa. The density of rock foundation is 2700 kg/m^3, and the Poisson's ratio is 0.25. The initial elastic modulus of rock foundation E=4.00 GPa, and the parameters α=0.9, β=0.1. The cohesion c=0.62 MPa and the internal friction angle φ=0.80. Applied load includes gravity load, hydrostatic and uplift pressure, and seismic load whose peak acceleration is 0.35g.

The acceleration time course is shown in Fig. 1 and the dam model is shown in Fig. 2. The probability distributions of all random parameters of each element are shown in Table 3.

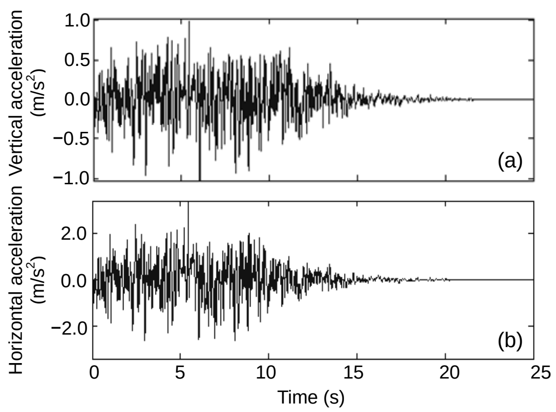

Fig. 1 Time course of (a) vertical acceleration and (b) horizontal acceleration

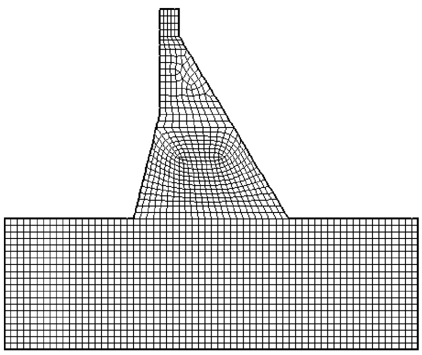

Fig. 2 Sub zone of materials in dam and its foundation

At time t, we can obtain the anti-slide limited state function of foundation plane $h(\boldsymbol{x},t)$ as

$$h(\boldsymbol{x},t)=\frac{\sum(c_i+\sigma_{Ni}\varphi_i)/(\tau_{Ni}L_i)}{\sum L_i}-1, \tag{21}$$

where c_i, σ_{Ni}, τ_{Ni}, φ_{Ni}, and L_i are the cohesion, normal stress, shear stress, internal friction angle, and projection length of elements on the foundation plane, respectively.

The implicit limited state function $g(\boldsymbol{x})$ is given as

$$g(\boldsymbol{x})=\min_{t=0}^{t_{\max}}\left(h(\boldsymbol{x},t)\right). \tag{22}$$

We regard $g(\boldsymbol{x})$ as the final implicit limited state function and use the proposed method to calculate the anti-slide reliability of the concrete gravity dam. In each iterative step of the proposed method, we use only 10 functions and use the proposed method to calculate the anti-slide reliability of the concrete gravity dam. In each iterative step of the proposed method, we use only 10 experimental points to approximate implicit limited state function $g(\boldsymbol{x})$ while the traditional RSM needs 4865 experimental points. Thus, the proposed method saves a lot of storage space and can be applied in analyzing large structures such as gravity dams. The iterative procedure is shown in Table 4, and the deviation factor iterative procedure is shown in Fig. 3.

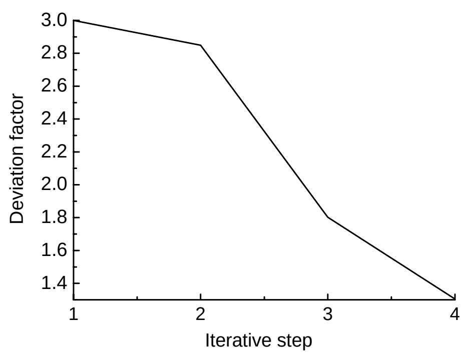

Fig. 3 Deviation factor iterative procedure

Table 3 Probability distribution for elastic modulus of rock foundation and dam

Elastic modulus	Probability distribution	Expected value (Pa)	Coefficient of variation
Rock foundation	Normal	4.00E+10	0.1
Dam	Normal	3.50E+10	0.1

Table 4 Iteration process of dam anti-slide reliability

v^k	k	β	Anti-slide reliability (%)
3.0000	1	3.5172	99.98
2.8500	2	1.3350	90.91
1.8028	3	1.2005	88.50
1.3041	4	1.1506	87.50

v^k: deviation factor; k: iterative step number; β: reliability index

The limit state function $h(\boldsymbol{x},t)$ time course under the condition of calculated elastic modulus is shown in Fig. 4. From Fig. 3, we can conclude that the convergence rate of the proposed method is rapid. From Fig. 4, we find that how the limit state function $h(\boldsymbol{x},t)$ is always greater than zero, and the value range is 0.35–0.55. From Table 4, we can obtain the anti-slide reliability as 87.50%.

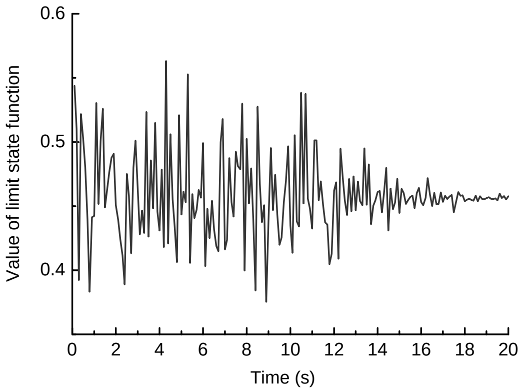

Fig. 4 Time course of the limit state function $h(x,t)$

4 Conclusions

In this paper, we briefly discussed the limitations and deficiencies of the traditional RSM. We first researched an improved RSM with singular value decomposition techniques. Furthermore, we improved the response surfaces method from the aspects of the regression of sample points, the selection of experimental points, the determined method of weight matrix, and the calculation method of checking point. Finally, we provided a test example to verify and analyze the convergence and stability of this method. Compared with other conventional algorithms, this method reduces the amount of arithmetic operations and greatly enhances calculation and storage efficiency.

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