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Ground-borne vibrations due to dynamic loadings from moving trains in subway tunnels*

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Abstract: In this study, ground vibrations due to dynamic loadings from trains moving in subway tunnels were investigated using a 2.5D finite element model of an underground tunnel and surrounding soil interactions. In our model, wave propagation in the infinitely extended ground is dealt with using a simple, yet efficient gradually damped artificial boundary. Based on the assumption of invariant geometry and material distribution in the tunnel's direction, the Fourier transform of the spatial dimension in this direction is applied to represent the waves in terms of the wave-number. Finite element discretization is employed in the cross-section perpendicular to the tunnel direction and the governing equations are solved for every discrete wave-number. The 3D ground responses are calculated from the wave-number expansion by employing the inverse Fourier transform. The accuracy of the proposed analysis method is verified by a semi-analytical solution of a rectangular load moving inside a soil stratum. A case study of subway train induced ground vibration is presented and the dependency of wave attenuation at the ground surface on the vibration frequency of the moving load is discussed.

Key words: Subway tunnel, Moving train loadings, Ground-borne vibration, 2.5D finite element, Gradually damped artificial boundary

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1 Introduction

In recent decades, ground-borne vibration caused by subway traffic has received increasing attention in most developed cities in the world. Ground-borne vibration arises when successive axles of a train pass by a specific observation position. Proper vibration assessment is needed not only for the control of environmental vibration along the rail line, but also to maintain the serviceability of tunnel structures embedded in soft soil against long-term settlement generated by train traffic loadings.

Many predictive models are available in the literature for simulating and analyzing ground vibrations generated by subway traffic. Some analytical and semi-analytical solutions have been proposed to assess moving load induced ground vibrations (Forrest and Hunt, 2006). The wave types generated and their propagation patterns in the ground can be clearly interpreted theoretically from such solutions. However, a common disadvantage of these solutions is that assumptions must be made about the geometry and material to obtain solutions of the ground vibrations caused by moving loads. To take account of the effects of tunnel structure and soil condition on ground-borne vibration, a lot of work has been done in recent years to develop numerical predictive models, using mainly a finite element approach (Stamos and Beskos, 1995; Sheng *et al.*, 2003; Andersen and Jones, 2006). However, finite element models with local absorbing boundary

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conditions become prohibitively large for applications when a wide frequency range is under consideration. Even for a hybrid finite element-boundary element (FE-BE) model computation would be also very expensive (Degrande *et al.*, 2006). By assuming that the geometry and material distribution in the tunnel' s direction are invariant, the 2.5D finite element approach has been applied to the problem of ground vibrations caused by traffic (Takemiya, 2003; Yang *et al.*, 2003; Sheng *et al.*, 2006; Bian *et al.*, 2008).

In this paper, the 2.5D finite element approach (Bian *et al.*, 2008; 2011) is adopted to research the dynamic interaction between the tunnel-track structure and its surrounding ground under moving train loadings. The tunnel structure and ground with complicated physical properties in cross-section are modeled by quadrilateral elements with three degrees of freedom (DOFs) per node. The track resting on the tunnel floor is modeled by a Euler beam. The dynamic responses in the cross-section perpendicular to the tunnel direction are first solved, and then the 3D ground responses are calculated from the wave-number expansion by employing the inverse Fourier transform. A gradually damped artificial boundary is adopted to absorb the waves propagated from the tunnel to distant fields. Finally, the ground vibration features are revealed by illustrative case studies of a train's series of wheel axle loads moving along the tunnel.

2 Physical model and mathematical formulations

2.1 2.5D tunnel-track structure and ground finite element model

The tunnel-track structure and its surrounding ground interaction model are presented in Fig. 1. The material and geometric properties of the track and tunnel are assumed to be invariant along the tunnel direction. The Fourier transform in the tunnel direction is applied to represent the nodal motions by the wave-number in that direction. The track resting on the tunnel floor is modeled by a Euler beam, and its stiffness can be readily considered in the adjoining finite elements of the tunnel using the track-ground coupling method proposed by Takemiya (2003). The tunnel structure and ground in the transversal-vertical section are modeled by quadrilateral elements. Each node has three DOFs (Fig. 2), therefore wave motions in 3D space can be faithfully taken into account.

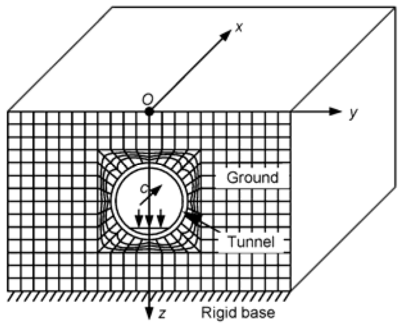

Fig. 1 Tunnel-track and surrounding soil interaction model with moving load

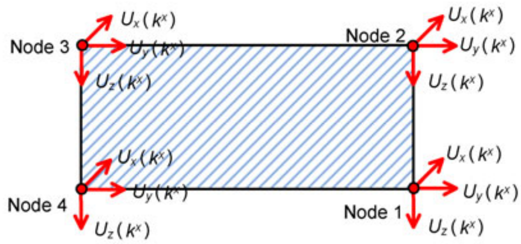

Fig. 2 Plain strain quadrilateral element with three DOFs per node

The adopted 2.5D finite element model in this study was described in detail by Bian *et al.* (2008). Here, we present some of the main equations. The Fourier transform in the x coordinate direction (along the tunnel direction) is used to simplify the equations governing ground motions:

$$u^x = \int_{-\infty}^{+\infty} u(x)\exp(\mathrm{i}k_x x)\mathrm{d}k_x, \tag{1}$$

and this equation's corresponding inverse transform is

$$u = \frac{1}{2\pi}\int_{-\infty}^{+\infty} u^x(k_x)\exp(-\mathrm{i}k_x x)\mathrm{d}k_x, \tag{2}$$

where the superscript 'x' means in wave-number domain and k_x is the wave-number along the x-directional. Navier's equation is used to describe the motions of the ground in the frequency domain:

$$\mu^* u_{i,jj}^t + (\lambda^* + \mu^*)u_{j,ji}^t + \omega^2 \rho u_i^t + f_i^t = 0, \tag{3}$$

in which $\lambda^*=(1+2\mathrm{i}\beta)\lambda$, $\mu^*=(1+2\mathrm{i}\beta)\mu$, where β is the damping ratio of the ground soil. The superscript 't' means in the frequency domain. In the light of the small strain assumption, the strain-displacement relationship in the wave-number and frequency domain can be given by

$$\boldsymbol{\varepsilon}^{xt}=\boldsymbol{B}\boldsymbol{u}^{xt}, \tag{4}$$

where the strain-displacement relationship matrix $\boldsymbol{B}$ is expressed in wave-number domain as

$$\boldsymbol{B}=\begin{bmatrix} -\mathrm{i}k_x & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & -\mathrm{i}k_x & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & -\mathrm{i}k_x \end{bmatrix}^{\mathrm{T}}. \tag{5}$$

The four-node quadrilateral element is used in our method. The discretized form of the governing equation in the frequency domain is given by

$$(\boldsymbol{K}^{xt}-\omega^2\boldsymbol{M})\boldsymbol{U}^{xt}=\boldsymbol{F}^{xt}, \tag{6}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}^{xt}$ is the stiffness matrix and $\boldsymbol{F}^{xt}$ is the equivalent nodal force vector.

2.2 Gradually damped artificial boundary

For static loading, a fictitious boundary may have little influence on ground responses as it is located at a proper distance from the vibration source. However, for dynamic loading, because of the wave reflection at this boundary, the responses obtained are not correct. We know that the finite element has a limited size and cannot deal with an infinite ground medium, so a kind of boundary at which little wave reflection occurs needs to be constructed. In this study, a gradually damped artificial boundary (Fig. 3) proposed by Liu and Quek Jerry (2003) is adopted which has already been implemented in the Abaqus program. This kind of boundary consists of N equal-width sets of damping layers. The kth (k=1, 2, …, N) damping layer's damping ratio can be calculated by

$$\beta_k=\beta_0\zeta^k. \tag{7}$$

where β_0 is the damping ratio of the ground, and ζ is a constant increasing factor.

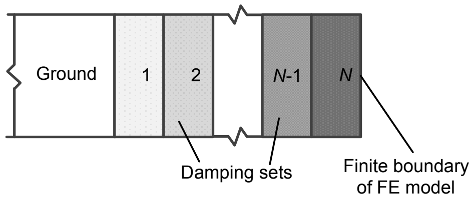

Fig. 3 Gradually damped artificial boundary

2.3 Verification of the present model

To verify the accuracy of our model, in this study we compared the responses due to a rectangular load moving inside a soil stratum computed by our proposed method with a semi-analytical solution. The computation model has a layer of soil 20 m deep with a rigid base (Fig. 4). The soil has a density of 1750 kg/m^3, a shear wave velocity of 100 m/s, a Poisson's ratio of 0.3, and a damping ratio of 0.05. A rectangular load (1 m×1 m) with a magnitude of 1 N is moving along the x coordinate at a speed of c inside the soil stratum at a depth of 10 m. In our model, the calculated width is 40 m, and there is a damping boundary with a width of 15 m on both sides. The damping boundary consists of five equal-width damping layers, with a constant increasing factor of ζ=2. The other parameters of the damping boundary are the same as those of the soil.

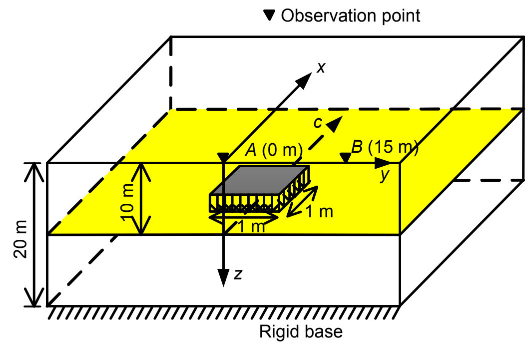

Fig. 4 Verification model

Figs. 5 and 6 show the vertical displacements at four different velocities at a point A (0, 0, 0) and a point B (0, 15, 0). From the figures we know that the responses obtained by our method are very close to the semi-analytical solutions. So the present 2.5D method is reliable for simulating the ground vibration.

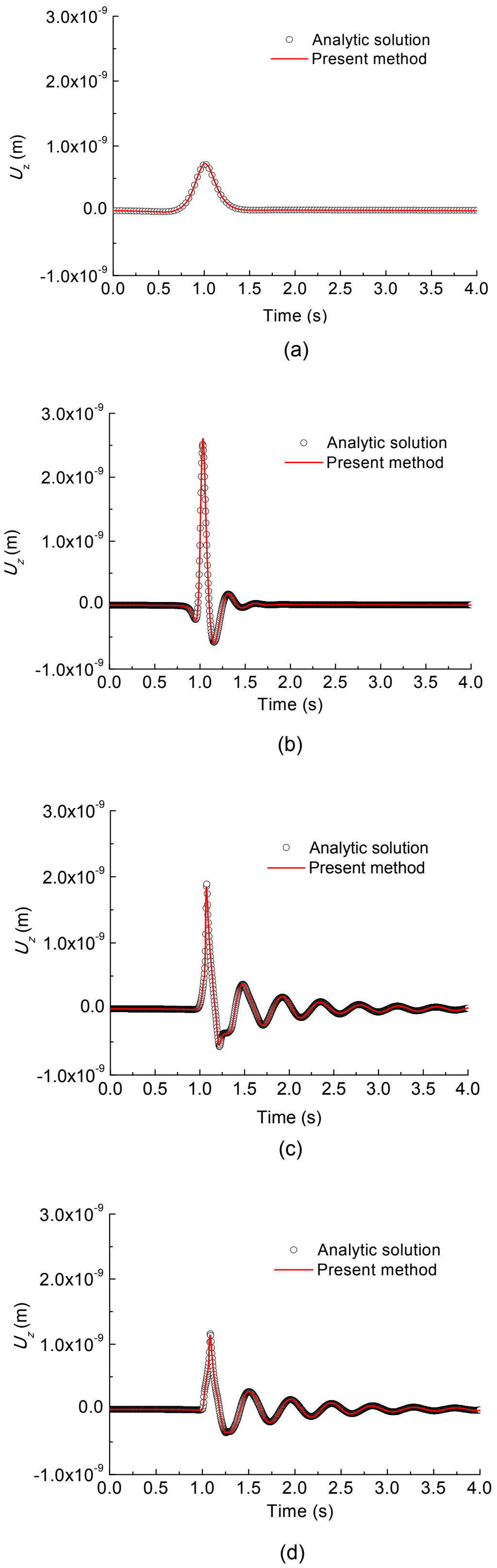

Fig. 5 Vertical displacements at point *A* at different velocities
(a) 50 m/s; (b) 95 m/s; (c) 150 m/s; (d) 200 m/s

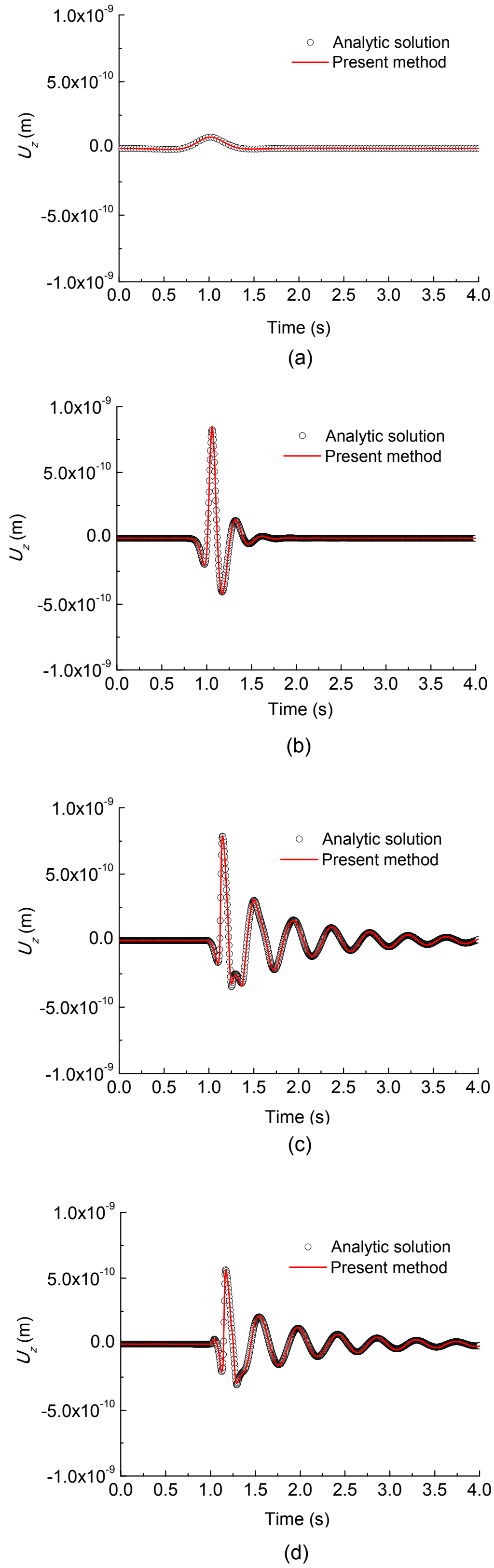

Fig. 6 Vertical displacements at point *B* at different velocities
(a) 50 m/s; (b) 95 m/s; (c) 150 m/s; (d) 200 m/s

3 Ground vibrations caused by moving train loadings

3.1 Parameters of the computation model

As an illustrative example, the actual geometry and physical properties of the tunnel-track structure and ground of a metro line in the Shanghai downtown area were used in the numerical computations. In the current analyses, to focus on the generation and propagation of wave motions generated by moving train loadings, the ground is assumed to have a depth of 50 m and a rigid base. The track has a unit length mass of 2370 kg/m and a bending stiffness of 200 MN·m^2. The thickness of the tunnel wall is 0.6 m, taking into account the two layers of tunnel lining. The outer radius of the tunnel is 3 m. The other properties of tunnel structure and surrounding soil are given in Table 1. Many observation points were selected on the ground surface along the transversal distance and within 60 m from the tunnel centerline (Fig. 7) to inspect the ground motions generated by the excitations of moving train loadings in the tunnel structure. The moving train loadings travel in the positive x direction (Fig. 2).

Table 1 Parameters of the ground soil and tunnel structure

Material	Density (kg/m^3)	V_s (m/s)	Poisson's ratio	Damping ratio
Ground soil	1800	110	0.250	0.05
Tunnel structure	3500	2500	0.150	0.02

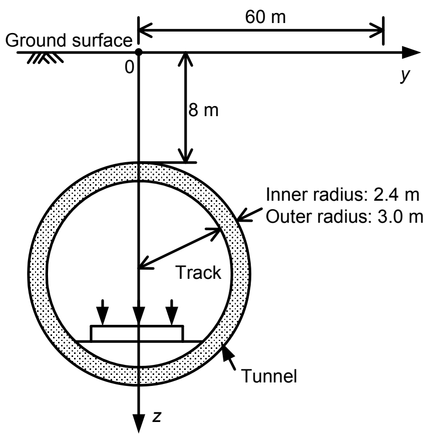

Fig. 7 Computation model of an underground tunnel and the extent of observation points on the ground surface

Currently, the running speed of most subway trains is relatively low, therefore, in this study, a normal operation speed of 20 m/s was considered. The vibration frequency of the moving load also needs special attention in studies of the dynamic effects of moving train loadings. In our computation, the train is considered to have M numbers of cars and runs with a constant velocity c, and its loading expression has been given by Takemiya and Bian (2005). Fig. 8 shows the geometry of the wheel axle weight distribution of the subway train used in this study.

3.2 Vibration attenuation at the ground surface

The wave propagation from the tunnel centerline to a far field is an important aspect of the ground vibrations caused by moving train loadings. In this section, the displacement response level (unit: dB) along the transversal distance from the tunnel centerline is interpreted, which can be computed by

$$L = 20\log\left(\frac{U}{U_0}\right), \tag{8}$$

where U is the computed displacement amplitude, and the reference value U_0=10^{-11} m. Attenuation of the displacement response level along the transversal distance from the tunnel centerline is depicted in Fig. 9. Obviously, the ground vibrations induced by moving train loadings in a subway tunnel can have a significant effect at the ground surface. That is why environmental vibration induced by subway trains causes quite a lot of complaints from residents living alongside the subway lines in metropolises. Ground vibrations generated by pseudo-static moving train loadings (f_0=0 Hz) have their strongest intensity at ground surface directly over the tunnel centerline, while the ground response generated by low

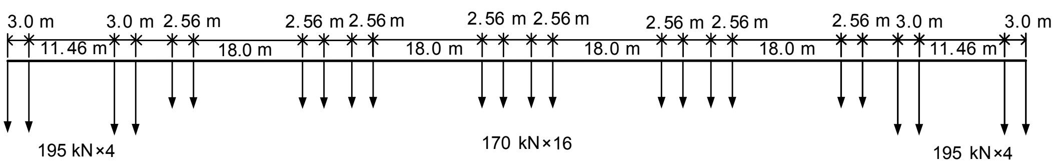

Fig. 8 Geometry of the wheel axle load distribution of a subway train

frequency (e.g., f_0=5 Hz) moving train loadings becomes the strongest at certain distances from the tunnel centerline. However, for moving train loadings with high vibration frequency (e.g., f_0=40 Hz), most wave energy will be absorbed during the propagation process in the adjacent soil medium.

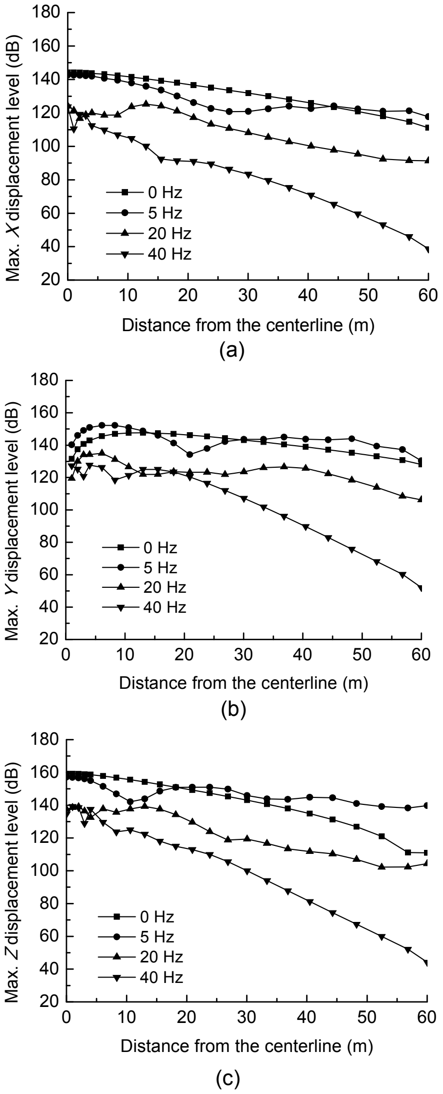

Fig. 9 Vibration attenuation at ground surface with distance from the tunnel centerline

(a) Horizontal displacement response level along the tunnel; (b) Horizontal displacement response level perpendicular to the tunnel; (c) Vertical displacement response level

3.3 Wave field generated by moving train loadings at the ground surface

Fig. 10 depicts the wave motions at the ground surface generated by moving train loadings in a tunnel at a speed of c=20 m/s. Four vibration frequencies are taken into account: f_0=0 Hz, 5 Hz, 20 Hz and 40 Hz, to consider the effects of both pseudo-static and dynamic components in moving train loadings.

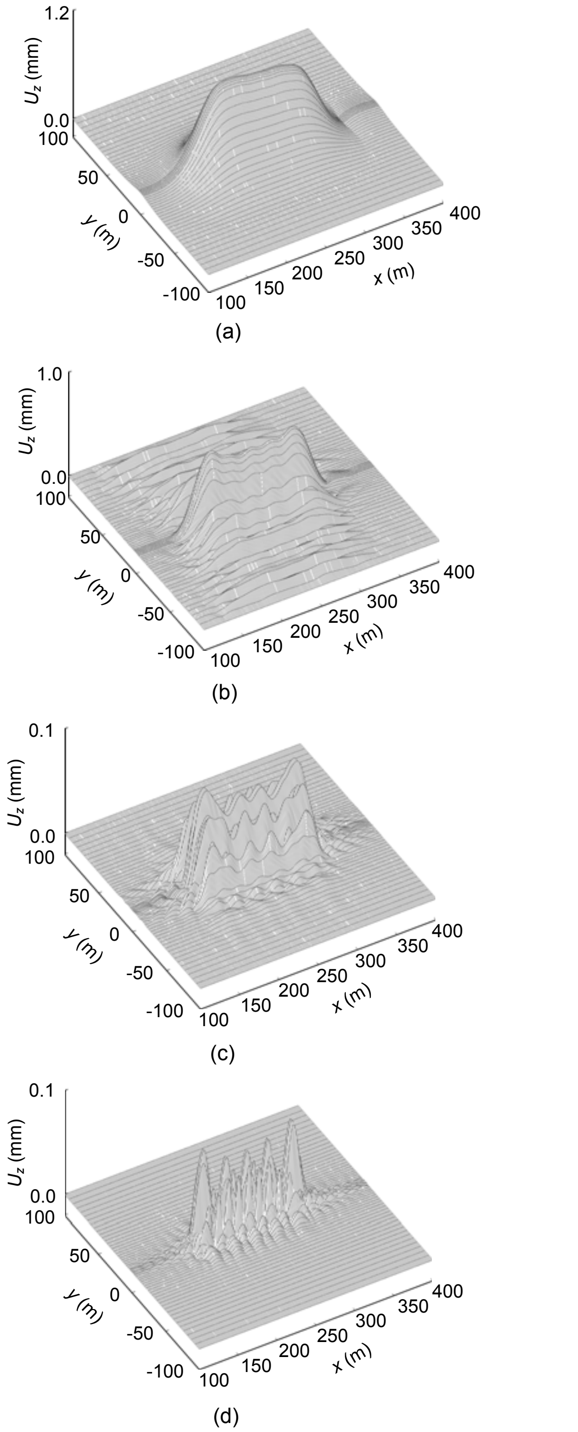

Fig. 10 Wave motions at the ground surface due to a train running at a speed of c=20 m/s in a tunnel with variable vibration frequencies

(a) f_0=0 Hz; (b) f_0=5 Hz; (c) f_0=20 Hz; (d) f_0=40 Hz

For moving train loadings without a vibration frequency, i.e., f_0=0 Hz, the ground vibration due to moving train loadings at a speed of c= 20 m/s can be regarded as the pseudo-static deformation induced by the total weight within the whole train geometry and no significant wave propagation phenomena can be observed at the ground surface. The ground motions generated by moving train loadings with certain vibration frequencies are quite different. At a relatively low vibration frequency of f_0=5 Hz, the wave motions can propagate a considerable extent from the tunnel centerline. With increasing vibration frequency, up to 20 Hz or 40 Hz, the vibration amplitudes at the ground surface decrease dramatically, and the effects of the distribution of wheel axle loads on ground responses become more apparent. In contrast to the results of a low vibration frequency, f_0=5 Hz, the ground motions due to moving train loadings with a high frequency, e.g., f_0=20 Hz or f_0=40 Hz, have a narrow propagation distance in the direction perpendicular to the tunnel centerline, but quite a large longitudinal propagation distance.

4 Conclusions

In this study, a high-efficiency 2.5D finite element approach incorporating a gradually-damped artificial boundary is proposed to calculate ground vibrations generated by subway trains. By comparing the computed responses of the ground surface due to a rectangular load moving inside soil stratum using our proposed method with the semi-analytical solution, we found that our 2.5D finite element method can calculate the ground response with very high accuracy.

Through the numerical computations, we found that the vibration frequency of moving train loadings has a great effect on the ground responses. The wave motions at the ground surface are confined to a narrow zone under pseudo-static moving train loadings without a vibration frequency, and are found in a much broader zone under moving train loadings with low frequency. But with increasing vibration frequency, the wave propagation zone at the ground surface becomes narrower and the effects of the distribution of wheel axle loads on ground responses become more apparent.

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