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Mathematical models for process commonality under quality and resources breakdown in multistage production

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Abstract: It is essential to manage customers' diverse desires and to keep manufacturing costs as low as possible for survival in competition and eventually in production. Sharing resources in manufacturing for different products is a vital method of accomplishing this goal. The advantages of using a common process in production are stated in the literature. However, the mathematical models as well as simulation or conceptual models are not sufficient. The main objective of this paper is to develop mathematical models for multiproduct and multistage production under quality and breakdown uncertainties. The idea of the process commonality is incorporated in the proposed models. The models are validated by primary data collected from a Malaysian company and comparison of the timely requirement schedules of earlier MRP II and the proposed models under stable and perfect production environments. An appreciable convergence of the outcomes is observed. However, the proposed models are carrying additional information about the available locations of the parts in a time frame. After validation, the effects of process commonality on cost, capacity and the requirement schedule under uncertainties are examined. It is observed that the use of common processes in manufacturing is always better than the non-commonality scenario in terms of production cost. However, the increase in capacity requirement for commonality designs is higher for an ideal system, while it is less when the system suffers from breakdowns and a quality problem.

Key words: Process commonality, Mathematical model, MRP II, Quality, Breakdown
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1 Introduction

The underlying ideas for commonality are not new. As early as 1914, the term "commonality" as "standardization" was coined by an automotive engineer to facilitate mix-and-matching of components and to reduce costs (Fixson, 2007). Two sources of commonality are identified in the literature, the component part commonality and the process commonality (Jiao and Tseng, 2000). Process commonality is the use of a single resource for multiple or a group of products. The details about the commonality, its measurements and models are reported in Wazed *et al.* (2010a). It has sufficient variables for use in manufacturing and thereby might allow a cost-effective development of a sufficient variety of products to meet customers' diverse demands. None of the systems in the production processes are perfect. Many factors are difficult in the planned execution of the operations. The factors and sources of various difficulties in the manufacturing can be found in Wazed *et al.* (2009; 2010c). This paper includes only the quality and machine breakdown uncertainties in the models development. These factors very often delay the anticipated production schedules.

To operate the partial or total parts for designing the process with commonality, platforms and families, multistage models are frequently used. Jiao and Tseng (1999) proposed a method to set up the product families. In addition, Jiao *et al.* (2000) anticipated a data structure integrating the bill-of-materials (BOM) and the bill-of-operations between product and process. Along with that, for incorporating the process flexibility, lot sizing and scheduling, Jiao and Tseng (2000) developed a process commonality index.

Similarity matrices were constructed for cluster formation using the real data on critical parameters to commonalize subsystems of a product (Qin *et al.*, 2005). Designing products including modular products as a common process is available in details (Ulrich and Eppinger, 2000; Kamrani and Salhieh, 2002). Farrell and Simpson (2003) suggested a five-step model for designing product family. Commonality across products is always viewed as a shared set of processing steps, for instance, the intermediate hot or cold forming steps from ingots during aluminum tube manufacturing (Balakrishnan and Brown, 1996). Wazed *et al.* (2011) observed the impact of quality and breakdowns using simulation. However, the mathematical models for these processes are not available in previous studies.

Machine breakdowns are a representation of machine failure. If machine breaks down during production, the parts that required the use of this machine will be accumulated in the queue. These parts could be from a variety of products. Hence, the effect can be expanded across the product range. This issue is addressed in Koh and Gunasekaran (2006), Balakrishnan and Cheng (2007), Xu and Li (2007), Arruda and do Val (2008), etc. In manufacturing, the quality problem may come from the procured raw material or machines may produce defective products. A certain proportion of products became defective due to poor production quality and material defects, and subsequently defective products are scrapped if they are not re-workable, or it is not cost-effective to do so. In multistage manufacturing, products move from one stage to the next, and every stage may yield a certain proportion of defective items. The proportion of defects may vary from stage to stage and also from cycle to cycle. Furthermore, during changeover, where there is effort required to switch from the production of one stock keeping unit (SKU) to a different one, some material may be destroyed (i.e., wasted), and some defective parts may be produced. The quality uncertainty is studied in some recent publications (Mayer and Nusswald, 2001; Kim and Gershwin, 2005; Heese and Swaminathan, 2006; Mukhopadhyay and Ma, 2009).

The quality and breakdown uncertainties are currently not covered by any mathematical model. The process commonality issue is completely ignored in the existing models. Hence, this paper proposes mathematical models under quality and breakdown uncertainties for multiproduct and multistage systems and finally the process commonality concept is integrated.

2 Models

The models are created using earlier MRP II models (Eqs. (1)–(8)) (Voß and Woodruff, 2003). They are used as a starting point for further modeling. The MRP and MRP II models have many limitations (Shenoy and Bhadury, 1998; Koh *et al.*, 2000; 2006). In fact, they are merely a good scheduling tool for production. Additionally, the MRP II model assumed that any part/component/module can be processed at any machine. The main shortcoming is that it has no realistic utilization in many areas. For example, in any manufacturing system, facilities/machines/stations are anchored at a suitable point according to the planning guidelines. Parts are also planned to follow specific routings. They cannot move arbitrarily. Therefore, a planned and controlled route of components is necessary. Secondly, the commonality dimensions (i.e., process commonality) are not considered in earlier models. The models introduced in this paper, incorporated process commonality and are able to locate the components at any stage.

Minimize

$$\sum_{i}^{N}\sum_{t}^{T}(T-t)x_{it}. \tag{1}$$

Subject to

$$I_{it-1}+\sum_{\tau=1}^{t-\mathrm{LT}(i)}x_{i\tau}-I_{it}\geq D(i,t),\quad i=1,\ldots,\mathrm{ENDP},\ t=1,\ldots,T, \tag{2}$$

$$I_{it-1}+\sum_{\tau=1}^{t-\mathrm{LT}(i)}x_{i\tau}-I_{it}\geq\sum_{j=1}^{N}R(i,j)x_{jt},\quad i=1,\ldots,N/\mathrm{ENDP},\quad t=1,\ldots,T, \tag{3}$$

$$\sum_{i}^{N}U(i,k)x_{it}\leq 1,\ k=1,\ldots,K,\ t=1,\ldots,T, \tag{4}$$

$$x_{it}-\delta_{it}\mathrm{LS}(i)\geq 0,\ i=1,\ldots,N,\ t=1,\ldots,T, \tag{5}$$

$$x_{it} \le M\delta_{it},\ i=1,\ldots,N,\ t=1,\ldots,T, \quad (6)$$

$$\delta_{it} \in \{0,1\},\ i=1,\ldots,N,\ t=1,\ldots,T, \quad (7)$$

$$x_{it} \ge 0,\ i=1,\ldots,N,\ t=1,\ldots,T, \quad (8)$$

where T is the number of time buckets (i.e., the planning horizon) subscripted as t; N is the number of SKUs/parts/components subscripted as i; ENDP is the number of end products (interchangeably used as P) subscripted as p; K is the number of resources or machines or stations, subscripted as k; x_{it} is the quantity of SKU i to start or order in period t; I_{it} is the inventory level of SKU i in period t; $D(i, t)$ is the external/internal demand for SKU i (including the end products) in period t; LT(i) is the lead time for SKU i; $R(i, j)$ is the number of part/component/module i needed to make one module/product j; $U(i, k)$ is the fraction of resource k needed to make/process one unit of SKU i (including the end products); LS(i) is the minimum lot size for SKU i; M is a large number at least 100 times greater than the value of any variable/parameter of the system; δ_{it} is 1 if SKU i (including the end products) is produced/processed in period t and 0 otherwise.

The constraints in Eqs. (2) and (3) require that the sum of initial inventory and production in each period has to be at least equal to the total of external demand and demand for assemblies respectively that use the SKU. The summation is to t−LT(i) for each period (there will be one constraint for each value of t) because work must be started LT periods before it can be used to satisfy demand. The product $R(i, j)x_{it}$ anticipates the demand for SKU i that results when it is a component of SKU j. Eq. (4) represents the capacity constraint. Lot size is confirmed by Eq. (5), and Eq. (6) ensures that the resource should be used if production is present. The objective function, Eq. (1), is to produce products as late as possible but no later.

2.1 Model environment

The model considers planning and scheduling of ENDP final products (independent items) and their sub-components (dependent items) over a discrete planning horizon of T periods (indexed by t) in a batch production environment. For each product at each time period, a proposed demand is specified based on the forecasted value or customers' orders. Associated sub-components are included to assemble or manufacture their end products. Their demands are dependent on the parent products and are obtained from the BOM and are timed by offsetting the manufacturers lead time. The inputs to the model are: end product demand by period (settled), resource capacities, BOM, routing and cost information. The decision variables are quantities of each end product and its sub-components for each time period at a specific point on the floor. It is indeed a multi-resource capacitated problem. It is assumed that

1. The time horizon is uniform with equal length, such as hour, day, week or month.
2. The demands for each final product for each time period are known in advance.
3. Lead time and processing times are an integer multiple of each stage. They are known and settled.
4. The processing time is constant for a resource, but repair time is added if the machine breaks down.
5. Machine requires setup when the system switches to another product/component and when it resumes from stoppage due to failure.
6. Quality (i.e., number of defectives) and breakdowns are completely randomly distributed. The defective items are simply rejected.
7. Shortages/backlogs are allowed for a penalty.
8. Each resource (machine) offers a limited capacity. Unless otherwise stated, machines are dedicated to produce a specific product and its sub-components.

If the assumptions, quality, breakdown issues and process commonality concept are integrated in the model, it is possible to prepare a dependable material requirement schedule for each point of consumption in the manufacture.

2.2 Multistage production models under deterministic conditions

The authors introduce a class of models based on the simplest assumptions known for multistage production under certain conditions. However, the assumptions of deterministic and stationary factors seem quite restrictive but play an important role in enriching the model because many results are quite robust with respect to the model parameters, such as the demand rate and costs. From another point of view, the results obtained from these simple models are often good starting solutions for more complex models. We consider a K-stage assembly or manufacturing line that produces ENDP end products as illustrated in Fig. 1. The production/assembly process of a product starts at stage 1. When a component moves along the line, a component (module) is added at some/all of the K stages. In general, each production line is specified for a product if the sharing of resources is not allowed. The resources are identified by the product P in the production stage K of the system. Component C_{pkit} is assembled to the product/module/SKU i (i=1, …, N) in period t (t=1, …, T) at resource WC(p, k) for p=1, …, ENDP and k=1, …, K, where C_{pkit} is the required number of component/module/SKU i at resource WC(p, k) in period t, and WC(p, k) is the identification of a machine which is at stage k in the line of product p.

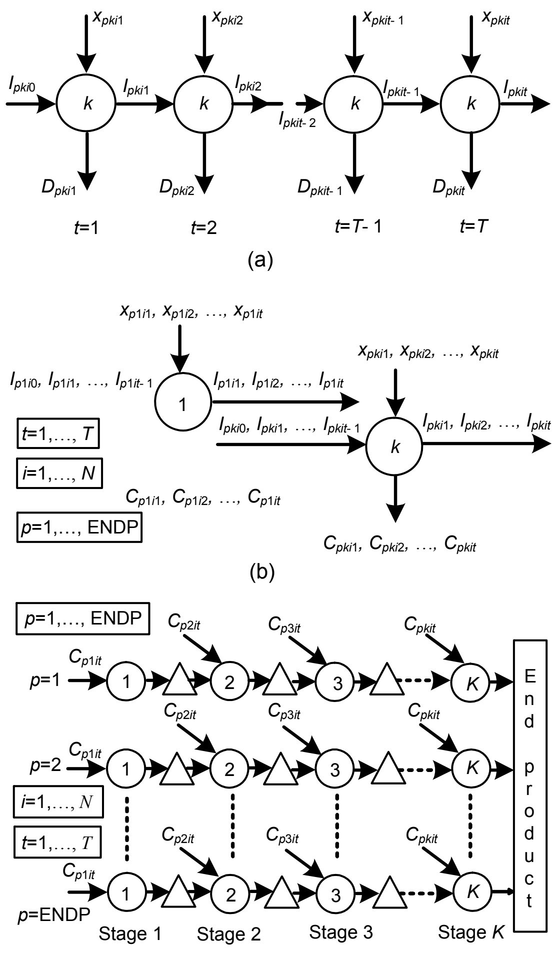

Fig. 1 Multistage production

(a) End product; (b) Component; (c) Manufacturing or assembly line. p is the number of end products; i includes all but end products

The complete model for a multistage system under ideal conditions is shown in Eqs. (9)–(21). In this model, parameters such as component purchasing cost, variable production cost and inventory costs for products and components and setup cost of the machines are taken into consideration in the cost function that has been modeled by Eq. (9). The demand and component requirement constraints can be written respectively as Eqs. (10) and (11). The required number of parts/modules needed at different periods and points in manufacturing can be generated by Eq. (12). Under capacity constraint in Eq. (16), processing, setup, over and under utilization of the resources have been taken into consideration. Over and under utilization cannot be positive for a resource in a period, which is assured by Eq. (17). Eq. (18) allows γ to be 1 for all SKU i on machine WC(p, k) only if there is a production of p in periods t and t−1. The constraint in Eq. (19) ensures that we only set γ to 1 for i that are to be routed to machine WC(p, k). This is done to avoid spurious values of γ that can be confusing when reading the solution. The constraint in Eq. (20) ensures that at most one product can span the time boundary on a specific resource WC(p, k).

Objective function:

$$\text{Minimize } z=\sum_{\mathrm{WC}(p,k)}\sum_{T}c_{\mathrm{WC}}\mathrm{OT}_{pkt}+\sum_{\mathrm{WC}(p,k)}\sum_{I}\sum_{T}(v_i x_{pkit}+q_i I_{pkit})+\sum_{\mathrm{WC}(p,k)}\sum_{I}\sum_{T}f_{pk}(y_{pkit}-\gamma_{pkit}). \tag{9}$$

Subject to

$$I_{pkit-1}+\sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)}x_{pki\tau}-I_{pkit}\geq D(p,k,i,t), \tag{10}$$

$$p=1,\ldots,\mathrm{ENDP},\ k=1,\ldots,K,\ i=P,\ t=1,\ldots,T,$$

$$I_{pkit-1}+\sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)}x_{pki\tau}-I_{pkit}\geq\sum_{\tau=1}^{t}\sum_{j=1}^{N}R(i,j)(x_{pkj\tau}+I_{pki\tau}),\ p=1,\ldots,\mathrm{ENDP}, \tag{11}$$

$$i=1,\ldots,N/\mathrm{ENDP},\ k=1,\ldots,K,\ t=1,\ldots,T,$$

$$C_{pkit}\geq\sum_{\tau=1}^{t}\sum_{j=1}^{N}R(i,j)(x_{pkj\tau}+I_{pki\tau}),$$

$$p=1,\ldots,\mathrm{ENDP},\ i=1,\ldots,N/\mathrm{ENDP}, \tag{12}$$

$$k=1,\ldots,K,\ t=1,\ldots,T,$$

$$x_{pkit} - ny_{pkit}\mathrm{LS}(i) = 0,\ p = 1, \ldots, \mathrm{ENDP}, \\ i = 1, \ldots, N/\mathrm{ENDP},\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{13}$$

$$x_{pkit} \le My_{pkit},\ p = 1, \ldots, \mathrm{ENDP}, \\ i = 1, \ldots, N/\mathrm{ENDP},\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{14}$$

$$\mathrm{LS}(i) \le My_{pkit},\ p = 1, \ldots, \mathrm{ENDP}, \\ i = 1, \ldots, N/\mathrm{ENDP},\ k = 1, \ldots, K,\ t = 1, \ldots, T. \tag{15}$$

The capacity constraints:

$$\sum_I \{U(p,k,i)x_{pkit} + \mathrm{ST}(p,k,i)(y_{pkit} - \gamma_{pkit})\} \\ -\mathrm{OT}_{pkt} + \mathrm{UT}_{pkt} \le 1, \\ p = 1, \ldots, \mathrm{ENDP},\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{16}$$

$$\mathrm{OT}_{pkt} \times \mathrm{UT}_{pkt} = 0, \\ p = 1, \ldots, \mathrm{ENDP},\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{17}$$

$$y_{pkit-1} + y_{pkit} \ge 2\gamma_{pkit},\ p = 1, \ldots, \mathrm{ENDP}, \\ i = 1, \ldots, N,\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{18}$$

$$\gamma_{pkit} \le MU(p,k,i),\ p = 1, \ldots, \mathrm{ENDP}, \\ i = 1, \ldots, N,\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{19}$$

$$\sum_{i=1}^{N} \gamma_{pkit} \le 1,\ p = 1, \ldots, \mathrm{ENDP}, \\ k = 1, \ldots, K,\ t = 1, \ldots, T. \tag{20}$$

Non-negativity constraints:

$$\text{All variables} \ge 0,\ y_{pkit} = \{0,1\},\ n = \text{Integer}, \tag{21}$$

where $D(p, k, i, t)$ is the external demand for product/component/module/SKU i in period t at WC(p, k); f_k is the cost for setting up resource WC(p, k); I_{pkit} is the inventory level of product/component/module/SKU i in front of resource WC(p, k) in period t; LT(p, k, i) is the lead time for product/component/module/SKU i at WC(p, k); q_i is the cost of carrying inventory of product/component/module/SKU i per unit time; ST(p, k, i) is the fraction of available time of resources used to set up machine WC(p, k) for product/component i; $U(p, k, i)$ is the fraction of available time of resource WC(p, k) needed to make one unit of SKU i; v_i is the production cost of product/component/module/SKU i; x_{pkit} is the number of component/module/SKU i produced at resource WC(p, k) in period t; y_{pkit}=1, if product/component/module/SKU i starts on resource WC(p, k) at period t, 0 otherwise; γ_{pkit}=1, if product/component/module/SKU i will be the last product produced on resource WC(p, k) in period t−1 and the first produced in time bucket t, 0 otherwise; c_{WC} is the unit cost of over-utilization of station WC(p, k); OT_{pkt} is the over utilization of station WC(p, k) in period t; UT_{pkt} is the under utilization of station WC(p, k) in period t; n is an integer number.

If a backlog is allowed, the demand/component requirement constraints and the cost function will be changed. The cost function in Eq. (22) includes the penalty for any backlog. The new demand and material requirement constraints are shown in Eqs. (23) and (24), respectively. Eq. (25) assures that inventory and backlog of a part/product cannot be positive in a period.

Minimize

$$z = \sum_{\mathrm{WC}(p,k)} \sum_I \sum_T (v_i x_{pkit} + q_i I_{pkit} + b_i B_{pkit}) \\ + \sum_{\mathrm{WC}(p,k)} \sum_I \sum_T f_{pk}(y_{pkit} - \gamma_{pkit}) + \sum_{\mathrm{WC}(p,k)} \sum_T c_{\mathrm{WC}} \mathrm{OT}_{pkt}, \tag{22}$$

$$I_{pkit-1} + \sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)} [x_{pki\tau} - I_{pkit} + B_{pkit} \\ -B_{pkit-1}] \ge D(p,k,i,t),\ p = 1, \ldots, \mathrm{ENDP}, \\ i = P,\ k = K,\ t = 1, .., T, \tag{23}$$

$$I_{pkit-1} + \sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)} x_{pki\tau} - I_{pkit} + B_{pkit} - B_{pkit-1} \\ \ge \sum_{\tau=1}^{t} \sum_{j=1}^{N} R(i,j)(x_{pkj\tau} + I_{pkit} + B_{pkit}), \\ p = 1, \ldots, \mathrm{ENDP},\ i = 1, \ldots, N/\mathrm{ENDP}, \\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{24}$$

$$I_{pkit} \times B_{pkit} = 0,\ p = 1, \ldots, \mathrm{ENDP}, \\ i = 1, \ldots, N/\mathrm{ENDP},\ k = 1, \ldots, K,\ t = 1, \ldots, T, \tag{25}$$

where B_{pkit} is the backlog of component i of product p at stage k in period t; b_i is the cost of backorder of component i of product p at stage k in period t.

3 Process commonality models under deterministic conditions

The multistage production models under known conditions are described and shown in the previous section. The models are used to incorporate a process commonality concept in the same environment. Two different process commonality layouts are proposed in Fig. 2. For the first layout (Fig. 2a), the resources (i.e., operations) are common for stage 1 to stage s. The final products are different so that it is possible to use at most $K-1$ (i.e., $0 \le s \le K-1$) common operations. Note that C is the number of common components or modules subscripted as c. s could be a decision variable representing the last common operation. It is assumed that common parts are used in these common processes. Afterwards the product specific components are added. For the second layout (Fig. 2b), few existing resources of a production line are designed to be shared with the others. Accordingly, the route of a component of one product is planned to receive the services of other common resources in another line, if the default resource is not available or there have been breakdowns. Components will return to their original line after completing the operation.

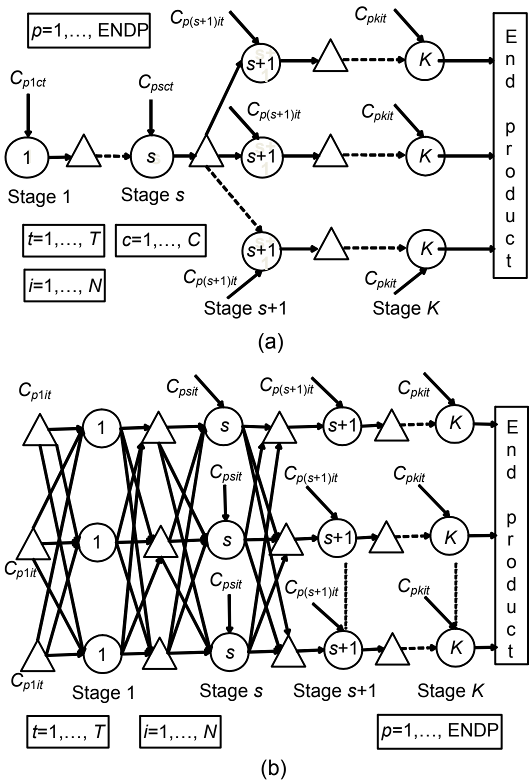

Fig. 2 Multistage production with process commonality
(a) First layout; (b) Second layout

The first layout (Fig. 2a) is a combination of common parts and processes. Thus, it is quite difficult to find only the effect of common processes on performance parameters. Therefore, the authors investigated the impact of process commonality on cost, capacity and material requirement scheduling in the second layout (Fig. 2b).

When common processes are introduced in the system, only the material requirement constraints will be changed. The material requirement constraint and the component requirements at various points in production in a period are shown respectively for the layout in Fig. 2b as follows:

$$\begin{aligned} & I_{pkit-1} + \sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)} x_{pki\tau} - I_{pkit} + B_{pkit} - B_{pkit-1} \\ & \ge \sum_{\tau=1}^{t}\sum_{j=1}^{N}\sum_{k\in p_i} R(i,j)(x_{pkj\tau} + I_{pkj\tau} + B_{pkj\tau}), \qquad (26) \\ & p=1,\ldots,\mathrm{ENDP},\ i=1,\ldots,N/\mathrm{ENDP}, \\ & k=1,\ldots,K,\ t=1,\ldots,T, \end{aligned}$$

$$\begin{aligned} & C_{pkit} \ge \sum_{\tau=1}^{t}\sum_{j=1}^{N}\sum_{k\in p_i} R(i,j)(x_{pkj\tau} + I_{pkj\tau} + B_{pkj\tau}), \\ & p=1,\ldots,\mathrm{ENDP},\ i=1,\ldots,N/\mathrm{ENDP}, \qquad (27) \\ & k=1,\ldots,K,\ t=1,\ldots,T, \end{aligned}$$

where p_i is the set of common processes which can process different types components/modules.

3.1 Models for known quality information

Let $W(i, j)$ represents the number of a defective product/component i when a system switches to product/component j. This means the machine requires setup. It could be either initiation or may resume from a breakdown or both. The number of defective parts is certainly known for each section in any stage. If β_i is the faction of defectives of all arrival part i, then $1-\beta_i$ represents the portion of usable raw materials. The capacity constraints will remain unaffected but the material requirement constraints will be changed. The new material requirement constraints is shown as

$$\begin{aligned} & I_{pkit-1} + \sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)} x_{pki\tau} - I_{pkit} + B_{pkit} - B_{pkit-1} \\ & \ge \sum_{\tau=1}^{t}\sum_{j=1}^{N}[(1+\beta_i)R(i,j)(x_{pkj\tau} + I_{pkj\tau} + B_{pkj\tau}) \qquad (28) \\ & +W(i,j)y_{pkj\tau}],\ p=1,\ldots,\mathrm{ENDP}, \\ & i=1,\ldots,N/\mathrm{ENDP},\ k=1,\ldots,K,\ t=1,\ldots,T. \end{aligned}$$

If common processes are applied as in Fig. 2b, the same will be true:

$$I_{pkit-1}+\sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)}x_{pki\tau}-I_{pkit}+B_{pkit}-B_{pkit-1}$$
$$\geq\sum_{\tau=1}^{t}\sum_{j=1}^{N}\sum_{k\in p_i}\Big[(1+\beta_i)R(i,j)(x_{pkj\tau}+I_{pkj\tau}+B_{pkj\tau}) \quad (29)$$
$$+W(i,j)y_{pkj\tau}\Big],\quad p=1,\ldots,\mathrm{ENDP},$$
$$i=1,\ldots,N/\mathrm{ENDP},\ \ k=1,\ldots,K,\ \ t=1,\ldots,T,$$

where β_i is the fraction of defectives in parts i that arrive in the system.

3.2 Models for known quality information and certain breakdown schedule

In this case, it assumes that the number of failures l_{pkt} of machine WC(p, k) in a time period t and the repair time r_{pk} of any breakdown the machine is known. The processing time T_{pki} is believed to remain constant regardless of the number of failures during the accomplishment of the task. The breakdowns increase the number of required setups and waste a fraction $V(p, k, t)$ of usable time of the machine. The number of defective items produced per unit breakdown due to regular setup can be functioned as $W(i, j)y_{pkit}$. If the system breaks down l_{pkt} times, it will be $W(i, j)y_{pkit}l_{pkt}$. Under the situations, the material requirement and capacity constraints can be considered as

$$I_{pkit-1}+\sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)}x_{pki\tau}-I_{pkit}+B_{pkit}-B_{pkit-1}$$
$$\geq\sum_{\tau=1}^{t}\sum_{j=1}^{N}[(1+\beta_i)R(i,j)(x_{pkj\tau}+I_{pkj\tau}+B_{pkj\tau}) \quad (30)$$
$$+(1+l_{pk\tau})W(i,j)y_{pkj\tau}],\ \ p=1,\ldots,\mathrm{ENDP},$$
$$i=1,\ldots,N/\mathrm{ENDP},\ \ k=1,\ldots,K,\ \ t=1,\ldots,T,$$

$$\sum_{I}[U(p,k,i)x_{pkit}+\mathrm{ST}(p,k,i)(y_{pkit}-\gamma_{pkit})$$
$$+\mathrm{ST}(p,k,i)l_{pkt}y_{pkit}+V(p,k,t)l_{pkt}y_{pkit}] \quad (31)$$
$$-\mathrm{OT}_{pkt}+\mathrm{UT}_{pkt}\leq CAP_{pkt},$$
$$p=1,\ldots,\mathrm{ENDP},\ \ k=1,\ldots,K,\ \ t=1,\ldots,T,$$

where CAP_{pkt} is the available capacity of station WC(p, k) in period t. When common processes are used in the system as in Fig. 2b, the material requirement constraint will be as

$$I_{pkit-1}+\sum_{\tau=1}^{t-\mathrm{LT}(p,k,i)}x_{pki\tau}-I_{pkit}+B_{pkit}-B_{pkit-1}$$
$$\geq\sum_{\tau=1}^{t}\sum_{j=1}^{N}\sum_{k\in p_i}[(1+\beta_i)R(i,j)(x_{pkj\tau}+I_{pkj\tau}+B_{pkj\tau}) \quad (32)$$
$$+(1+l_{pk\tau})W(i,j)y_{pkj\tau}],\ \ p=1,\ldots,\mathrm{ENDP},$$
$$i=1,\ldots,N/\mathrm{ENDP},\ \ k=1,\ldots,K,\ \ t=1,\ldots,T.$$

4 Validation of mathematical models

The original MRP II models (Eqs. (1)–(8)) are used to make a requirement list of parts/components/products with deterministic information like demand, lead time of products and component, etc., on an existing production layout of a Malaysian company. The company, namely ABC (a given name), is producing air filter products for a diverse air filtration system. The details of the company are found in (Wazed *et al.*, 2010b). The primary data collected from the company with the layout information is also employed in proposed mathematical models to prepare a timely requirement schedule of the systems under stable conditions. Both the existing (original MRP II) and proposed models are solved in Lingo systems with a global solver, and their outputs are compared. The product structure and manufacturing cell layout of the system are shown in Fig. 3. The model validation is performed to test the overall accuracy of the model and the ability to achieve the real value. Tables 1 and 2 show the timely requirements of components generated respectively by the MRP II and proposed mathematical models of the company. The models are further employed to multiple lines multistage production (a live case of a Malaysian company) for checking convergence of their outputs. The company namely XDE (a given name) located in Malaysia produces bicycle wheels. The details of the company and its production information are available in (Wazed *et al.*, 2011). The product structures and production layout of the company is shown in Fig. 4. Tables 3 and 4 show the timely requirement schedules for a multistage, multiproduct system generated by the basic MRP II and the proposed mathematical models, respectively.

It is shown that there is a good match between the two schedules generated by the MRP II and proposed models respectively. The latter is carrying

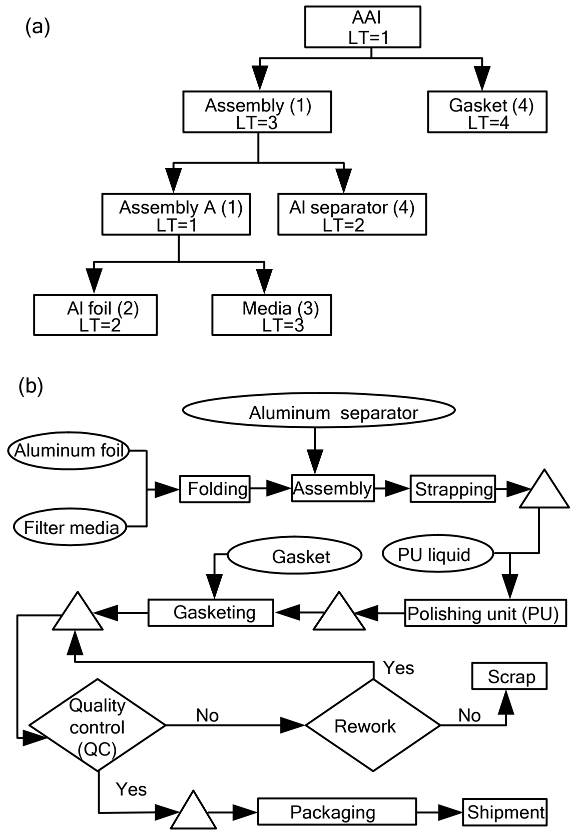

Fig. 3 Product structure (a) and manufacturing cell (b)

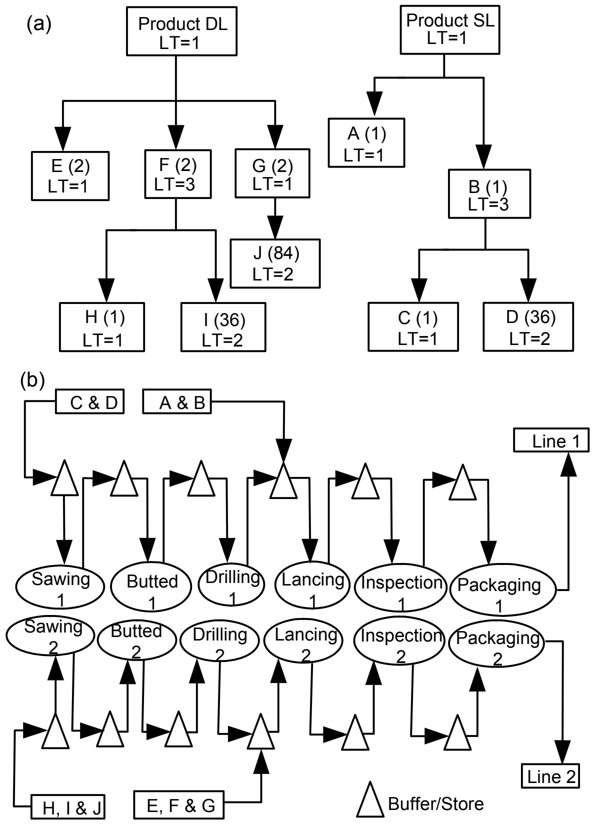

Fig. 4 Product structure (a) and manufacturing cell (b)

Table 1 Timely requirement schedule generated by MRP II for single line production

Part/Product	Period								
	1	2	3	4	5	6	7	8	9
AAI	0	0	0	0	0	0	0	50	0
Assembly	0	0	0	0	50	0	0	0	0
Gasket	0	0	0	0	0	0	200	0	0
Assembly A	0	0	0	50	0	0	0	0	0
Al separator	0	0	200	0	0	0	0	0	0
Al foil	0	100	0	0	0	0	0	0	0
Media	150	0	0	0	0	0	0	0	0

additional information of the location available for the parts in a time frame.

5 Effect of process commonality in production

In this section, the effect of process commonality is observed using the proposed commonality models, and the outcomes are compared with their basic forms. The models are executed for 18 periods under

Table 2 Timely requirement schedule generated by proposed mathematical models for single line production

Machine/ Stage	Part/Product	Period								
		1	2	3	4	5	6	7	8	9
Folding	Al foil	0	100	0	0	0	0	0	0	0
Folding	Media	150	0	0	0	0	0	0	0	0
Assembly	Assembly A	0	0	0	50	0	0	0	0	0
Assembly	Al separator	0	0	200	0	0	0	0	0	0
Strapping	Assembly	0	0	0	0	50	0	0	0	0
Gasketing	Gasket	0	0	0	0	0	0	200	0	0
Packaging	AAI	0	0	0	0	0	0	0	50	0

various created scenarios. For the commonality models, we assumed two different scenarios (Fig. 5).

The authors have executed the models in the Lingo system to observe the impact of the common process under the individual and joint actions of quality and breakdown in production. It is considered that the demand (Table 5) and procurement lead time (Table 6) are known and constant. For models with a quality problem, it is supposed that 3% of all incoming raw materials are defective and finally 1% nonconforming products are produced. The number of defective parts/products produced in a machine or process or resource during starting/changeover or when resuming from breakdowns are completely random in the range of 1–4 in a period. The breakdown pattern of the processes is also random in the range of 0–3 in a period. The ranges are selected from the collected primary data of the company. The cost of components, setup and processing times, demand, lead times are known. It is presumed that 0.9% of the processing time is used to repair when a machine/process/resource breaks down. It is assumed that the

Table 3 Timely requirement schedule generated by MRP II for double line production

Part/ Product	Period 1	2	3	4	5	6	7	8	9
Prod-SL	0	0	0	0	0	0	0	120	0
Prod-DL	0	0	0	0	0	0	0	140	0
A	0	0	0	0	0	0	120	0	0
B	0	0	0	0	120	0	0	0	0
C	0	0	0	120	0	0	0	0	0
D	0	0	4320	0	0	0	0	0	0
E	0	0	0	0	0	0	280	0	0
F	0	0	0	0	280	0	0	0	0
G	0	0	0	0	0	0	280	0	0
H	0	0	0	280	0	0	0	0	0
I	0	0	10 080	0	0	0	0	0	0
J	0	0	0	0	23 520	0	0	0	0

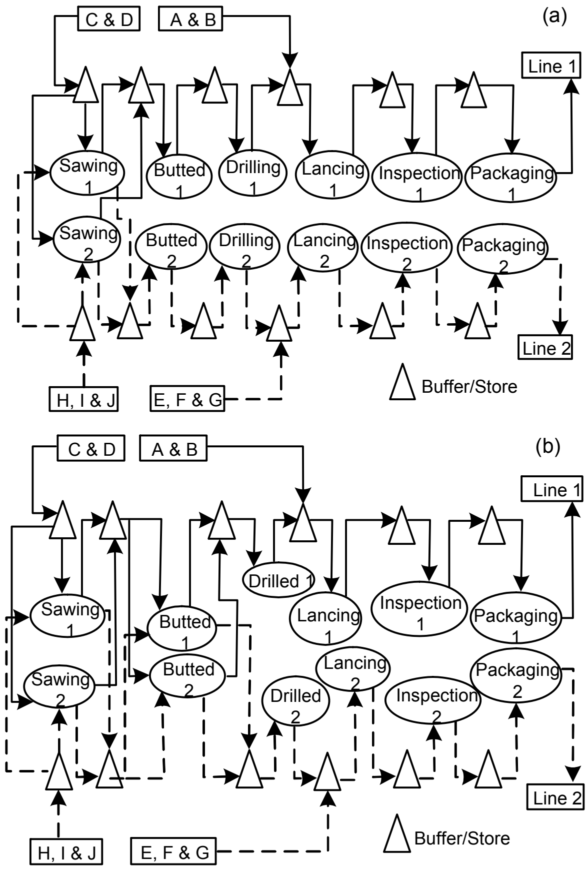

Fig. 5 Process commonality layout
(a) Single process; (b) Two common processes

Table 4 Timely requirement schedule generated by proposed models for double line production

Part/ Product	Machine/Stage	Period 1	2	3	4	5	6	7	8	9
Prod-SL	Packaging 1	0	0	0	0	0	0	0	120	0
Prod-DL	Packaging 2	0	0	0	0	0	0	0	140	0
A	Lancing 1	0	0	0	0	0	0	120	0	0
B	Lancing 1	0	0	0	0	120	0	0	0	0
C	Sawing	0	0	0	120	0	0	0	0	0
D	Sawing	0	0	4320	0	0	0	0	0	0
E	Lancing 2	0	0	0	0	0	0	280	0	0
F	Lancing 2	0	0	0	0	280	0	0	0	0
G	Lancing 2	0	0	0	0	0	0	280	0	0
H	Sawing	0	0	0	280	0	0	0	0	0
I	Sawing	0	0	10 080	0	0	0	0	0	0
J	Sawing	0	0	0	0	23 520	0	0	0	0

common processes are able to work on all parts that routed towards them and common processes require higher processing time than the others. Figs. 6 and 7 show the effect of processing cost for common processes on the total cost incurred and the capacity requirements at different stages, respectively.

Fig. 6 shows that the cost of production is always less for commonality cases for the same processing cost. The cost increases with the cost ratio for both commonality designs. Cost ratio represents how much more expensive the processing cost at common processes/resources is in comparison to the others. For example, 1.10 means that the processing cost at common processes is 10% more than the cost of processing for the others. It is observed from Fig. 6 that commonality offers a better choice, even if the

Table 5 Timely demand of the end products

Product	Period 9	10	11	12	13	14	15	16	17	18
Prod-SL	120	120	120	120	120	120	120	120	120	120
Prod-DL	140	140	140	140	140	140	140	140	140	140

Table 6 Lead time of products and components

Part/Product	Lead time	Part/Product	Lead time
Prod-SL	1	E	1
Prod-DL	1	F	3
A	1	G	1
B	3	H	1
C	1	I	2
D	2	J	2

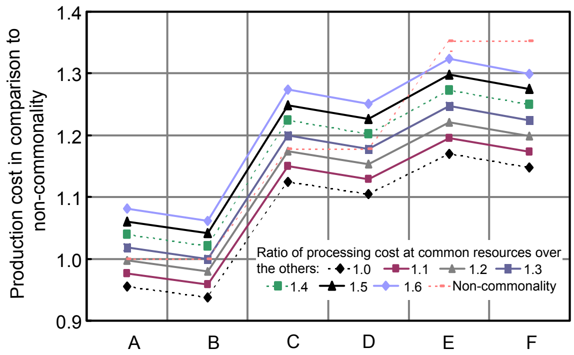

Fig. 6 Effect of common processes on total cost incurred under various scenarios

A: single common process; B: two common processes; C: single common process with quality problem; D: two common processes with quality problem; E: single common process with breakdowns and quality problem; F: two common processes with breakdowns and quality problem

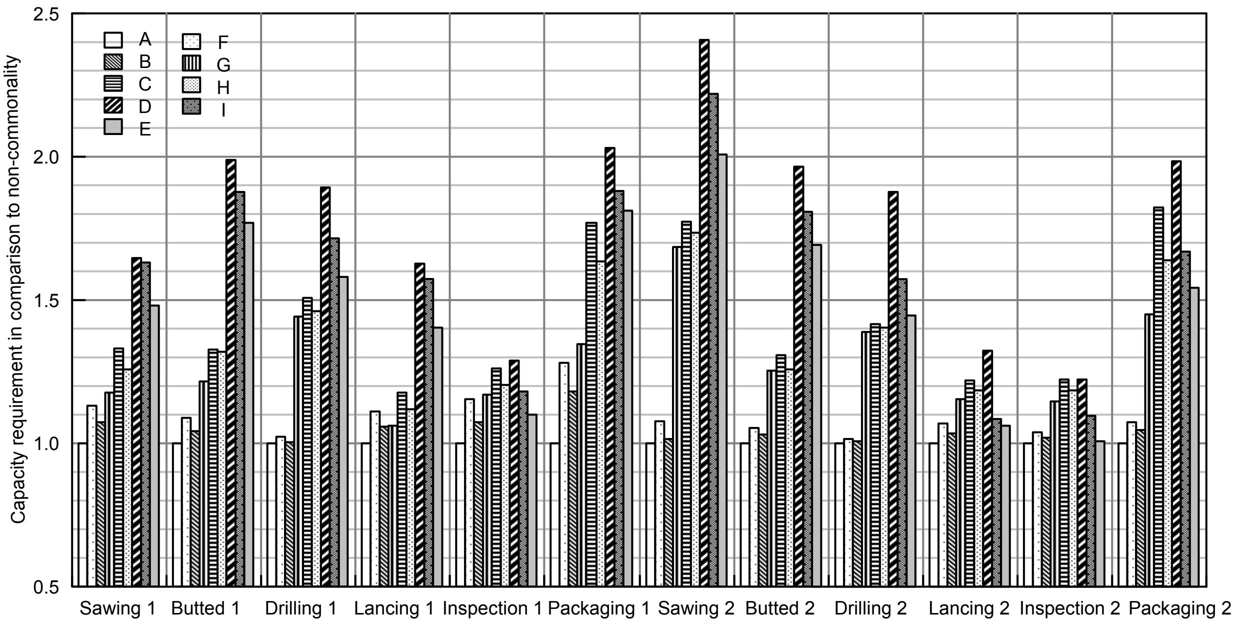

Fig. 7 Effect of common processes on capacity requirement in a multistage manufacturing

A: non-commonality; B: two common processes; C: single common process with quality problem; D: non-commonality with breakdowns and quality problem; E: two common processes with breakdowns and quality problem; F: single common process; G: non-commonality with quality problem; H: two common processes with quality problem; I: single common process with breakdowns and quality problem processing cost is 30% higher for common processes for both of the commonality scenarios (Fig. 5). The disparity in production cost with the cost ratio shows a similar impact for both cases. The least cost offers come from the second layout (i.e., two common resources) for a specific condition. It is clear that two common resources offer concurrent processing and alternative routes, and hence the average cycle times will decrease. These trends are valid even under quality and breakdown occurrences for a multistage production. However, the cost increases under the actions of quality and combination of both quality and breakdowns. The cost saving in commonality models mainly comes from the concurrent processing (alternate route) of operations. The cost reduction rate is larger under quality and breakdown uncertainties when there are two common resources in the system. The total cost is far less than their basic form. It is observed that the process commonality is more effective when the system suffers from frequent breakdowns. The process commonality shows better control on the resource breakdown than the quality problem.

Fig. 7 illustrates the capacity requirements of the manufacturing system under various situations. The basic form of non-commonality design is used as the basis to display the bar chart. When common processes are introduced into the system, an increasing trend in capacity is observed. The capacity requirement of common resources is higher for a system using a single common process. It reduces when two consecutive common operations are in the same system (Fig. 5b) for the same planning period. Process commonality offers flexibility and concurrent operations (alternate routes) and hence increases utilization of the resources. Clearly, the subsequent resources require higher capacity, and this fact can be reflected in Fig. 7. It is also true when the system suffers from a quality problem. Process commonality is incapable of handling a quality issue in manufacturing. However, the capacity requirement declines from its basic form under the breakdown and quality problem when there is process commonality. This is because of alternative routes offered by the common resources. Indeed, the capacity needed is always higher when the system suffers from quality problems, breakdowns or both.

6 Conclusions

From this study and analysis, we can conclude that:

1. The proposed models can provide exact planning like MRP II under stable and stationary conditions. Additionally, the parts routes are easily traced on the floor for each planning period, even with quality problems and resources breakdowns.

2. Use of common processes in manufacturing is always better over the non-commonality scenario in terms of production cost.

3. The capacity requirements for common resources are always less than those for their non-commonality state when the system suffers from uncertainties. Process commonality in a system has more impact on breakdowns than quality uncertainty.

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