Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Estimating the time-dependent reliability of aging structures in the presence of incomplete deterioration information*

Cao WANG[†1,2], Quan-wang LI[†‡1], Long PANG[1], A-ming ZOU[1]

([1]*Department of Civil Engineering, Tsinghua University, Beijing 100084, China*)

([2]*School of Civil Engineering, The University of Sydney, Sydney 2006, Australia*)

[†]E-mail: wangcao12@tsinghua.org.cn; li_quanwang@tsinghua.edu.cn

Received Dec. 20, 2015; Revision accepted Mar. 24, 2016; Crosschecked Aug. 23, 2016

Abstract: The performance of an aging structure is commonly evaluated under the framework of reliability analysis, where the uncertainties associated with the structural resistance and loads should be taken into account. In practical engineering, the probability distribution of resistance deterioration is often inaccessible due to the limits of available data, although the statistical parameters such as mean value and standard deviation can be obtained by fitting or empirical judgments. As a result, an error of structural reliability may be introduced when an arbitrary probabilistic distribution is assumed for the resistance deterioration. With this regard, in this paper, the amount of reliability error posed by different choices of deterioration distribution is investigated, and a novel approach is proposed to evaluate the averaged structural reliability under incomplete deterioration information, which does not rely on the probabilistic weight of the candidate deterioration models. The reliability for an illustrative structure is computed parametrically for varying probabilistic models of deterioration and different resistance conditions, through which the reliability associated with different deterioration models is compared, and the application of the proposed method is illustrated.

Key words: Time-dependent reliability, Deterioration model, Error quantification, Averaged reliability, Structural safety

http://dx.doi.org/10.1631/jzus.A1500342 **CLC number:** TU312

1 Introduction

Civil infrastructures are often exposed to severe operating or environmental conditions in service. The strength or stiffness of these structures may deteriorate in time due to the service conditions and even fall beyond the baseline assumed for new ones, impairing the structural safety and serviceability significantly. Despite the impact of degradation in service on structural safety, because of socio-economic constraints, many degraded structures are still in use. The safety evaluation and service-life prediction of deteriorating structures should be based on reliability concepts and methods, aiming at preventing the use of these structures beyond an acceptable safety level. There has been a considerable amount of literature published in past decades on the safety evaluation of existing structures (Mori and Ellingwood, 1993; Hong, 2000; Stewart and Mullard, 2007), taking into account the uncertainties associated with

‡ Corresponding author

* Project supported by the National Natural Science Foundation of China (No. 51578315) and the Major Projects Fund of Chinese Ministry of Transport (No. 201332849A090)

ORCID: Cao WANG, http://orcid.org/0000-0002-2802-1394; Quan-wang LI, http://orcid.org/0000-0002-6967-1940

structural resistance and applied load models. The work by Mori and Ellingwood (1993) was one of the first attempts to assess time-dependent reliability of structures considering both the randomness of resistance and the stochastic nature of load, and was used to predict the remaining service-life of deteriorating concrete structures (Enright and Frangopol, 1998; Mori and Ellingwood, 2006). Li Q.W. *et al.* (2015) improved the work by Mori and Ellingwood (1993) and proposed an explicit method for time-dependent reliability analysis of aging structures subjected to non-stationary loads. In these existing methods, the presence of the probability distribution of resistance deterioration is essentially necessary for structural reliability analysis when taking into account the uncertainty associated with the time-variant resistance. Practically, the main characteristics of resistance deterioration (e.g., the mean value and/or the standard deviation) can be obtained by either experimental laboratory data (Paik and Kim, 2012; Mohd and Paik, 2013; Ma *et al.*, 2013; Mohd *et al.*, 2014) or *in-situ* inspection following established standards or manuals (AASHTO, 2008; MOT, 2011; Wang *et al.*, 2015). However, the probabilistic distribution of degraded resistance often remains unaddressed due to the limits of available data (Ellingwood, 2005). While this rough estimate of deterioration condition is widely employed for reliability analysis of aging structures in engineering practice, the question arises then regarding (1) the error of reliability induced if one chooses empirically the deterioration probabilistic model and (2) how to estimate the structural reliability under incomplete deterioration information.

The scope of this paper is to discuss the estimate of time-dependent reliability of aging civil structures under incomplete deterioration information. The role of deterioration model choice in structural reliability is investigated, where five candidate deterioration models, namely uniform, normal, Beta, after-mentioned inverse-lognormal, and Gamma distributions, are discussed. A new method is developed for the estimate of averaged structural reliability under incomplete deterioration information, which is independent of the probabilistic weight of the candidate models. Illustrative examples are also presented to demonstrate the application of the proposed method and to compare the reliabilities associated with different deterioration models.

2 Structural reliability analysis

Consider the reliability of a structure over time interval $(0, T]$, where significant load events occur randomly in time with random intensities. Suppose that the load intensity varies negligibly during the interval in which it occurs without dynamic response, and the duration of each load is small enough. With this, the sequence of loads may be treated as a Poisson point process (a discrete stochastic process). If the sequence of loads, $S_1, S_2, \ldots, S_n$, occur at time points $t_1, t_2, \ldots, t_n$ during $(0, T]$, then the structural time-dependent reliability, $L(T)$, is written as

$$L(T) = \Pr[R(t_1) > S_1 \cap \cdots \cap R(t_n) > S_n], \quad (1)$$

where $\Pr[\cdot]$ denotes the probability of the event in the bracket, and $R(t_1), R(t_2), \ldots, R(t_n)$ are the resistances corresponding to times $t_1, t_2, \ldots, t_n$, respectively. While the structural deterioration process is indeed a complex non-stationary stochastic process (Dieulle *et al.*, 2003; Saassouh *et al.*, 2007; Wang *et al.*, 2015), neither fully-correlated nor statistically independent by nature, Li Q.W. *et al.* (2015) revealed that the assumption of fully-correlated deterioration is reasonable for reliability analysis producing acceptable error. As a result, in this study, the resistance deterioration process is assumed to be a fully-correlated one, which means that the overall trajectory of deterioration will be determined as soon as the deterioration type and the deterioration value at any time are given.

Now, if the sequence of load effects, S_1, S_2, ..., S_n, is modeled as a stationary Poisson process with time-invariant occurrence rate λ, and each load effect is statistically independent with cumulative density function (CDF) of $F_S(s)$, under the assumption that the deterioration process is deterministic, we have (Mori and Ellingwood, 1993)

$$L(T) = \exp\left\{-\int_0^T \lambda[1 - F_S(r(t))]\mathrm{d}t\right\}, \quad (2)$$

where $r(t)$ is the deterministic resistance at time t. More generally, if the load process is a non-stationary one with time-variant occurrence rate $\lambda(t)$ and time-dependent CDF of load corresponding to time t, $F_S(s, t)$, Eq. (2) becomes (Li Q.W. *et al.*, 2015)

$$L(T) = \exp\left\{-\int_0^T \lambda(t)[1 - F_S(r(t), t)]\mathrm{d}t\right\}. \quad (3)$$

3 Probabilistic model of resistance deterioration

The varying of material properties of structures over time impacts structural performance and service life. A frequently-used model to describe the fully-correlated resistance deterioration of aging structures is

$$R(t) = R_0 \cdot [1 - G(t)], \tag{4}$$

where $G(t)$ denotes the deterioration function, which is a mechanism-related process by nature. $G(t)$ takes the form of $G(t) = K \cdot t^{\alpha}$, where K and α are parameters determining the deterioration rate and shape, respectively. The parameter K is a random variable reflecting the uncertainty associated with $G(t)$, while the value of α is related to the dominant deterioration mechanism of interest. Deterioration of concrete due to expansive aggregate reactions or sulfate (or other chemical) attack is nonlinear; when the loss of rebar diameter due to active general corrosion is governed by Faraday's law, and the loss of rebar area is approximately linear for small losses due to corrosion, the deterioration of flexural capacity is approximately linear (Clifton and Knab, 1989; Mori and Ellingwood, 1993). Correspondingly, α equals 1, 2, and 0.5, respectively, for the dominant mechanisms of corrosion, sulfate attack, and diffusion-controlled aging. The initial resistance, R_0, is assumed deterministic in this study (i.e., $R_0 = r_0$) since its variation is negligible compared with the uncertainty associated with the deterioration and loading processes. For the case where R_0 is a random variable and the probability density function (PDF) of R_0 is known, the structural reliability may be reassessed based on the total probability theorem, for which some analytical techniques such as Latin Hypercube sampling (Mckay *et al.*, 2000) may be employed to improve the calculation efficiency.

In practice, the structural resistance is seldom evaluated during the service life partially because of the relatively high cost involved; as a result, available deterioration models are often associated with great epistemic uncertainties due to the limited amount of reliable data. For example, when performing reliability analysis for a structure during a service period of T, modeling the resistance deterioration as fully correlated, the probabilistic distribution of $G(T)$ is first predicted approximately with limited information such as mean value and empirical coefficient of variation (COV, defined as the ratio of standard deviation to the mean value), which further determines the overall deterioration rate within the whole service period. That is, the distribution of $G(T)$ cannot be determined with high accuracy using these limited data. With this in mind, several common distribution types including uniform, normal, Beta, inverse-lognormal (for random variable X, if $1 - X$ follows lognormal distribution, then X is deemed to follow inverse-lognormal distribution), and Gamma distributions are often selected to model the uncertainty associated with $G(T)$. These distribution types will be employed in this study to describe the probabilistic behaviour of $G(T)$ in the deterioration model-related reliability analysis.

A brief explanation for the five candidate probabilistic models is presented as follows. (1) The uniform and normal distributions are two of the most common and widely-used models providing unbiased probabilistic descriptions for random variables; they may be employed to model the variation associated with resistance deterioration in reliability analysis when there is insufficient information available. (2) The Beta distribution is used in some studies to model $G(T)$ because a random variable following Beta distribution is defined strictly in [0, 1] (Li and Wang, 2015). (3) The classical probability theorem demonstrates that the resistance at time T, $R(T)$, may be modeled as lognormal distribution since it is the product of several random variables, indicating that $1 - G(T)$ also follows lognormal distribution with deterministic R_0 (or lognormal-distributed R_0 referring to Eq. (4)); in such a case, $G(T)$ follows an inverse-lognormal distribution. (4) Some studies have also used the Gamma process to model the resistance deterioration (Dieulle *et al.*, 2003; Li Q.W. *et al.*, 2015; Wang *et al.*, 2015), with which $G(T)$ follows Gamma distribution.

Now, taking into account the uncertainty associated with $G(T)$, supposing that the PDF of $G(T)$ is $f_G(g)$, Eq. (2) is rewritten as follows with the help of the total probability theorem:

$$L(T) = \int_{-\infty}^{+\infty} \exp\left[- \int_0^T \lambda(t)(1 - F_S(r(t|g), t))\mathrm{d}t \right] \cdot f_G(g)\mathrm{d}g, \tag{5}$$

where $r(t|g)$ is the resistance at time t given that $G(T)$ equals g. However, note that Eq. (5) works only in terms of mathematical derivation; rather, $G(T)$ should not be less than 0 for structures without maintenance or repair measures because the resistance process is non-increasing, nor be greater than 1 because the resistance of a structure never become a negative value. Thus, Eq. (5) is rewritten as

$$L'(T)=\int_0^1 \exp\left[-\int_0^T \lambda(t)(1-F_S(r(t|g),t))\mathrm{d}t\right] \cdot f_G(g)\mathrm{d}g. \tag{6}$$

With Eq. (6),

$$\begin{aligned}\lim_{T\to 0} P_\mathrm{f}'(T) &= 1-\int_0^1 \exp(0) f_G(g)\mathrm{d}g \\ &= 1-F_G(1)+F_G(0),\end{aligned} \tag{7}$$

where $P_\mathrm{f}'(T)=1-L(T)$, and $F_G(\cdot)$ is the CDF of $G(T)$. Eq. (7) implies that Eq. (6) may lead to significant error in structural reliability if $1-F_G(1)+F_G(0)$ is non-negligible compared with the structural failure probability. For example, supposing that the mean value and COV of $G(T)$, $\mu_{G(T)}$ and $c_{G(T)}$, equal 0.2 and 0.4, respectively, if $G(T)$ is modeled as normally distributed, $1-F_G(1)+F_G(0)$ approximates 0.006 21, leading to a significant error when the structural failure probability is around 0.001 (corresponding to a reliability index of 3.1). In such cases, Eq. (6) is further revised as

$$L_R(T)=\frac{L'(T)}{F_G(1)-F_G(0)}. \tag{8}$$

The item $1/(F_G(1)-F_G(0))$ in Eq. (8) accounts for the truncated distribution of $f_G(g)$ defined in $[0,1]$; it is introduced so that $\lim_{T\to 0} P_\mathrm{f}^R(T)=1-\lim_{T\to 0} L_R(T)=0$. As will be demonstrated in Section 5.2, the structural reliability estimated by Eqs. (5) and (8) are close, while Eq. (6) may underestimate the reliability significantly with great COV of $G(T)$ and short service period. Thus, the structural reliability or failure probability will be estimated using Eq. (8) in the following when $G(T)$ is assumed as one of the five candidate probabilistic models.

4 Reliability assessment under incomplete deterioration information

Probability-based civil engineering problems are often associated with incomplete information due to the limits of available data (Ellingwood, 2005; Tang *et al.*, 2013; 2015). In terms of structural reliability analysis, from the viewpoint of deterioration model choice, it is noted that there are various possible solutions associated with different deterioration models and some of them may even be biased towards the non-conservative side. Since the structural reliability itself can be regarded as a random variable (Zhang *et al.*, 2010), whose uncertainty is related to the choice of probabilistic models of resistance or load, the objective is to select an appropriate approach to estimate the reliability properly.

For a set of candidate models Ω = {uniform, normal, inverse-lognormal, Beta, Gamma}, let $L_R(T|\omega_j)$ denote the structural reliability associated with the candidate distribution ω_j from Ω, where $\Omega=\{\omega_1,\omega_2,\ldots,\omega_k\}$ (in this study, $k=5$). Traditionally, the concept of 'averaged reliability' is routinely used to account for the multiple choices of potential probabilistic models (Cao and Wang, 2014; Li D.Q. *et al.*, 2015). With this, using the Bayesian method,

$$L_\mathrm{a}(T)=\sum_{j=1}^{k} q_j L_R(T|\omega_j), \tag{9}$$

where $\sum_{j=1}^{k} q_j=1$, $L_\mathrm{a}(T)$ is the averaged structural reliability, which is a robust estimate of structural reliability because it takes into account the possibility that each candidate deterioration model may be the realistic one. With Eq. (9), the failure probability, $P_{\mathrm{f,a}}(T)$, equals $1-L_\mathrm{a}(T)$.

However, this traditional and well-documented method relies heavily on the weight of each candidate model, q_j. As a result, $L_\mathrm{a}(T)$ may be significantly biased when the selection of candidate models is empirical or inappropriate. In this regard, a new method is developed herein for the estimate of averaged reliability of aging structures with incomplete deterioration information.

As before, let $f_G(g)$ denote the PDF of $G(T)$. Supposing that the mean value and standard deviation of $G(T)$ are assessed as μ and σ, respectively, we have

$$E(G(T))=\int_0^1 g\cdot f_G(g)\mathrm{d}g=\mu, \tag{10a}$$

$$E(G(T)^2)=\int_0^1 g^2\cdot f_G(g)\mathrm{d}g=\mu^2+\sigma^2, \tag{10b}$$

where $E(\cdot)$ denotes the mean value of the random variable in the bracket. Now we reconsider Eq. (5), which is rewritten as follows provided that $G(T)$ is strictly defined in $[0,1]$:

$$L(T)=\int_0^1 h(g)\cdot f_G(g)\mathrm{d}g, \tag{11}$$

where

$$h(g)=\exp\left[-\int_0^T \lambda(t)(1-F_S[R(t|g),t])\mathrm{d}t\right]. \tag{12}$$

Consider the Fourier expansion of $h(g)$, i.e.,

$$h(g)=\frac{a_0}{2}+\sum_{m=1}^{\infty}[a_m\cos(mg\pi)+b_m\sin(mg\pi)], \tag{13}$$

where

$$a_j=\int_{-1}^1 h(g)\cos(jg\pi)\mathrm{d}g,\ j=0,1,2,\cdots, \tag{14a}$$

$$b_j=\int_{-1}^1 h(g)\sin(jg\pi)\mathrm{d}g=0,\ j=1,2,\cdots, \tag{14b}$$

in which $h(-g)=h(g)$ for $\forall g\in[0,1]$. By noting that for any real number x, $\cos x=(\mathrm{e}^{\mathrm{i}x}+\mathrm{e}^{-\mathrm{i}x})/2$, where $\mathrm{i}=\sqrt{-1}$, we have

$$h(g)=\frac{a_0}{2}+\frac{1}{2}\sum_{m=1}^{\infty}a_m\left[\exp(\mathrm{i}mg\pi)+\exp(-\mathrm{i}mg\pi)\right]. \tag{15}$$

Substituting Eq. (15) into Eq. (11),

$$L(T)=\frac{a_0}{2}+\frac{1}{2}\sum_{m=1}^{\infty}a_m\left[\phi(\mathrm{i}m\pi)+\phi(-\mathrm{i}m\pi)\right], \tag{16}$$

where $\phi(x)$ is the moment generating function of $G(T)$, $\phi(x)=E\left(\mathrm{e}^{G(T)\cdot x}\right)$ ($\phi(\mathrm{i}x)$ is called the characteristic function of $G(T)$). A generalized form of $\phi(x)$ is

$$\phi(x)=\sum_{j=0}^{\infty}c_jx^j. \tag{17}$$

Since $\phi(0)=1$, $c_0=1$. Substituting Eq. (17) into Eq. (16), $L(T)$ becomes

$$L(T)=\frac{a_0}{2}+\sum_{m=1}^{\infty}\left(a_m\sum_{j=0}^{\infty}c_{2j}\cdot(m\pi)^{2j}(-1)^j\right). \tag{18}$$

An important property of the moment generating function is that $\phi^{(j)}(0)=E[G(T)^j]$ for $j=0,1,2,\cdots$, where $\phi^{(j)}(x)$ is the jth order derivative of $\phi(x)$. According to Eq. (17), $\phi^{(j)}(0)=c_j\cdot j!$. However, $E[G(T)^j]$ for $j>2$ remains unaddressed because of the incomplete probabilistic information of $G(T)$. Generally, $E[G(T)^j]$ declines as j increases, and $E[G(T)^j]\to 0$ when $j\to\infty$. Moreover, $E[G(T)^{j+1}]/E[G(T)^j]$ increases with j and $\sup\left\{E[G(T)^{j+1}]/E[G(T)^j]\right\}=1$ (see Appendix A for detailed explanation). With this, $E[G(T)^{j+1}]/E[G(T)^j]$ is expected to take the form of

$$\frac{E[G(T)^{j+1}]}{E[G(T)^j]}=1-\exp(\gamma_1\cdot j+\gamma_2), j=0,1,\cdots, \tag{19}$$

where the two parameters, γ_1 and γ_2, are obtained as follows according to Eq. (10):

$$\gamma_1=\ln\left(1-\frac{\mu^2+\sigma^2}{\mu}\right)-\ln(1-\mu), \tag{20a}$$

$$\gamma_2=\ln(1-\mu). \tag{20b}$$

Thus,

$$E[G(T)^j]=\prod_{k=0}^{j-1}\left[1-\exp(\gamma_1k+\gamma_2)\right]. \tag{21}$$

With Eq. (21), the parameters c_j ($j\geq 1$) can be determined as

$$c_j=\frac{\phi^{(j)}(0)}{j!}=\frac{\prod_{k=0}^{j-1}\left[1-\exp(\gamma_1k+\gamma_2)\right]}{j!}. \tag{22}$$

Substituting Eq. (22) into Eq. (18), it is seen that all the parameters in Eq. (18) have been addressed, with which Eq. (18) is ready for structural reliability analysis taking into consideration the uncertainty associated with the resistance deterioration, providing the mean value and standard deviation of $G(T)$. In summary, Eq. (18) is the proposed averaged time-dependent reliability of aging structures in the presence of incomplete deterioration information, which is independent of the probability mass (weight) of the candidate deterioration models. The solution of Eq. (18) is discussed in Appendix B and the application of Eq. (18) is found in Section 5.3.

Finally, it is emphasized that in many cases, the expression of $h(g)$ can be simplified with no integral operation involved so that the efficiency of Eq. (18) can be significantly improved. For the case where the load process is stationary, and each load effect follows an Extreme type I distribution (if the mean value and standard deviation of load effect are μ_S and σ_S, respectively, then $a=\sqrt{6}\sigma_S/\pi$ and $m=$

$\mu_S - 0.5772a$) with position parameter of m and scale parameter of a, $h(g)$ is simplified as follows with the assumption of linear deterioration (Wang *et al.*, 2016):

$$h(g) = \exp(-\lambda \cdot \xi), \tag{23}$$

where

$$\xi = \begin{cases} \exp\left(\dfrac{m-r_0}{a}\right)T, & g=0, \\ \exp\left(\dfrac{m-r_0}{a}\right)\dfrac{aT}{r_0 g}\left[\exp\left(\dfrac{r_0 g}{a}\right)-1\right], & g \neq 0. \end{cases} \tag{24}$$

Further, taking into account the non-stationarity in load process, if the mean value of load increases linearly with time, i.e., $\mu_S(t) = \mu_S(0) + \kappa_m t$, while the standard deviation is constant, ξ is revised as

$$\xi = \exp\left(\frac{m_0 - r_0}{a}\right)\frac{aT}{r_0 g + \kappa_m T} \times \left[\exp\left(\frac{r_0 g + \kappa_m T}{a}\right) - 1\right], \tag{25}$$

where $m_0 = \mu_S(0) - 0.5772a$. Next, if the mean occurrence rate of load increases linearly with time, $\lambda(t) = \lambda_0 + \kappa_\lambda t$, while its mean value and standard deviation are constant, $h(g)$ is rewritten as (Wang *et al.*, 2016)

$$h(g) = \exp(-\lambda_0 \cdot \xi - \kappa_\lambda \cdot \psi), \tag{26}$$

where

$$\psi = \begin{cases} \exp\left(\dfrac{m-r_0}{a}\right)\cdot\dfrac{1}{2}T^2, & g=0, \\ \exp\left(\dfrac{m-r_0}{a}\right)\left\{\exp\left(\dfrac{r_0 g}{a}\right)\cdot\left[\dfrac{aT^2}{r_0 g} - \left(\dfrac{aT}{r_0 g}\right)^2\right] + \left(\dfrac{aT}{r_0 g}\right)^2\right\}, & g \neq 0. \end{cases} \tag{27}$$

It is seen that the simplified expressions of $h(g)$ as in Eqs. (23) and (26) are beneficial for finding the coefficient parameters a_j in Eq. (14).

5 Illustrative examples

5.1 Structural configuration

The role of deterioration model choice in structural time-dependent reliability and the application of Eq. (18) are illustrated in this section using several simple parametric examples. The structural initial resistance is assumed to be deterministic and equal to 1.05 times of R_n (R_n is the nominal or code-specified resistance). The dead load, D, equals its nominal value (D_n), and the variability of D is assumed to have a negligible impact on reliability in comparison with the uncertainty in the time-varying live load. The mean load occurrence rate of the live load is set equal to 1/year, and a reference period of 40 years is considered in our analysis. The deterioration function, $G(40)$, is assumed to have a mean value of 0.2 or 0.4 at 40 years (i.e., Mean$[G(40)] = 0.2$ or 0.4). Taking into account the non-stationarity in the live load (L), the mean value of L, μ_L, is assumed to vary linearly in time:

$$\mu_L(t) = (0.4 + 0.005t) \cdot L_n, \tag{28}$$

where L_n is the nominal live load. Eq. (28) implies that the mean value of the live load increases from $0.4L_n$ at initial time to $0.6L_n$ at the end of 40 years. The live load is assumed to follow the Extreme type I distribution with standard deviation being $0.12L_n$ constantly for all cases. The combination of nominal live load L_n and dead load D_n is considered according to the following design equation (ACI Committee, 2014):

$$0.9R_n = 1.2D_n + 1.6L_n. \tag{29}$$

For simplicity, it is assumed that $D_n = L_n$. The structure is subjected to both live load and dead load, and since the dead load is assumed to be deterministic, whose effect can be subtracted from the resistance, the items $F_S(R(t|g), t)$ in Eqs. (5), (6), and (8) should be replaced by $F_S(R(t|g) - D, t)$. Similarly, if Eq. (23) or (26) is used, the item $\exp((m - r_0)/a)$ in Eqs. (24) and (27) should be replaced by $\exp((m + D - r_0)/a)$ and $\exp((m_0 - r_0)/a)$ in Eq. (25) should be modified as $\exp((m_0 + D - r_0)/a)$.

5.2 Comparison of Eqs. (5), (6), and (8)

The difference between structural time-dependent failure probabilities obtained from Eqs. (5), (6), and (8) are presented in Fig. 1, assuming that the resistance degrades linearly to 0.6 times of initial resistance (i.e., $\mu_{G(40)} = 0.4$) with COV of $G(40)$ being 0.2 or 0.4. In such cases, the Beta and uniform distributions satisfy that $G(40)$ is defined in

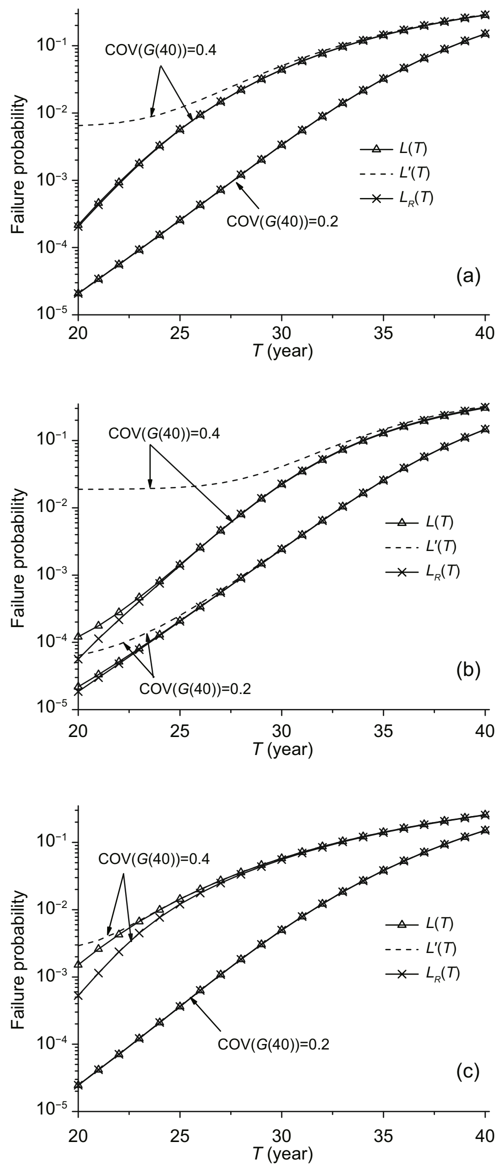

Fig. 1 Comparison of failure probabilities associated with different analysis methods: (a) normal distribution, (b) inverse-lognormal distribution, and (c) Gamma distribution

[0, 1] or a subdomain of [0, 1], and thus $L(T)$, $L'(T)$, and $L_R(T)$ are equivalent. For normal and inverse-lognormal distribution types, it is seen from Fig. 1 that the failure probability may be overestimated using Eq. (6) and this overestimation is enhanced with greater COV of $G(40)$. This is explained by the fact that with Eq. (6) where $G(40)$ was assumed to be defined in $\mathbb{R} = (-\infty, +\infty)$, the structural reliability will be underestimated since the probability of $G(40) < 0$ or $G(40) > 1$ is ignored in Eq. (6); such probability becomes larger as a result of the increment of the COV of $G(40)$. For example, for a service period of 22 years, modeling $G(40)$ as inverse-lognormal distributed with $c_{G(40)}$ being 0.4, $P_{\mathrm{f}}'(22)$ equals 0.019, about 68 times of $L(22)$ and 87 times of $L_R(22)$. However, for the case of the Gamma distribution, the error among $L(T)$, $L'(T)$, and $L_R(T)$ is insignificant.

For all cases, the lines of $L'(T)$ and $L_R(T)$ are almost overlapping, indicating that the reliabilities obtained according to Eqs. (5) and (8) are very close. This means that when performing reliability analysis in engineering practice, one may choose either Eq. (5) or (8). Moreover, it is seen that the structural failure probability increases with greater COV of $G(40)$, because the uncertainty associated with resistance becomes larger with greater value of $c_{G(40)}$, leading to a reduction in structural service reliability.

5.3 Reliability assessment under incomplete deterioration information

First, to investigate the impact that the choice of resistance deterioration model has on the time-dependent probability of failure, $P_{\mathrm{f}}^R(T)$, Fig. 2 plots the structural failure probabilities for reference periods up to 40 years, utilizing the five distribution types as discussed in Section 3. The mean value of $G(40)$ is assumed to equal 0.2 or 0.4 while the COV of $G(40)$, $c_{G(40)}$, is 0.2 or 0.4. For all cases, the resistance degrades linearly in time. It is seen that the failure probability increases with greater $\mu_{G(40)}$ as expected, because more severely degraded resistance leads to larger service risk. The probabilities of failure associated with different deterioration models are close to each other when the COV of $G(40)$ is small, regardless of the magnitude of the failure probability. This means that if the deterioration process is associated with small uncertainty, the choice of probabilistic model of resistance deterioration only has a slight impact on the structural reliability. However, with increasing $c_{G(40)}$, the difference between the reliabilities associated with different deterioration models becomes significant. Obviously, the structural reliability cannot be determined uniquely due to the multi-choice of deterioration models. For the case of $\mu_{G(40)} = 0.2$, $c_{G(40)} = 0.4$, the failure probability associated with the Gamma distributed $G(40)$ is the largest, followed by those associated with Beta, normal, inverse-lognormal, and uniform distributions. However, comparing Figs. 2c and 2d,

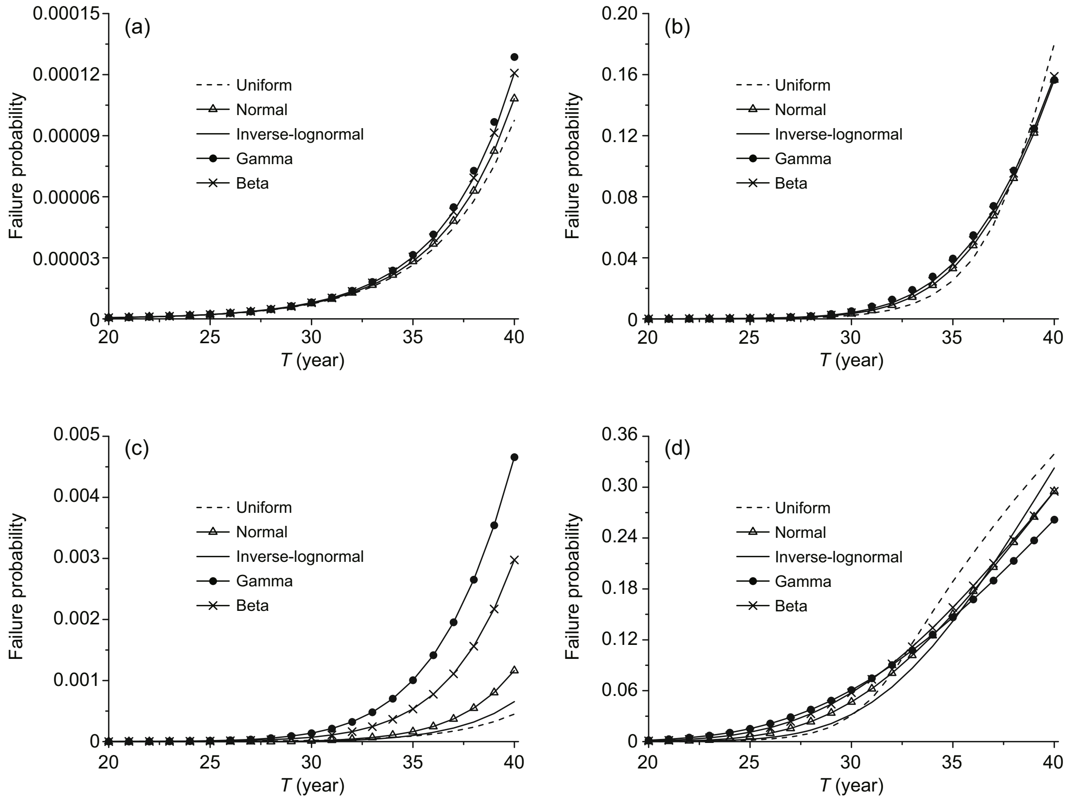

Fig. 2 Dependence of failure probability on resistance deterioration: (a) $\mu_{G(40)}=0.2, c_{G(40)}=0.2$; (b) $\mu_{G(40)}=0.4, c_{G(40)}=0.2$; (c) $\mu_{G(40)}=0.2, c_{G(40)}=0.4$; (d) $\mu_{G(40)}=0.4, c_{G(40)}=0.4$

it is found that the Gamma distribution for $G(40)$ does not produce the greatest failure probabilities for all cases; in fact, among the five candidate probability models of deterioration, no such model exists which always leads to the greatest or smallest failure probability.

Corresponding to the four cases shown in Fig. 2, the averaged time-dependent failure probabilities, $P_{\rm f}(T)$ (Eq. (18)), for service periods up to 40 years are presented in Fig. 3, where the up and low bounds of $P_{\rm f,a}(T)$ are also plotted for the purpose of comparison. According to Eq. (9), $\min_{j=1}^{k} L_R(T|\omega_j) \leq L_a(T) \leq \max_{j=1}^{k} L_R(T|\omega_j)$, and thus the up and low bounds of $P_{\rm f,a}(T)$ are $1-\min_{j=1}^{k} L_R(T|\omega_j)$ and $1-\max_{j=1}^{k} L_R(T|\omega_j)$, respectively. Generally, with the service period fixed, $P_{\rm f,a}(T)$ varies because of the different choices of probabilistic mass (weight) of the candidate deterioration models (i.e., q_j in Eq. (9)). For the case of small variation associated with $G(40)$ (e.g., $c_{G(40)}=0.2$), the difference between the two bounds of $P_{\rm f,a}(T)$ is negligible, which is consistent with the observation from Fig. 2 that the deterioration model choice has a slight impact on the structural reliability if the COV of $G(40)$ is small. However, as the uncertainty associated with $G(40)$ increases, the difference between the up and low bounds of $P_{\rm f,a}(T)$ becomes increasingly significant. For instance, for a service period of 40 years, when $\mu_{G(40)}=0.2, c_{G(40)}=0.4$, the up bound of $P_{\rm f,a}(40)$ is 0.004 66, about 10 times of the low bound. For all cases, $P_{\rm f}(T)$ lies between the up and low bounds of $P_{\rm f,a}(T)$, indicating that Eq. (18) provides a reasonable estimate of structural averaged reliability. For the case of small $c_{G(40)}$, the difference between $P_{\rm f}(T)$ and $P_{\rm f,a}(T)$ is expected to be insignificant since the up and low bounds of $P_{\rm f,a}(T)$ are very close; however, as the variation associated with the deterioration process increases, the two estimates of structural averaged failure probability, $P_{\rm f,a}(T)$ and $P_{\rm f}(T)$, may become remarkably different. Keeping in mind that $P_{\rm f}(T)$ is independent of the probabilistic mass function of the candidate deterioration models, the structural averaged time-dependent reliability estimated using Eq. (18) is associated with reduced epistemic uncertainty, compared with the reliability obtained from Eq. (9).

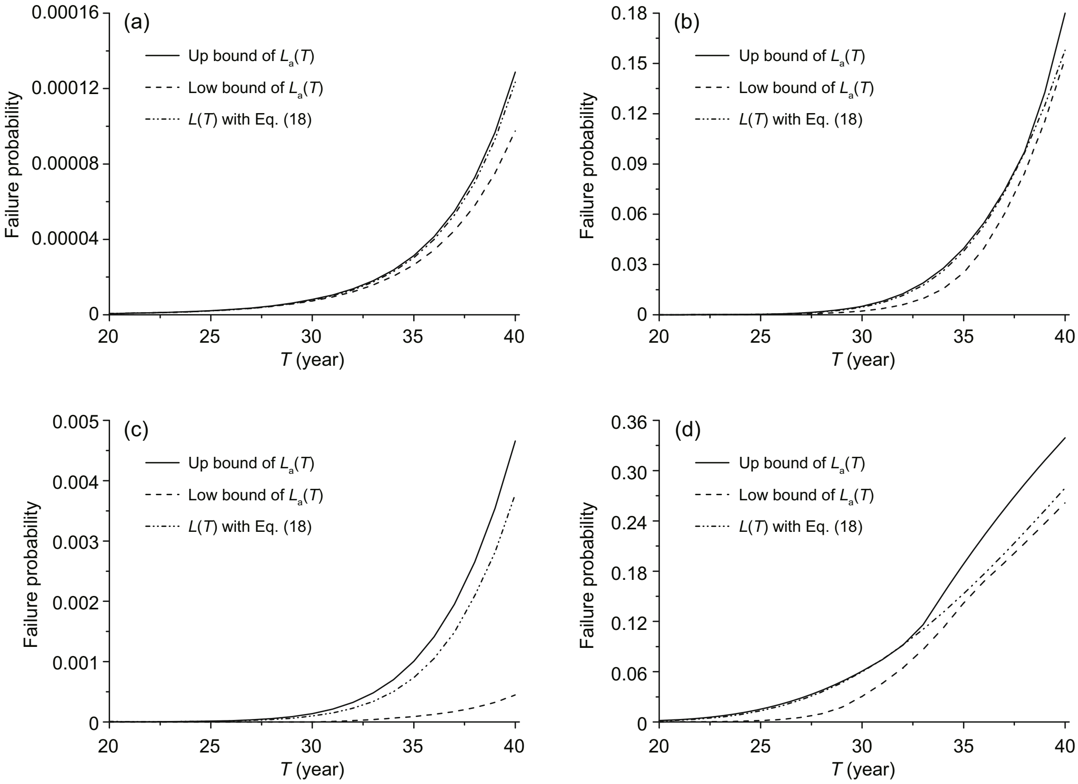

Fig. 3 Impact of resistance deterioration on structural averaged failure probability: (a) $\mu_{G(40)} = 0.2, c_{G(40)} = 0.2$; (b) $\mu_{G(40)} = 0.4, c_{G(40)} = 0.2$; (c) $\mu_{G(40)} = 0.2, c_{G(40)} = 0.4$; (d) $\mu_{G(40)} = 0.4, c_{G(40)} = 0.4$

6 Conclusions

In this paper, the amount of error in structural time-dependent reliability introduced by different probabilistic models of resistance deterioration is investigated, and a novel method is developed for the estimate of averaged reliability under incomplete deterioration information. An illustrative structure is studied by comparing the reliabilities associated with different deterioration models. Depending on the case investigated, the following conclusions can be drawn:

1. Employing the five common probabilistic models for resistance deterioration (namely uniform, normal, inverse-lognormal, Beta, and Gamma distributions), the reliability analysis method for aging structures is discussed. When the fully-correlated deterioration process is associated with small uncertainty, the choice of probabilistic models of resistance deterioration impacts the structural reliability slightly. However, with the increase of the variation associated with resistance deterioration, the reliabilities associated with different deterioration models differ significantly. Moreover, no deterioration model exists that results in the greatest or smallest failure probability for all deterioration conditions. As a result, one should compare all the failure probabilities associated with different candidate deterioration models to obtain the maximum probability of failure for the service periods, which indeed provides a relatively conservative estimate of structural safety.

2. A new method is developed to estimate the averaged time-dependent reliability of aging structures. It is independent of the probabilistic mass scenario of the candidate deterioration models, and thus reduces the epistemic (knowledge-based) uncertainties associated with the model weight, providing a practical tool for evaluating the reliability of structures under incomplete deterioration information.

Acknowledgements

The authors would like to thank Mr. Kai-rui FENG (Department of Civil Engineering, Tsinghua University) for his assistance in the programming.

References

AASHTO (American Association of State Highway and Transportation Officials), 2008. The Manual for Bridge Evaluation. AASHTO, Washington DC, USA.

ACI Committee, 2014. Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary. American Concrete Institute, Farmington Hills, MI, USA.

Cao, Z., Wang, Y., 2014. Bayesian model comparison and selection of spatial correlation functions for soil parameters. *Structural Safety*, **49**:10-17. http://dx.doi.org/10.1016/j.strusafe.2013.06.003

Clifton, J.R., Knab, L.I., 1989. Service Life of Concrete. Technical Report No. NUREG/CR-5466 and NISTIR-89-4086, Nuclear Regulatory Commission, Washington DC, USA. National Institute of Standards and Technology, Gaithersburg, USA.

Comenetz, M., 2002. Calculus: the Elements. World Scientific Publishing Press, Singapore.

Dieulle, L., Bérenguer, C., Grall, A., *et al.*, 2003. Sequential condition-based maintenance scheduling for a deteriorating system. *European Journal of Operational Research*, **150**(2):451-461. http://dx.doi.org/10.1016/S0377-2217(02)00593-3

Ellingwood, B.R., 2005. Risk-informed condition assessment of civil infrastructure: state of practice and research issues. *Structure and Infrastructure Engineering*, **1**(1):7-18. http://dx.doi.org/10.1080/15732470412331289341

Enright, M.P., Frangopol, D.M., 1998. Service-life prediction of deteriorating concrete bridges. *Journal of Structural Engineering*, **124**(3):309-317. http://dx.doi.org/10.1061/(ASCE)0733-9445(1998)124:3(309)

Gilbert, S., 1988. Linear Algebra and Its Applications. Harcourt Brace Jovanovich, USA.

Hong, H., 2000. Assessment of reliability of aging reinforced concrete structures. *Journal of Structure Engineering*, **126**(12):1458-1465. http://dx.doi.org/10.1061/(ASCE)0733-9445(2000)126:12(1458)

Li, D.Q., Zhang, L., Tang, X.S., *et al.*, 2015. Bivariate distribution of shear strength parameters using copulas and its impact on geotechnical system reliability. *Computers and Geotechnics*, **68**:184-195. http://dx.doi.org/10.1016/j.compgeo.2015.04.002

Li, Q.W., Wang, C., 2015. Updating the assessment of resistance and reliability of existing aging bridges with prior service loads. *Journal of Structural Engineering*, **141**(12):04015072. http://dx.doi.org/10.1061/(asce)st.1943-541X.0001331

Li, Q.W., Wang, C., Ellingwood, B.R., 2015. Time-dependent reliability of aging structures in the presence of non-stationary loads and degradation. *Structural Safety*, **52**:132-141. http://dx.doi.org/10.1016/j.strusafe.2014.10.003

Ma, Y.F., Zhang, J.R., Wang, L., *et al.*, 2013. Probabilistic prediction with Bayesian updating for strength degradation of RC bridge beams. *Structural Safety*, **44**:102-109. http://dx.doi.org/10.1016/j.strusafe.2013.07.006

Mckay, M.D., Beckman, R.J., Conover, W.J., 2000. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics*, **42**(1):55-61. http://dx.doi.org/10.1080/00401706.2000.10485979

Mohd, M.H., Paik, J.K., 2013. Investigation of the corrosion progress characteristics of offshore subsea oil well tubes. *Corrosion Science*, **67**:130-141. http://dx.doi.org/10.1016/j.corsci.2012.10.008

Mohd, M.H., Kim, D.K., Kim, D.W., *et al.*, 2014. A time-variant corrosion wastage model for subsea gas pipelines. *Ships & Offshore Structures*, **9**(2):161-176. http://dx.doi.org/10.1080/17445302.2013.770724

Mori, Y., Ellingwood, B.R., 1993. Reliability-based service-life assessment of aging concrete structures. *Journal of Structural Engineering*, **119**(5):1600-1621. http://dx.doi.org/10.1061/(ASCE)0733-9445(1993)119:5(1600)

Mori, Y., Ellingwood, B.R., 2006. Reliability assessment of reinforced concrete walls degraded by aggressive operating environments. *Computer Aided Civil Infrastructure Engineering*, **21**:157-170.

MOT (Ministry of Transport of the People's Republic of China), 2011. Specification for Inspection and Evaluation of Load-bearing Capacity of Highway Bridges, JTG/T J21-2011. China Communications Press, China (in Chinese).

Paik, J.K., Kim, D.K., 2012. Advanced method for the development of an empirical model to predict time-dependent corrosion wastage. *Corrosion Science*, **63**:51-58. http://dx.doi.org/10.1016/j.corsci.2012.05.015

Prince, P.J., Dormand, J.R., 1981. High order embedded Runge-Kutta formulae. *Journal of Computational & Applied Mathematics*, **7**(1):67-75. http://dx.doi.org/10.1016/0771-050X(81)90010-3

Saassouh, B., Dieulle, L., Grall, A., 2007. Online maintenance policy for a deteriorating system with random change of mode. *Reliability Engineering & System Safety*, **92**(12):1677-1685. http://dx.doi.org/10.1016/j.ress.2006.10.017

Stewart, M.G., Mullard, J.A., 2007. Spatial time-dependent reliability analysis of corrosion damage and the timing of first repair for RC structures. *Engineering Structures*, **29**(7):1457-1464. http://dx.doi.org/10.1016/j.engstruct.2006.09.004

Tang, X.S., Li, D.Q., Rong, G., *et al.*, 2013. Impact of copula selection on geotechnical reliability under incomplete probability information. *Computers and Geotechnics*, **49**:264-278. http://dx.doi.org/10.1016/j.compgeo.2012.12.002

Tang, X.S., Li, D.Q., Zhou, C.B., *et al.*, 2015. Copula-based approaches for evaluating slope reliability under incomplete probability information. *Structural Safety*, **52**:90-99. http://dx.doi.org/10.1016/j.strusafe.2014.09.007

Wang, C., Li, Q.W., Zou, A.M., *et al.*, 2015. A realistic resistance deterioration model for time-dependent reliability analysis of aging bridges. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, **16**(7):513-524. http://dx.doi.org/10.1631/jzus.A1500018

Wang, C., Li, Q.W., Ellingwood, B.R., 2016. Time-dependent reliability of aging structures: an approximate approach. *Structure and Infrastructure Engineering*, Online First. http://dx.doi.org/10.1080/15732479.2016.1151447

Zhang, H., Mullen, R.L., Muhanna, R.L., 2010. Interval Monte Carlo methods for structural reliability. *Structural Safety*, **32**(3):183-190. http://dx.doi.org/10.1016/j.strusafe.2010.01.001

Appendix A: Properties of series $E[G(T)^j]$

First, we briefly introduce the Cauchy-Schwarz inequality taking the form of integral (Gilbert, 1988).

For two functions $s(g)$ and $t(g)$ defined in domain D_0, we have

$$\left[\int_{D_0} s(g)t(g)\mathrm{d}g\right]^2 \le \int_{D_0} s^2(g)\mathrm{d}g\cdot\int_{D_0} t^2(g)\mathrm{d}g, \quad \text{(A1)}$$

where the equality holds if and only if $s(g)$ and $t(g)$ are linearly dependent. With this, letting $D_0=[0,1]$, $s(g)=\sqrt{g^{j+1}f_G(g)}$, and $t(g)=\sqrt{g^{j-1}f_G(g)}$ for $j\ge 2$,

$$\left[\int_0^1 g^j f_G(g)\mathrm{d}g\right]^2 \le \int_0^1 g^{j+1} f_G(g)\mathrm{d}g\cdot\int_0^1 g^{j-1} f_G(g)\mathrm{d}g. \quad \text{(A2)}$$

That is,

$$\frac{E[G(T)^j]}{E[G(T)^{j-1}]} \le \frac{E[G(T)^{j+1}]}{E[G(T)^j]}, \quad j\ge 2. \quad \text{(A3)}$$

Eq. (A3) indicates that $E[G(T)^{j+1}]/E[G(T)^j]$ increases with j. Moreover,

$$\lim_{j\to\infty}\left|\frac{E[G(T)^{j+1}]}{E[G(T)^j]}-1\right| = \lim_{j\to\infty}\left|\frac{\int_0^1 g^j(1-g)f_G(g)\mathrm{d}g}{\int_0^1 g^j f_G(g)\mathrm{d}g}\right|. \quad \text{(A4)}$$

As j is large enough, there exist two positive constants $\theta_1\to 1^-$ and $\theta_2\to 1^-$ such that

$$\lim_{j\to\infty}\int_0^1 g^j(1-g)f_G(g)\mathrm{d}g = \lim_{j\to\infty}\int_{\theta_1}^1 g^j(1-g)f_G(g)\mathrm{d}g, \quad \text{(A5a)}$$

$$\lim_{j\to\infty}\int_0^1 g^j f_G(g)\mathrm{d}g = \lim_{j\to\infty}\int_{\theta_2}^1 g^j f_G(g)\mathrm{d}g. \quad \text{(A5b)}$$

With the 'mean value theorem for definite integrals' (Comenetz, 2002), there exists a constant $\theta_3\in(\theta_1,1)$ which satisfies

$$\lim_{j\to\infty}\int_{\theta_1}^1 g^j(1-g)f_G(g)\mathrm{d}g = (1-\theta_3)\lim_{j\to\infty}\int_{\theta_1}^1 g^j f_G(g)\mathrm{d}g. \quad \text{(A6)}$$

Thus,

$$\lim_{j\to\infty}\left|\frac{E[G(T)^{j+1}]}{E[G(T)^j]}-1\right| = \lim_{j\to\infty}|1-\theta_3| = 0. \quad \text{(A7)}$$

Now, by noting that $E[G(T)^{j+1}]/E[G(T)^j]<1$, we have

$$\sup\left\{\frac{E[G(T)^{j+1}]}{E[G(T)^j]}\right\} = 1. \quad \text{(A8)}$$

Appendix B: Solving $L(T)$ with Eq. (18)

First, Eq. (18) is rewritten as

$$L(T) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m f(m\pi), \quad \text{(B1)}$$

where

$$f(x) = \sum_{j=0}^{\infty} c_{2j}x^{2j}(-1)^j = \sum_{j=0}^{\infty}\frac{\phi^{(2j)}(0)}{(2j)!}x^{2j}(-1)^j. \quad \text{(B2)}$$

Obviously, $f(x)$ is an even function and $f(0)=1$. To obtain the values of $f(x)$ for $x=\pi, 2\pi, ...$, one may add the terms $c_{2j}\cdot(m\pi)^{2j}(-1)^j$ directly according to Eq. (B2), which, unfortunately, may result in significant error in calculation; alternatively, one may employ the after-mentioned method.

The second derivative of $f(x)$ is

$$f''(x) = \sum_{j=2}^{\infty}\frac{\phi^{(2j)}(0)}{(2j-2)!}x^{2j-2}(-1)^j - \phi^{(2)}(0). \quad \text{(B3)}$$

According to Eq. (19),

$$\phi^{(2j)} = \phi^{(2j-2)}\cdot\left(1-\gamma_3\gamma_4^{2j-1}-\gamma_3\gamma_4^{2j-2}+\gamma_3^2\gamma_4^{4j-3}\right), \quad \text{(B4)}$$

where $\phi^{(\cdot)}=\phi^{(\cdot)}(0)$, $\gamma_3=\exp(\gamma_2)$, and $\gamma_4=\exp(\gamma_1)$. With this, Eq. (B3) becomes

$$\begin{aligned} f''(x) = &-(\mu^2+\sigma^2) + \sum_{j=2}^{\infty}\frac{\phi^{(2j-2)}}{(2j-2)!}x^{2j-2}(-1)^j \\ &- \sum_{j=2}^{\infty}\frac{\phi^{(2j-2)}}{(2j-2)!}\gamma_3\gamma_4^{2j-1}x^{2j-2}(-1)^j \\ &- \sum_{j=2}^{\infty}\frac{\phi^{(2j-2)}}{(2j-2)!}\gamma_3\gamma_4^{2j-2}x^{2j-2}(-1)^j \\ &+ \sum_{j=2}^{\infty}\frac{\phi^{(2j-2)}}{(2j-2)!}\gamma_3^2\gamma_4^{4j-3}x^{2j-2}(-1)^j. \end{aligned} \quad \text{(B5)}$$

By noting the fact that

$$\sum_{j=2}^{\infty}\frac{\phi^{(2j-2)}}{(2j-2)!}x^{2j-2}(-1)^{j}=1-f(x), \tag{B6}$$

we have

$$f''(x)=1-(\mu^2+\sigma^2)-f(x)-\gamma_3\gamma_4[1-f(\gamma_4 x)] - \gamma_3[1-f(\gamma_4 x)]+\gamma_3^2\gamma_4[1-f(\gamma_4^2 x)]. \tag{B7}$$

For simplicity, let $A=\gamma_3\gamma_4+\gamma_3$ and $B=-\gamma_3^2\gamma_4$, then we have

$$f''(x)=-f(x)+Af(\gamma_4 x)+Bf(\gamma_4^2 x). \tag{B8}$$

With Eq. (B8), $f(x)$ can be solved employing some numerical techniques such as the Runge-Kutta method (Prince and Dormand, 1981).

中文概要

题　目：非完整衰减信息下的劣化结构时变可靠度评估

目　的：研究衰减函数的概率分布对结构可靠度的影响；在非完整衰减信息下，对结构的时变可靠度进行评估。

创新点：1. 假定衰减函数服从5种常见的概率分布，讨论可靠度分析结果的差异；2. 当衰减函数的概率分布（或各概率分布的权重）未知时，提出平均可靠度的分析方法。

方　法：假定承载力是完全相关的随机过程，在其概率分布未知的情况下:（1）假定5种常见的概率分布，对结构可靠度进行分析和比较；（2）将时变可靠度的计算公式Taylor展开，转化为关于衰减函数一阶矩和二阶矩的函数，推导平均可靠度的计算方法（公式18）。

结　论：1. 不同的衰减函数概率分布对结构可靠度的影响显著；2. 利用本文提出的方法，在衰减函数概率的分布（或各概率分布的权重）未知时，可以得出结构可靠度的合理评估。

关键词：时变可靠度；劣化模型；平均可靠度；误差分析；结构安全