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Nonparametric modeling on random uncertainty and reliability analysis of a dual-span rotor*

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Abstract: A general procedure is proposed to estimate the reliability of a dual-span rotor based on nonparametric modeling on random uncertainty. First, the vibration equation of the rotor with random uncertainty is constructed based on random matrices through the nonparametric modeling approach. Second, the reliability estimation is then performed by response spectral analysis and the moment method. By making full use of the advantages of nonparametric method and response spectral analysis, not only is the requirement on probability density function (PDF) avoided, but also the first and second moments are no longer needed to be estimated or assumed for calculating the reliability. Finally, the statistical index Z^*-value based on short-term predictability is introduced to investigate the influence of random uncertainties on the reliability of the dual-span rotor. Illustrating examples show that the results obtained from the proposed procedure are consistent with those from short-term predictability, such that dangerous ranges can be well identified during the start-up process of the rotor.

Key words: Random uncertainty; Nonparametric model; Reliability; Response spectral analysis
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1 Introduction

Recently, more and more research has focused on the dynamical characteristics of the multi-span rotor with strong nonlinearity. This usually requires substantial modifications of the analysis of the rotor with single span, and a low computational efficiency is often produced due to the large dimension of the system (Shiau et al., 2009). Most of the contributions on the multi-span rotor are based on a deterministic model, which means the dynamical equation can fully reflect the structure of system and the parameters are all accurately measured.

In practical application, there are inevitably many uncertain factors, and this means that an uncertain model is more reasonable. Narendra and Parthasarathy (1990) established the dynamical model of a 500 MW supercritical steam turbine and pointed out that there was a significant gap between the theoretical predictions and the actual measured unit data. The Siemens Corporation also found that the key problem in the calculation of complex valve flow is that the pressure loss cannot be calculated accurately (Deckers and Doerwald, 1997). Murugan et al. (2008a, 2008b) used the test data to determine the strength of uncertainty in studying the effect of uncertainty of composite material properties on the rotor blade of

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helicopters. Rinehart and Simon (2014) collected aircraft engine data and verified that the uncertainties of actuators, sensors, and other factors caused inaccurate monitoring of the operating status. Beck (2015) proposed a new 3D coupled model to analyze the dynamical response characteristics of a hydraulic turbine to investigate the increase of the hydraulic transmission coefficient during mining or liquid transportation. Pichler et al. (2015) proposed a novel detection method to analyze the vibration phenomenon caused by the changing environment and dynamic load in the study of an air compressor.

Most previous work is based on the assumption that the probability density function (PDF) of the data may be precisely obtained and different data components are independent, which is almost impossible in practice (Song et al., 2006). Spinato et al. (2008) investigated more than 6000 modern rotating machines and used industry data to calculate their reliability, and suggested that the existing methods cannot make accurate assessments and more rigorous reliability measures should be developed. Au et al. (1999) proposed an asymptotic expansion technique in reliability analysis for uncertain dynamical systems to achieve any desired level of accuracy provided by a sufficiently large number of samples. Some alternative techniques have also been proposed to solve the reliability problem of structure systems with random uncertainties, such as the ensemble crossing (Beck and Melchers, 2004), the probability density evolution (Li and Chen, 2005), and the statistical fourth moment (Zhang et al., 2003). Nevertheless, these methods generally have two disadvantages in analyzing uncertainties. One is that the PDF or lots of statistical data should be provided, and the other is that the probabilistic reliability assessment or the Monte Carlo simulation usually encounters the problem of a large amount of calculation.

In general, there are two typical situations in dynamical modeling on uncertain systems, i.e. data uncertainty and model uncertainty (Soize, 2000), and most previous studies were concerned with the former. The nonparametric approach proposed by Soize (2000) can carry out the calculations and simulations without identifying the type and number of uncertainties and what is needed is only the basic mean model which can be established by some customary methods. Recently, this method has been applied to analyze the vibration characteristics of a Jeffcott rotor with single span affected by disc offset, unbalanced force, and bounded noise excitation (Gan et al., 2014). To the best of our knowledge, the nonparametric approach has not been used in the field of reliability.

Here, a general procedure is proposed for reliability estimation on a dual-span Jeffcott rotor with random uncertainties. The method is based on the results of a nonparametric approach, and the variance value of system response is deduced by solving the response spectral density. Here, an important goal is to carry out the reliability analysis of a multiple span rotor system containing both parameter and model uncertainties without knowing the first several moments. Another aim is to avoid solving complicated equations, especially several sets of multivariate equations. The proposed method makes full use of the advantages of the nonparametric method and the response spectral analysis, to ensure that the calculation of reliability can be performed without enough information of the uncertain system, quickly and efficiently determining the probability of failure.

2 Nonparametric modeling

The dual-span rotor system is shown in Fig. 1. It consists of two identical shafts rigidly coupled together and connected by a flexible coupling in the middle. Each individual shaft has a disc, midway between two hydro-dynamic bearing supports where balance masses can be applied (Fig. 1). The parameters involved are explained in Table 1. There will be a slight angle displacement which varies between the two shafts formed at the beginning and determined by the torsion stiffness of the coupling. This angle does not change during the operation, so the damping of the coupling is generally ignored and only the stiffness is taken into consideration.

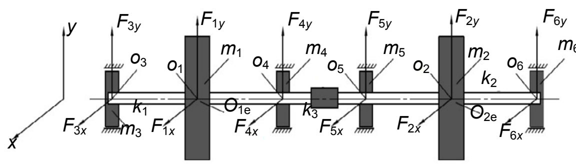

Fig. 1 A schematic of the dual-span rotor

Table 1 Parameters involved in the dual-span rotor model

Parameter	Description
o_1, o_2	Geometric centers of disc 1 and disc 2, respectively
o_3, o_4, o_5, o_6	Geometric centers of bearings
m_3, m_4, m_5, m_6	Equivalent lump masses at bearings
k_1, k_2	Stiffnesses of shafts
c_1, c_2	Damping coefficients of discs
L	Length of bearing
C	Bearing clearance
F_{ix}, F_{iy} (i=1, 2, …, 6)	Nonlinear oil film forces on shaft from the bearing
(x_1, y_1), (x_2, y_2)	Disc positions in the coordinate system
O_{1e}, O_{2e}	Centroids of disc 1 and disc 2, respectively
m_1, m_2	Masses of disc 1 and disc 2, respectively
e_1, e_2	Eccentricities of disc 1 and disc 2, respectively
k_3	Stiffness of the coupling
c_3, c_4, c_5, c_6	Damping coefficients of bearings
R	Radius of bearing
(x_3, y_3), (x_4, y_4), (x_5, y_5), (x_6, y_6)	Positions of bearings in the coordinate system

The hydrodynamic bearings forces can be evaluated from the pressure distribution applying the finite difference solution of Reynolds equation and can be linearized by the finite difference method (Liu and Noyak, 1995), i.e.

$$F_{ix}=-k_{xx}x_i-k_{xy}y_i-c_{xx}\dot{x}_i-c_{xy}\dot{y}_i, \tag{1a}$$

$$F_{iy}=-k_{yx}x_i-k_{yy}y_i-c_{yx}\dot{x}_i-c_{yy}\dot{y}_i, \tag{1b}$$

where x_i and y_i denote the corresponding displacement components of the shaft at the ith bearing, k_{xx}, k_{xy}, k_{yx}, k_{yy}, c_{xx}, c_{xy}, c_{yx}, and c_{yy} are the eight stiffness and damping coefficients, and F_i is the hydrodynamic bearing force.

As for the Jeffcott rotor, it can be simply seen as composed of three parts: the rotating discs, the elastic but massless shaft, and the supporting bearings. Since only the most important and fundamental movement forms are the objective of this study, the torsion and longitudinal vibrations (Lal and Tiwari, 2012) are not considered here. Thus, by introducing a generalized coordinate vector $\boldsymbol{U}=\begin{bmatrix}u_1 & u_2 & \cdots & u_6\end{bmatrix}^{\mathrm{T}}$ (u_l=x_l+jy_l, l=1, 2, …, 6) in the complex domain, where "j" is used to indicate an imaginary term, the finite element model of the dual-span rotor can be expressed as

$$\underline{\boldsymbol{M}}\ddot{\boldsymbol{U}}+\underline{\boldsymbol{C}}\dot{\boldsymbol{U}}+\underline{\boldsymbol{K}}\boldsymbol{U}=\underline{\boldsymbol{F}}, \tag{2}$$

where

$\underline{\boldsymbol{M}}=\mathrm{diag}\begin{bmatrix}m_1 & m_2 & \cdots & m_6\end{bmatrix}$,

$\underline{\boldsymbol{C}}=\mathrm{diag}\left[c_1, c_2, c_3-c_{33}, c_4-c_{44}, c_5-c_{55}, c_6-c_{66}\right]$,

$$\underline{\boldsymbol{K}}=\begin{bmatrix}2k_1 & 0 & -k_1 & -k_1 & 0 & 0\\ 0 & 2k_2 & 0 & 0 & -k_2 & -k_2\\ -k_1 & 0 & k_1+k_{33} & 0 & 0 & 0\\ -k_1 & 0 & 0 & k_1+k_{44}+k_3 & -k_3 & 0\\ 0 & -k_2 & 0 & -k_3 & k_2+k_{55}+k_3 & 0\\ 0 & -k_2 & 0 & 0 & 0 & k_2+k_{66}\end{bmatrix}.$$

$\underline{\boldsymbol{K}}$ is the stiffness matrix, where k_l (l=1, 2, …, 6) is the equivalent stiffness of the rotor and k_{ii} (i=1, 2, …, 6) is the stiffness coefficient of the bearings. Moreover, $\underline{\boldsymbol{F}}=\underline{\boldsymbol{Q}}+\underline{\boldsymbol{G}}+\boldsymbol{F}_i$, $\underline{\boldsymbol{G}}=-\mathrm{j}g\begin{bmatrix}m_1 & m_2 & \cdots & m_6\end{bmatrix}^{\mathrm{T}}$, $\underline{\boldsymbol{Q}}=$ $[m_1e_1\omega^2(\cos(\omega t)+\mathrm{j}\sin(\omega t))\quad m_2e_2\omega^2(\cos(\omega t)+\mathrm{j}\sin(\omega t))\quad 0\quad 0\quad 0\quad 0]^{\mathrm{T}}$, g is the acceleration of gravity, t is the time, and ω is the rotating speed of shaft.

$\underline{\boldsymbol{M}}$, $\underline{\boldsymbol{F}}$, and $\underline{\boldsymbol{C}}$ are all symmetric matrices. The dynamical model given by Eq. (2) is just a deterministic one in which the mass, stiffness, and damping matrices are all assumed to be accurately measured, i.e. the system is assumed to have no approximation nor simplification, and the material properties are also assumed to have no change during the running of the system. Therefore, this can be seen as the mean model of the rotor.

As $\boldsymbol{C}$ and $\boldsymbol{K}$ are positive-definite matrices, we use ψ_α and ψ_β (α, β=1, 2, …, 6) to express the eigenvectors of the system. Then we can obtain $\underline{\boldsymbol{M}}_{\alpha\beta}=\delta_{\alpha\beta}$, $\underline{\boldsymbol{C}}_{\alpha\beta}=\langle\underline{\boldsymbol{C}}\psi_\alpha, \psi_\beta\rangle$, $\underline{\boldsymbol{K}}_{\alpha\beta}=\omega_\alpha^2\delta_{\alpha\beta}$, and $\underline{\boldsymbol{F}}_{\alpha\beta}=\langle\underline{\boldsymbol{F}}\psi_\alpha, \psi_\beta\rangle$, where δ is the Dirac function, $\langle y, x\rangle=y_1x_1+y_2x_2+\ldots+y_mx_m$, and ω_α is the eigenfrequency of structural mode ψ_α (Ohayon and Soize, 1998). Thus, the reduced matrix model of this system can be expressed as

$$\underline{\boldsymbol{M}}\ddot{\boldsymbol{Z}}+\underline{\boldsymbol{C}}\dot{\boldsymbol{Z}}+\underline{\boldsymbol{K}}\boldsymbol{Z}=\underline{\boldsymbol{F}}, \tag{3}$$

where $\boldsymbol{Z}=\boldsymbol{\psi}_{\alpha}^{-1}\boldsymbol{U}$.

In reality, errors always exist between the design parameters and real working conditions. For example, some parameters will alter in some operational processes, such as creep and differential expansion, and they cannot be obtained with an accurate value except for the empirical data, such as Young's modulus. All these imply that the mean model is not an accurate reflection of the actual situation. To represent the real operational state, uncertain factors must be taken into account, and consequently, the random matrix model is of more practical significance, which can be expressed as

$$\boldsymbol{M}_{\mathrm{r}}\ddot{\boldsymbol{Z}}+\boldsymbol{C}_{\mathrm{r}}\dot{\boldsymbol{Z}}+\boldsymbol{K}_{\mathrm{r}}\boldsymbol{Z}=\boldsymbol{F}_{\mathrm{r}}, \tag{4}$$

where $\boldsymbol{M}_{\mathrm{r}}$, $\boldsymbol{C}_{\mathrm{r}}$, and $\boldsymbol{K}_{\mathrm{r}}$ are the random matrices of mass, damping, and stiffness in the reduced model, respectively. $\boldsymbol{F}_{\mathrm{r}}=\boldsymbol{Q}_{\mathrm{r}}+\boldsymbol{G}_{\mathrm{r}}$ has also been randomized due to the relationship of the mass matrix.

In the following, several key equations are briefly introduced from Soize (2000)'s work to derive the dispersion control parameter and explain the simulation process for a random matrix. Here, they will be applied to investigate the reliability of a dual-span rotor in Section 3. Assuming $\boldsymbol{A}$ is a $n\times n$ random matrix of mass, damping, or stiffness, it can be easily deduced that the PDF $p_{A}(\boldsymbol{A})$ of $\boldsymbol{A}$ must fulfill the following three basic constraints according to the statistical properties of a random matrix:

$$\int_{M_n^+(R)} p_A(\boldsymbol{A})\,\tilde{\mathrm{d}}\boldsymbol{A}=1, \tag{5a}$$

$$\int_{M_n^+(R)} \boldsymbol{A}p_A(\boldsymbol{A})\,\tilde{\mathrm{d}}\boldsymbol{A}=\underline{\boldsymbol{A}}\in M_n^+(R), \tag{5b}$$

$$\int_{M_n^+(R)} \ln(\det\boldsymbol{A})p_A(\boldsymbol{A})\,\tilde{\mathrm{d}}\boldsymbol{A}=\nu,\quad |\nu|<+\infty, \tag{5c}$$

where $M_n^{+}(R)$ is constituted by positive definite symmetric matrices.

There are many methods for obtaining the PDF provided that there is enough information, but the information one can obtain here is just the constraints. In the circumstances, the entropy is usually used to represent the degree of uncertainty of the system, while the maximum entropy principle can permit the function to be constructed with the available information simultaneously ensuring physical significance, and the PDF can be deduced by constructing the Lagrangian and expressed as

$$p_A(\boldsymbol{A})=c_{\mathrm{A}}\times(\det\boldsymbol{A})^{\lambda-1}\times\exp\left(-\frac{n-1+2\lambda}{2}\mathrm{tr}\{\underline{\boldsymbol{A}}^{-1}\boldsymbol{A}^{\mathrm{T}}\}\right), \tag{6}$$

where $\underline{\boldsymbol{A}}$ is the mathematical expectation of the random matrix $\boldsymbol{A}$, $\mathrm{tr}\,\boldsymbol{A}=\sum_{j=1}^{n}\boldsymbol{A}_{jj}$, and c_{A} is a positive constant calculated by

$$c_{\mathrm{A}}=\frac{(2\pi)^{-n(n-1)/4}\left(\dfrac{n-1+2\lambda}{2}\right)^{n(n-1+2\lambda)/2}}{\left\{\prod_{l=1}^{n}\Gamma\left(\dfrac{n-l+2\lambda}{2}\right)\right\}(\det\underline{\boldsymbol{A}})^{(n-1+2\lambda)/2}}, \tag{7}$$

where $\Gamma(x)$ is the gamma function defined by $\Gamma(x)=\int_0^{+\infty}t^{x-1}\mathrm{e}^{-x}\mathrm{d}t\ \ (x>0)$. The variance of $\boldsymbol{A}$ can be expressed as

$$\sigma_{jk}=\frac{1}{n-1+2\lambda}\{\underline{\boldsymbol{A}}_{jk}^2+\underline{\boldsymbol{A}}_{jj}\underline{\boldsymbol{A}}_{kk}\},\quad 0<j\le k<n. \tag{8}$$

The Frobenius norm of matrix $\boldsymbol{A}$ is defined by $\|\boldsymbol{A}\|_{\mathrm{F}}=\{\mathrm{tr}(\boldsymbol{A}\cdot\boldsymbol{A}^{*})\}^{1/2}$. Since $E\{\|\boldsymbol{A}-\underline{\boldsymbol{A}}\|_{\mathrm{F}}^2\}=\sum_{j=1}^{n}\sum_{k=j}^{n}\sigma_{jk}^2$, if we define a dispersion control parameter δ_{A} as

$$\delta_{\mathrm{A}}=\left\{\frac{E\{\|\boldsymbol{A}-\underline{\boldsymbol{A}}\|_{\mathrm{F}}^2\}}{\|\underline{\boldsymbol{A}}\|_{\mathrm{F}}^2}\right\}^{1/2}=\left\{\frac{1}{n-1+2\lambda}\left(1+\frac{(\mathrm{tr}\,\underline{\boldsymbol{A}})^2}{\mathrm{tr}(\underline{\boldsymbol{A}})^2}\right)\right\}^{1/2}, \tag{9}$$

then the parameter λ in Eqs. (6)–(9) can be deduced as

$$\lambda=\frac{1}{2\delta_{\mathrm{A}}^2}\left(1-\delta_{\mathrm{A}}^2(n-1)+\frac{(\mathrm{tr}\,\underline{\boldsymbol{A}})^2}{\mathrm{tr}(\underline{\boldsymbol{A}})^2}\right). \tag{10}$$

It should be noted that for n fixed, λ increases as δ_A decreases. If $\lambda\to+\infty$, then $\delta_A\to 0$ and therefore $\boldsymbol{A}\to\underline{\boldsymbol{A}}$ in probability. For each δ_A, the random matrix $\boldsymbol{A}$ can be obtained by the Monte Carlo simulation of $\boldsymbol{A}$.

For any positive symmetric matrix $\boldsymbol{A}$, applying the Cholesky factorization yields

$$\underline{\boldsymbol{A}} = \boldsymbol{L}_A^T \boldsymbol{L}_A, \tag{11}$$

where $\boldsymbol{L}_A$ is an upper triangular real matrix. Assuming λ is a positive integer, let $m_A=n-1+2\lambda$, and the random matrix $\boldsymbol{A}$ can be written by

$$\boldsymbol{A} = \frac{1}{m_A}\sum_{j=1}^{m_A}(\boldsymbol{L}_A^T\boldsymbol{U}_j)(\boldsymbol{L}_A^T\boldsymbol{U}_j)^T, \tag{12}$$

where $\boldsymbol{U}_j$ (j=1, 2, …., m_A) with the size of $1\times n$ is the vector consisting of the Gaussian random variables with zero mean and unit variance. Thus, the samples of the random matrix $\boldsymbol{A}$ are yielded by Eq. (12) and the random matrix model Eq. (4) can then be constructed (Soize, 2000). The PDF of random matrix $\boldsymbol{A}$ is consistent with Eq. (6). Eq. (12) is deduced from Eq. (6) in the nonparametric method.

3 Reliability analysis

Reliability is defined as the ability of completing the predetermined function under the given time and conditions, and there are many numerical quantities, usually called reliability indices, such as the Cornell index and the H-L index (Mbarka et al., 2010). The statistical index based on short-term prediction is usually used to monitor the performance of machines. Both the reliability and statistical indices are employed to assess the operational state, the former mainly from the macro point of view while the latter is from the micro point of view. Therefore, they not only can verify the effectiveness but also, in some ways, complement each other.

3.1 Reliability based on the response spectral analysis

The first- and second-order reliability methods have been the most popular for several decades since they can be used when just the former two moments are given. However, because of the disturbance caused by uncertainties, the reliability calculation is difficult unless there is a comprehensive understanding of random parameters. Fortunately, the response spectral analysis can effectively deduce the former two moments without solving the equations, and the calculation speed is higher than that of solving the equations. Therefore, the proposed method in this study makes full use of the advantages of the nonparametric method and the response spectral analysis.

In practice, usually only a prior few order modes play a leading role. Therefore, only these modes are considered in the calculation of responses. The complex frequency response function matrix is given by

$$\boldsymbol{H}_r(\omega) = (\boldsymbol{K}_r - \boldsymbol{M}_r\omega^2 + \mathrm{j}\omega\boldsymbol{C}_r)^{-1}, \tag{13}$$

and the mean response of Eq. (4) can be given by

$$\boldsymbol{U}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\boldsymbol{\psi}\boldsymbol{H}_r(\omega)\boldsymbol{F}_r(\omega)\mathrm{e}^{\mathrm{j}\omega t}\mathrm{d}\,\omega. \tag{14}$$

The correlation matrix is usually used to indicate the relationship between the variables. For the mean response $\boldsymbol{U}(t)$, its correlation matrix is defined as

$$\begin{aligned}\boldsymbol{R}_u(\tau) &= \lim_{T^*\to\infty}\frac{1}{T}\int_{-\frac{T^*}{2}}^{+\frac{T^*}{2}}\boldsymbol{U}(t)\boldsymbol{U}^T(t+\tau)\mathrm{d}\,t \\ &= \boldsymbol{\psi}\lim_{T^*\to\infty}\frac{1}{T}\int_{-\frac{T^*}{2}}^{+\frac{T^*}{2}}Z(t)Z^T(t+\tau)\mathrm{d}\,t\,\boldsymbol{\psi}^T,\end{aligned} \tag{15}$$

where τ is the time interval, and T^* is the observation time. While in the complex frequency domain, from previous study (Wan et al., 2008), we have

$$\boldsymbol{R}_z(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\boldsymbol{H}_r^*(\omega)\boldsymbol{S}_r(\omega)\boldsymbol{H}_r(\omega)\mathrm{e}^{\mathrm{j}\omega\tau}\mathrm{d}\,\omega, \tag{16}$$

where $\boldsymbol{H}_r^*(\omega)$ is the complex conjugate matrix of $\boldsymbol{H}_r(\omega)$, $\boldsymbol{S}_r(\omega)$ is the Fourier transform of the correlation matrix of the excitation $\boldsymbol{R}_r(\tau)$ and

$$\begin{aligned}\boldsymbol{S}_r(\omega) = \int_{-\infty}^{+\infty}\lim_{T^*\to\infty}\frac{1}{T}\int_{-\frac{T^*}{2}}^{+\frac{T^*}{2}}&\{\boldsymbol{F}_r(t)\} \\ &\cdot\{\boldsymbol{F}_r(t+\tau)\}^T\mathrm{e}^{-\mathrm{j}\omega\tau}\mathrm{d}\,t\,\mathrm{d}\,\tau.\end{aligned} \tag{17}$$

Consequently, the correlation matrix of the response of the original system is expressed as

$$\boldsymbol{R}_u(\tau)=\frac{1}{2\pi}\boldsymbol{\psi}\int_{-\infty}^{+\infty}\boldsymbol{H}_{\mathrm{r}}^{*}(\omega)\boldsymbol{S}_{\mathrm{r}}(\omega)\boldsymbol{H}_{\mathrm{r}}(\omega)\mathrm{e}^{\mathrm{j}\omega\tau}\mathrm{d}\omega\,\boldsymbol{\psi}^{\mathrm{T}}. \quad (18)$$

For the reliability analysis in this study, we focus on the deviation distance of the shaft and not the coordinate values of vibration, so the object variable $\boldsymbol{U}$ changes with time but is not a periodic variation. Therefore, it can be deduced that when the time T tends to infinity, as τ=0, $\boldsymbol{R}_u(0)$ tends to the square value of the object variable. Thus,

$$\boldsymbol{U}^2=\frac{1}{2\pi}\boldsymbol{\psi}\int_{-\infty}^{+\infty}\boldsymbol{H}_{\mathrm{r}}^{*}(\omega)\boldsymbol{S}_{\mathrm{r}}(\omega)\boldsymbol{H}_{\mathrm{r}}(\omega)\mathrm{d}\omega\,\boldsymbol{\psi}^{\mathrm{T}}. \quad (19)$$

Hence, the variance of the response of the system is deduced as

$$\sigma^2=E[\boldsymbol{U}^2]-E^2[\boldsymbol{U}]. \quad (20)$$

Here, the uncertain factors involved have no relationship with time, and will not change once produced in the subsequent numerical simulation. Thus, it will belong to the stationary random process. The intermediate variables in the process of calculating response make computing variance easier and faster with the above equations.

Statistical characteristics are usually not enough for determining the operating status and trend in many engineering structural problems. In a normal state, only the first two moments can be roughly estimated, and accordingly, the first-order second-moment method is generally adopted to calculate the probability of failure in reliability analysis (Liu and Peng, 2012). We define γ and Θ as two variables of the randomly uncertain systems, and $g(\Theta, \gamma)$ as the working state structure function or structure performance function, which can express the three statuses of the system as

$$g(\Theta,\gamma)\begin{cases}<0, & \text{failure},\\ =0, & \text{critical},\\ >0, & \text{safe}.\end{cases} \quad (21)$$

Here, we define the first-time destruction function $g(\Theta, \gamma)$ as

$$g(\Theta,\ \gamma)=\Theta-\gamma \quad (22)$$

for the dual-span rotor system with random uncertainties, where γ is defined as the examining objective and Θ the threshold, and usually the target and threshold variables are assumed to be mutually independent.

According to probability theory, the first two moments of the state function $g(\Theta, \gamma)$ can be deduced as

$$\begin{cases}\mu=E[g(\Theta,\gamma)]=E(\Theta)-E(\gamma)=\mu_{\Theta}-\mu_{\gamma},\\ \sigma^2=\mathrm{Var}[g(\Theta,\gamma)]=\sigma_{\Theta}^2+\sigma_{\gamma}^2.\end{cases} \quad (23)$$

Then, the Cornell index β is calculated by (Liu and Peng, 2012)

$$\beta=\frac{\mu}{\sigma}, \quad (24)$$

which represents the minimum distance between the surface of the critical state performance function and the origin in the state space. The probability of failure is generally an inverse function of the minimum distance, and a large distance indicates that the system has a small failure probability or a high reliability. As the arbitrary distribution function of the standard random variable can be approximately expressed by the standard normal distribution function using the Edgeworth series, the reliability degree R_{d} can be expressed as (Zhang et al., 1998)

$$R_{\mathrm{d}}=\rho(\beta)-\frac{\varsigma}{3!}\rho^{(3)}(\beta)-\frac{\zeta-3}{4!}\rho^{(4)}(\beta)-\frac{10\varsigma^2}{6!}\rho^{(6)}(\beta) \quad (25)$$

by a normalization processing for easier analysis, where $\rho(\beta)$ is the standard normal distribution function, ς is the slanting peak coefficient to describe the distribution of the deviation degree from the symmetry, ζ is the kurtosis coefficient reflecting the relative degree of the variables distributed around the mean values, and $\rho^{(i)}(\beta)$ is the ith differentiation of

$\rho(\beta)$. The Edgeworth series method can approximate the real distribution of the random variable with any level of precision, and typically the first fourth-order of results can meet the accuracy requirement to some extent. The reliability degree represents the probability of operating safely and Eq. (32) can help us get the probability.

As mentioned in the previous section, some researchers began their calculation with a given approximation of the PDF for each uncertain parameter, while others simulated the PDF with measured or empirical data. However, the approximation cannot meet the demands of engineering, and is tedious to calculate. Reliability analysis based on response spectrum analysis not only avoids the demand for much information but also simplifies the tedious calculations. Only the calculation of the eigenvectors and correlation matrix is involved in the process of deducing the variance and mean value. This greatly reduces the time on calculations and releases computer resources, bringing considerable convenience to solving engineering problems.

We then conclude the process by carrying out the reliability analysis as follows:

1. Analyze the uncertainties and construct the reduced finite mean model from the structure model expressed in matrix form.

2. Define a dispersion control parameter according to the actual situation, and then produce a sufficient number of samples to construct the uncertain dynamical model according to the nonparametric technology.

3. Calculate the complex frequency response function, the Fourier transform of excitation, the correlation coefficient matrix, and the spectral density function.

4. Calculate the mean value of response. Let τ=0, and deduce the mean square value. Then, get the variance of the uncertain system, and calculate the reliability of the uncertain structure system by use of Eq. (25).

From the above procedure, both data and model uncertainties are included, the requirement of unknown information is avoided, and the computation workload is clearly reduced. It should be noted that all the processes involved can avoid the demand for statistical data and prior information. We believe that the present procedure is instructive for designing this kind of rotating machine, and fault diagnosis can also be conducted based on the variation of the reliability index of the rotor system.

3.2 Z^*-value based on short-term predictability

To adequately illustrate the effect of uncertainties, the next task of this study is to further assess the operating status by short-term predictability based on the Monte Carlo simulation, as a supplement and verification to the proposed procedure in Section 3.1.

In nonlinear theory, performance evaluation methods are usually based on the reconstruction technology of the state space. The quality of the dynamics can be clearly displayed in the state space of the original time series or measured data, quantified with the short-term predictability and expressed as the final evaluation by the statistical index Z^*-value. The principle of this evaluation is as follows. First, formulate a null hypothesis that the original time series and the measured data are produced by the same mechanism, namely they are from the same underlying distribution function. Then, determine the statistic of the variation of the distance between two close points in the state space during a short period of time, denoted by Z^*-value. Generally, a larger value of $|Z^*|$ than 2.33 signifies that the null hypothesis does not hold at the 99% confidence level (Kennel and Isabelle, 1992). If the calculated $|Z^*|$ is beyond this limit, one can confirm that the original and measured time series are not produced by the same mechanism and the essence of the system has been varied.

Under the influence of uncertainties to the rotor system, it can be confirmed that the performance of the system has been varied if the above mentioned null hypothesis does not hold. Therefore, this statistical index can also be used to evaluate the reliability of the present rotor.

We first express the time series of shaft vibration of the design model without uncertainties as $\{x_i\}$ (i=1, 2, …, N), where N is the number of points in the series. Next, the most important step is the state space reconstruction to intuitively represent the dynamic properties which are hardly seen just from the time-domain waveform, in which two crucial parameters are involved, i.e. the embedding dimension m_0 and the delay interval ξ, both playing an important role in unfolding the attractor in the state space. Too small a result of the two parameters will result in an incomplete presentation of the attractor, while too large a result may bring much redundant information to reconstruct vectors making some properties overestimated and the computation being significantly increased. We adopt two acknowledged reasonable methods, i.e. the G-P algorithm (Grassberger and Procaccia, 1983) and the C-C method (Kim et al., 1999), to obtain the two parameters. Then the vectors in the state space can be expressed as $\boldsymbol{X}_i=[x_i, x_{i+\xi}, x_{i+2\xi}, \ldots, x_{i+(m_0-1)\xi}]^{\mathrm{T}}$, where i=1, 2, …, M and $M=N-(m_0-1)\xi$ is the number of vectors. $\boldsymbol{X}_i$ can also be seen as the points in the state space.

Commonly, two points are considered to be initially close together if the distance between them is smaller than the average absolute deviation, which is defined as

$$d_0=\frac{1}{N}\sum_{i=1}^{N}\left|x_i-\bar{x}\right|, \tag{26}$$

where $\bar{x}$ is the average of the time series obtained from

$$\bar{x}=\frac{1}{N}\sum_{i=1}^{N}x_i. \tag{27}$$

If two vectors or points in the state space are close together, then the distance between them must meet the following formula expressed in supremum norm as

$$\max_{0\le s\le m_0-1}\left|x_{i+s}-x_{j+s}\right|\le d_0. \tag{28}$$

Then, we consider the fluctuation of the distance during a short period of time, say 10% of the average cycle P (generally m_0 is big enough to take place of P) as only the initial growth of the distance between the points considered in short-term predictability. Thus, the distance is changed to

$$d_{i,j}=\max_{0\le s\le m_0+m_{\mathrm{d}}-1}\left|x_{i+s}-x_{j+s}\right|, \tag{29}$$

where $m_{\mathrm{d}}=0.1m_0$. The interval between the points is required to be larger than one average circle to avoid the spurious effects of dynamic correlations, reflected in the coordination as $|i-j|>m_0$.

To obtain the statistical index, the distances $d_{i,j}$ of the theoretical and actual series are compared during the short period of evolution of the trajectory. The set of distances obtained from the theoretical time series is denoted as A, formed by $d_{i,j}$ with the number of N_A. Likewise, a set B is formed by N_B distances from the actual series. Now the Mann-Whitney U test is defined as

$$U_\varepsilon=\sum_{i=1}^{N_A}\sum_{j=1}^{N_B}H_\varepsilon(A_i-B_j), \tag{30}$$

where $H_\varepsilon(\cdot)$ is the Heaviside step function. Finally, the statistical index Z^*-value is defined as

$$Z^*=\frac{U_\varepsilon-N_AN_B/2}{\sqrt{\frac{1}{12}N_AN_B(N_A+N_B+1)}}. \tag{31}$$

This index will have a distribution with zero mean and unit variance as long as the hypothesis is tenable that the two series are generated by the same mechanism (van Ommen et al., 1999). In this study, the actual time series come from the system with random uncertainties and are compared with the ideal series from the mean model, which is used to assess the influence of uncertainties. Normally, a larger value for $|Z^*|$ than 2.33 will help one confirm that the hypothesis does not hold at the 99% confidence level. It should be noted that this process has no concern with any information about the uncertainty influence, but it signifies that the system is experiencing changes in the essence of its performance.

The calculation process of the statistical index Z^*-value is concluded in detail as follows:

1. Perform numerical simulation with the Monte Carlo method to generate the time series of the mean and uncertain models. M denotes the time series from the mean model and R denotes that from the random model.

2. Choose proper parameters m_0, ξ, and P of M and R, and reconstruct the state space.

3. Determine the mean value $\bar{x}$ and the average absolute variance d_0, and find the points initially

closed with Eq. (28) between which the intervals are larger than P.

4. Calculate the variation of distance within a short time period with Eq. (29) to form the set A obtained from M and B from R.

5. Carry out the Mann-Whitney U test with Eq. (30) and calculate the statistical index Z^*-value with Eq. (31).

4 Numerical examples

To illustrate the application and verify the efficiency of the present analysis, numerical simulation is employed, and the result is discussed in detail. First, the reliability index is calculated by the proposed method, and then the assessment is done by the statistical index based on short-term predication at each given time. It is very important that the degree of uncertainties should be selected according to the actual circumstances.

Indeed, as stated in the introduction, there are many uncertain factors involved in a rotor system, such as external interference and the physical structure of the system. In most cases, some parameters in the model cannot be directly measured, and even the mathematical model may be uncertain since it depends on the system's physical structure and the modeling approach. Therefore, the levels of the uncertainty involved in the system are usually difficult to determine. Here, we choose three different values of the divergence control parameter shown in Eq. (9) as in (Soize, 2000), i.e. δ=0.02, 0.05, and 0.08, to investigate the fluctuations of the vibration response of the rotor system and further perform the reliability estimation on the rotor system. For other uncertainty levels, a similar analysis can be performed.

4.1 Cornell index

The rotor system is an asymmetric damped system, and the design values of each part are set as given in Table 2. Thirty two samples are produced in the simulation process of random uncertainties to ensure the ergodicity by the nonparametric approach introduced in Section 3. In order to illustrate the operational states in detail, we firstly resort to the fourth-order Runge Kutta method to obtain the numerical solution of Eq. (4), setting the time step as 0.01 s. To guarantee the veracity of the results, the transient response is abandoned and 3000 points are adopted in the stability status of each sample at each rotating speed, and the results obtained are shown in Fig. 2.

Table 2 Some parameter values in the mean model of the dual-span rotor

Parameter	Value	Parameter	Value
m_1, m_2 (kg)	32.1	c_1, c_2 (N·s/m)	2100
m_3, m_4, m_5, m_6 (kg)	4	c_3, c_4 (N·s/m)	1050
R (mm)	25	c_5, c_6 (N·s/m)	1150
L (mm)	12	e_1, e_2 (mm)	0.05
k_1, k_2 (N/m)	2.5×10^7	k_3 (N/m)	2.5×10^5
C (mm)	0.11	Θ	1.3

Fig. 2a shows the sample responses for three different dispersion parameters, i.e. δ=0.02, 0.05, and 0.08, at a speed of 1000 rad/s, while Figs. 2b–2d are the vibration amplitude fluctuations of disc 1 during the speed-up process for the three cases. From Figs. 2b–2d, it can be seen that the rotor is rising to the second vibration peak range at 1000 rad/s caused by oil whip. Three cases of vibration in Fig. 2a are mostly different in general, so different uncertainties can lead to different sample responses.

Moreover, the vibration response contains two peaks in the speed range of 200–1500 rad/s, and the average curves of amplitude in the three cases are almost the same, while large differences exist in the range of response amplitude. The increase of the dispersion parameter will extend the amplitude distribution intervals. Therefore, considering that the response of the actual system is certainly one of these uncertain samples, instead of the mean model, large differences exist between the reliability of the actual system and the design value. It should be pointed out that although there is a big gap between the vibration amplitudes in the resonance region, the values of the critical speed remain essentially constant. Thus, it can be concluded that the random uncertainty has little influence on the critical speed, and the uncertainties can be ignored in determining the critical speed value but not in examining the reliability.

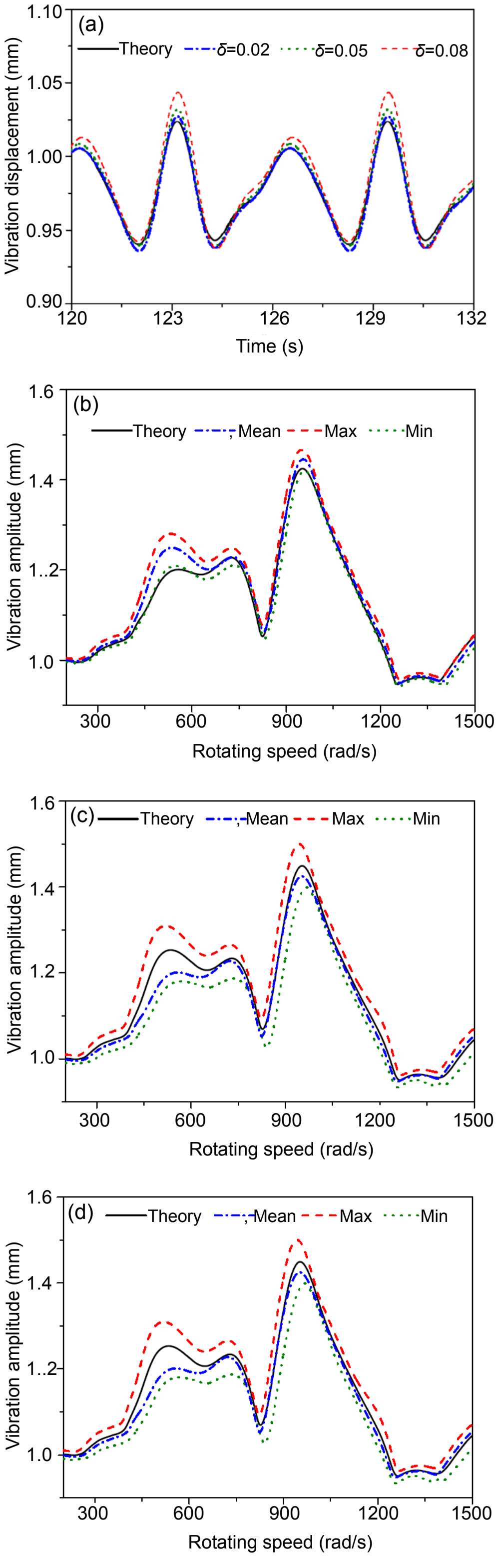

Fig. 2 Response of disc 1 of the dual-span rotor system with random uncertainties: (a) time series of vibration; (b), (c), and (d): vibration amplitude fluctuations with *δ*=0.02, 0.05, and 0.08, respectively

The vibration amplitude of the dual-span rotor is about 0.9 mm in the non-resonance region (Fig. 2). In actual situations, 30% outside the normal range is seen as the boundary, so we use 1.15 as the threshold in the following reliability analysis. In Fig. 3, the reliability degree R_d calculated by the proposed method denoted by δ_r and by the Monte Carlo method denoted by δ_0 in the above three cases are compared. As can be seen from the figure, the results of the two methods are quite close, indicating that the proposed method can effectively be used to calculate reliability. Special attention should be paid to the fact that the reliability in the case of δ=0.08 is very low but higher than others when the system speeds up to the first and second critical speeds. That is because in regions of vibration exceeding the threshold, the samples of small uncertainties mostly exceed the threshold while the samples with large uncertainties may be more under the threshold reducing the probability of system failure.

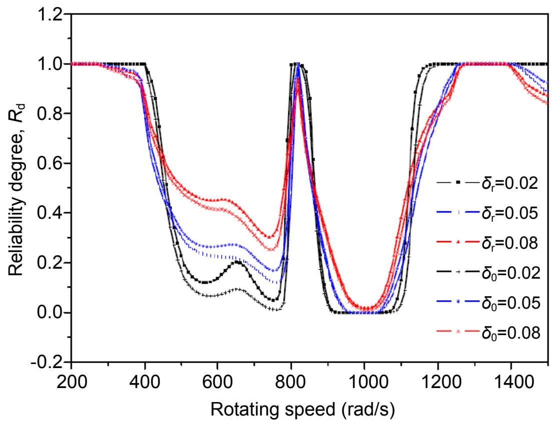

Fig. 3 Reliability degrees R_d of the two methods for the three different dispersion parameters, i.e. *δ*=0.02, 0.05, and 0.08

Only 32 samples are simulated in this paper, and the error between the two methods will be gradually reduced with the increase of sample number. Even so, the maximum deviation of the three cases is only 0.10 at 650 rad/s, and in most cases the error is very small. The reliability indices of the three cases are almost the same, being low near the first or the second critical speed. At this point, the uncertainties do not afford a critical role but the inherent nature of the system does. Thus, the resonance region should be quickly crossed over in any case to avoid deformation, fracture, or other accidents caused by excessive vibration.

4.2 Z^*-value

Although there has been much research investigating the influence of uncertainties, no evaluation method has yet been accepted extensively. In the following, the Z^*-value based on short-term predictability is employed to assess the influence of uncertainties on the present rotor. In this subsection, 32 samples for each case, i.e. δ=0.02, 0.05, or 0.08, are generated with the help of the Monte Carlo simulation, and 4000 data points are taken at each time instant to ensure sufficient evaluation. For large N_A and N_B, which in practice mean several tens (here we use 100), the statistical approach will result in a more precise result.

As described above, the Z^*-value has a distribution with a zero mean and a unit variance as long as the two sets of series are from the same mechanism, while a larger Z^*-value, greater than 2.33, will ensure that the essential differences have been generated due to random uncertainties with a 99% confidence level. Usually, this threshold value is extended to 3.0 in practice with no particular requirements, while a smaller one may be adopted for precision instruments or early warning.

The results form four curves as shown in Fig. 4. The solid line labeled 'sample' represents a random sample of $|Z^*|$, the dashed line and dash-dotted line indicate the maximum and minimum values of the 32 samples, respectively, and the area between them is the distribution range of $|Z^*|$, while the dotted line denotes the average value. When δ=0.02, the average value fluctuates in the region [2.23, 3.01], implying that the influence of random uncertainties is not large enough to deteriorate the performance of the rotor. When δ=0.05 and δ=0.08, the ranges are about [3.06, 3.85] and [3.43, 4.10]. This says that the two cases have exceeded their desired ranges of working states and also means the system has a great possibility of the fault occurrence.

It should be noted that about 90% of the samples fall down below the safe limit of 3.0 when δ=0.02, indicating that the reliability can meet the production requirement in general with random uncertainties in this case. This proportion decreases to 50% and 30% when δ=0.05 and δ=0.08, respectively, suggesting that most samples have deviated from the predetermined operating state. These proportions can help us estimate the safety of the system in different cases of uncertainties which are consistent with the results of reliability in Fig. 3, validating the efficiency of the proposed method and the predictions in the previous

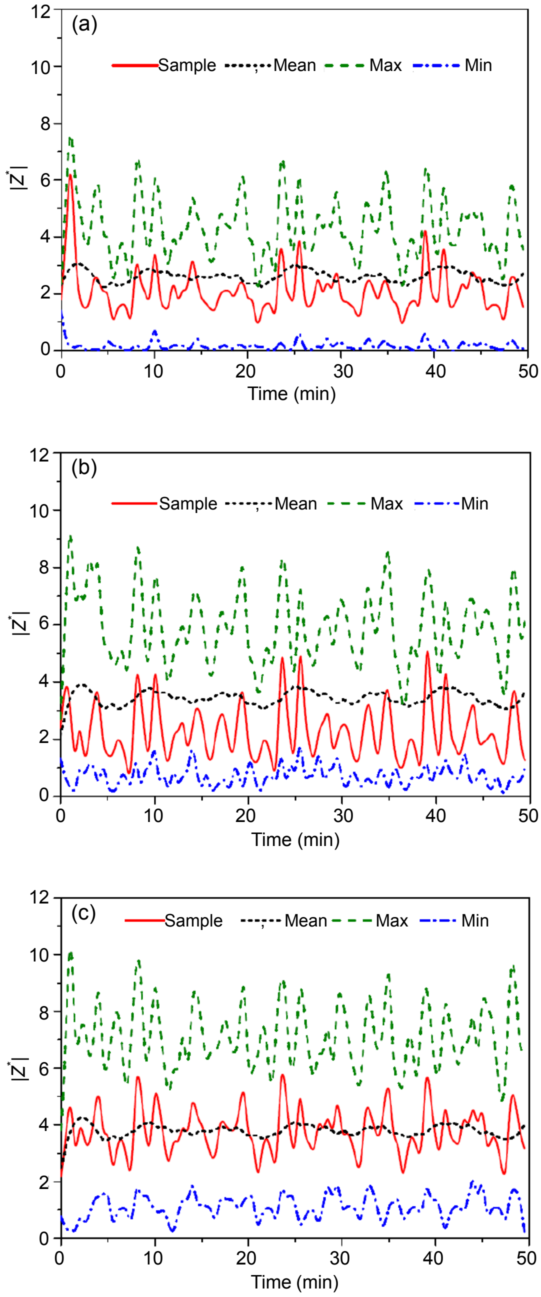

Fig. 4 Statistical indices in three cases: (a) δ=0.02, (b) δ=0.05, and (c) δ=0.08

sections. In addition, the samples are set to be deterministic before calculation in this study, but actually, they may be changing all the time due to the wear of components, immunity decrease of the disturbance, or the creep of material. Thus, the tendency of the curves may not be approximately horizontal but rather oblique, and constant observations are usually required even for rotors with initially high reliability index.

From the above analysis, it can be concluded that the procedure proposed here can be employed to obtain a relatively accurate reliability estimation for the present uncertain rotor system. The Cornell index can clearly exhibit the change of reliability as the rotating speed increases in an integrative manner, and the comparisons between the results for several randomly uncertain situations can also be observed at any speed of the shaft. The statistical index builds its analysis on the output data only at some given time, but can precisely identify the differences of reliability under various situations. Moreover, fluctuation ranges of the system's vibration response are directly related to the strengths of randomly uncertain perturbations and usually increase in proportion to the strengths of such perturbations. This will make the vibration amplitude exceed the predetermined threshold and result in system instability and even failure. Thus, the previous deterministic models under ideal conditions cannot accurately reveal the running status of the rotor. It is necessary to take the uncertain factors into consideration to grasp and control the system, and a sufficient margin of safety should be made to effectively avoid the low reliability area of the system.

5 Conclusions

A reliability estimation procedure combining a nonparametric model with response spectrum analysis is proposed in this study. The mean vibration equation of a dual-span rotor with 12 degrees of freedom (DOFs) is first established, and then the random matrix model is presented by the nonparametric approach containing both data and model uncertainties. Thereafter, reliability analysis is carried out by the proposed procedure to gauge the short-term predictability. From this the reliability indices such as the Cornell index and Z^*-value are then given to make a reliability estimation on the dual-span rotor.

Numerical examples are employed to verify the proposed method. This validates the fact that the reliability calculation can be carried out without knowing discrete movements. Moreover, a large amount of information needed for determining the PDF is avoided in the reliability analysis. Furthermore, solving tedious and consuming equations is avoided using this method. From the simulation results, the safe or failure ranges of the dual-span rotor system can be clearly found from the reliability curves.

Random uncertainties are essentially unavoidable, and existing deterministic dynamic models cannot accurately represent the operational states of machines due to the incompleteness and inaccuracy of data and the simplification of the machine structure. All the calculations involved in the present study can avoid the demands of statistical data and prior information, and have the advantages of simplicity and low computational complexity.

References

Au SK, Papadimitriou C, Beck JL, 1999. Reliability of uncertain dynamical systems with multiple design points. *Structural Safety*, 21(2):113-133. https://doi.org/10.1016/S0167-4730(99)00009-0

Beck AT, Melchers RE, 2004. On the ensemble crossing rate approach to time variant reliability analysis of uncertain structures. *Probabilistic Engineering Mechanics*, 19(1-2): 9-19. https://doi.org/10.1016/j.probengmech.2003.11.018

Beck D, 2015. Applications of hydro-mechanically coupled 3D mine and reservoire scale, discontinuous, strain-softening dilatant models with damage. Proceedings of the 49th US Rock Mechanics/Geomechanics Symposium.

Deckers M, Doerwald D, 1997. Steam turbine flow path optimizations for improved efficiency. Proceedings of PowerGen Asia, p.160-185.

Gan CB, Wang YH, Yang SX, et al., 2014. Nonparametric modeling and vibration analysis of uncertain Jeffcott rotor with disc offset. *International Journal of Mechanical Science*, 78:126-134. https://doi.org/10.1016/j.ijmecsci.2013.11.009

Grassberger P, Procaccia I, 1983. Characterization of strange attractors. *Physical Review Letters*, 50(5):346-349. https://doi.org/10.1103/PhysRevLett.50.346

Kennel MB, Isabelle S, 1992. Method to distinguish possible chaos from colored noise and to determine embedding parameters. *Physical Review A*, 46(6):3111-3118. https://doi.org/10.1103/PhysRevA.46.3111

Kim HS, Eykholt R, Salas JD, 1999. Nonlinear dynamics, delay times, and embedding windows. *Physica D: Nonlinear Phenomena*, 127(1-2):48-60. https://doi.org/10.1016/S0167-2789(98)00240-1
Lal M, Tiwari R, 2012. Multi-fault identification in simple rotor-bearing-coupling systems based on forced response measurements. *Mechanism and Machine Theory*, 51:87-109. https://doi.org/10.1016/j.mechmachtheory.2012.01.001
Li J, Chen JB, 2005. Dynamic response and reliability analysis of structures with uncertain parameters. *International Journal for Numerical Methods in Engineering*, 62(2): 289-315. https://doi.org/10.1002/nme.1204
Liu DS, Peng YH, 2012. Reliability analysis by mean-value second-order expansion. *Journal of Mechanical Design*, 134(6):0061005. https://doi.org/10.1115/1.4006528
Liu WM, Noyak M, 1995. Dynamic behaviour of turbine generator foundation systems. *Earthquake Engineering and Structural Dynamics*, 24(3):339-360. https://doi.org/10.1002/eqe.4290240304
Mbarka S, Baroth J, Ltifi M, et al., 2010. Reliability analyses of slope stability. *European Journal of Environmental and Civil Engineering*, 14(10):1227-1257. https://doi.org/10.1080/19648189.2010.9693293
Murugan S, Ganguli R, Harursampath D, 2008a. Aeroelastic response of composite helicopter rotor with random material properties. *Journal of Aircraft*, 45(1):306-322. https://doi.org/10.2514/1.30180
Murugan S, Harursampath D, Ganguli R, 2008b. Material uncertainty propagation in helicopter nonlinear aeroelastic response and vibratory analysis. *AIAA Journal*, 46(9): 2332-2344. https://doi.org/10.2514/1.35941
Narendra KS, Parthasarathy K, 1990. Identification and control of dynamical systems using neural networks. *IEEE Transactions on Neural Networks*, 1(1):4-27. https://doi.org/10.1109/72.80202
Ohayon R, Soize C, 1998. Structural Acoustics and Vibration. Academic Press, New York, USA.
Pichler K, Lughofer E, Pichler M, et al., 2015. Fault detection in reciprocating compressor valves under varying load conditions. *Mechanical Systems and Signal Processing*, 70:104-119. https://doi.org/10.1109/ISSPIT.2011.6151564
Rinehart AW, Simon DL, 2014. An integrated architecture for aircraft engine performance monitoring and fault diagnostics: engine test results. 50th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, AIAA Propulsion and Energy Forum. https://doi.org/10.2514/6.2014-3924
Shiau TN, Huang KH, Wang FC, et al., 2009. Dynamic response of a rotating multi-span shaft with elastic bearings subjected to a moving load. *Journal of System Design and Dynamics*, 3(1):107-119. https://doi.org/10.1299/jsdd.3.107
Soize C, 2000. A nonparametric model of random uncertainties for reduced matrix models in structural dynamics. *Probabilistic Engineering Mechanics*, 15(3):277-294. https://doi.org/10.1016/S0266-8920(99)00028-4
Song Y, Feng HL, Liu SY, 2006. Reliability models of a bridge system structure under incomplete information. *IEEE Transactions on Reliability*, 55(2):162-168. https://doi.org/10.1109/TR.2006.874944
Spinato F, Tavner PJ, van Bussel GJW, et al., 2008. Reliability of wind turbine subassemblies. *IET Renewable Power Generation*, 3(4):387-401. https://doi.org/10.1049/iet-rpg.2008.0060
van Ommen JR, Schouten JC, Coppens MO, et al., 1999. Monitoring fluidization by dynamic pressure analysis. *Chemical Engineering & Technology*, 22(9):773-775. https://doi.org/10.1002/(SICI)1521-4125(199909)22:9<773::AID-CEAT773>3.0.CO;2-I
Wan F, Zhu WP, Swamy MNS, 2008. A frequency-domain correlation matrix estimation algorithm for MIMO-OFDM channel estimation. IEEE 68th Vehicular Technology Conference, p.1-5. https://doi.org/10.1109/VETECF.2008.72
Zhang YM, Wen BC, Liu QL, 1998. First passage of uncertain single degree-of-freedom nonlinear oscillators. *Computer Methods in Applied Mechanics and Engineering*, 165(1-4): 223-231. https://doi.org/10.1016/S0045-7825(98)00042-5
Zhang YM, Wen BC, Liu QL, 2003. Reliability sensitivity for rotor–stator systems with rubbing. *Journal of Sound and Vibration*, 259(5):1095-1107. https://doi.org/10.1006/jsvi.2002.5117

中文概要

题　目：双跨转子的随机不确定性非参数建模与可靠性分析

目　的：旋转机械由于工作环境复杂，在运行过程中会不可避免地受到各种不确定性因素的影响，从而引发转子系统的异常振动。因此，迫切需要对系统工作状态开展可靠性分析。本文将外部扰动不确定性与模型不确定性考虑在内，旨在建立转子系统运行状态的可靠性评估指标，丰富转子动力学理论体系，为工程应用提供参考。

创新点：1. 采用非参数法进行建模，能够将外部扰动不确定性与模型不确定性同时包含在内；2. 在非参数建模基础上，结合响应谱分析法进行可靠性计算，可避免对系统先验知识的需求并降低计算过

程的复杂性；3. 将短周期预测理论扩展应用于可靠性分析验证。

方 法： 1. 借助非参数法建立转子系统的随机不确定性模型；2. 结合响应谱分析法推导出系统可靠性指标计算式；3. 采用短周期预测方法对模拟数据统计指标进行计算与验证。

结 论： 1. 本方法可用于评估大型复杂旋转机械系统的可靠性，尤其对于服役时间较长导致系统参数出现不确定性变化的情形；2. 本研究结果可为大型复杂旋转机械的设计、运行和控制提供理论基础，同时也可以为其他类型机械设备的可靠性分析和预测方法提供参考。

关键词： 随机不确定性；非参数建模；可靠性；响应谱分析