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Effect of the uncertainty in soil-water characteristic curve on the estimated shear strength of unsaturated soil*

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Abstract: Most failures or instabilities of geotechnical structures commonly result from shear failure in soil. In addition, many infrastructures are constructed within the unsaturated zone. Therefore, the determination of shear strength of unsaturated soil is crucial in geotechnical design. The soil-water characteristic curve (SWCC) is commonly used to estimate the shear strength of unsaturated soil because the direct measurement is time-consuming and costly. However, the uncertainty associated with the determined SWCC is rarely considered in the estimation of the shear strength. In this paper, the uncertainties of SWCC resulted from different factors are reviewed and discussed. The variability of the estimated shear strength for the unsaturated soil due to the uncertainty of SWCC associated with the best fit process is quantified by using the upper and lower bounds of the determined SWCC. On the other hand, the uncertainties of the estimated shear strength due to different initial void ratios or different confining pressures are quantified by adopting different SWCCs. As a result, it is recommended that the measured SWCC from the conventional Tempe cell or pressure plate needs to be corrected by considering different stress levels in the estimation of the shear strength of unsaturated soil.

Key words: Unsaturated shear strength; Pore-size distribution function; Variability; Soil-water characteristic curve (SWCC); Confidence limits
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1 Introduction

The shear strength of unsaturated soil is the key parameter needing to be considered in geotechnical design. Experimental measurements of the unsaturated shear strength are not commonly adopted in practice because of the long testing time. Instead, the unsaturated shear strength is commonly estimated from the soil-water characteristic curve (SWCC). Fredlund and Rahardjo (1993) indicated that the SWCC is commonly considered as the controlling parameter in the estimation of the unsaturated shear strength of soil. As a result, there are various models that have been proposed (Fredlund et al., 1996; Vanapalli et al., 1996; Khalili and Khabbaz, 1998; Toll and Ong, 2003; Sheng et al., 2008; Goh et al., 2010; Sheng et al., 2011; Schnellmann et al., 2015; Zhai et al., 2019a; Cai et al., 2020) for the estimation of the unsaturated shear strength from the SWCC.

Uncertainty associated with SWCC was reported by Yaldo (1999), Zapata et al. (2000), Gaharagheer

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(2009), Phoon et al. (2010), Rahardjo et al. (2012), Ye et al. (2012), and Zhai and Rahardjo (2013). Agus and Schanz (2007), Patrick et al. (2007), Nam et al. (2010), and Rahardjo et al. (2019) indicated that the variability of SWCC might result from different measurement techniques. Zapata (1999) and Zhai and Rahardjo (2013) stated that the number of data points, measured suction range, and the best fit equation selected for the regression analysis might also lead to uncertainty. In addition, Fredlund (1964), Birle et al. (2008), Tarantino (2009), Zhou et al. (2012), Cai et al. (2014), Mendes and Toll (2016), Wijaya and Leong (2017), and Gao et al. (2019) indicated that the SWCC can vary with different initial saturated water contents or initial void ratios. All these factors, which may lead to the variability of SWCC, may also result in the uncertainty in the estimated unsaturated shear strength.

In this paper, the effect of the variability of SWCC on the estimated unsaturated shear strength is evaluated. Methods for the estimation of the uncertainty that incorporate the variability of SWCC are proposed. The proposed method was verified with experimental data. It is recommended that the variability of SWCC on the estimated unsaturated shear strength should be taken into account in order to increase the confidence level in the geotechnical design.

2 Variability of soil-water characteristic curve

The variability of SWCC may result from the following factors: (1) natural variation in soil textures; (2) experimental measurement error; (3) different measurement equipment or techniques; (4) experience of the operator; (5) number of data points collected; (6) range of suction covered; (7) the best fit equation selected; (8) different forms of SWCC (SWCC expressed in the form of the gravimetric water content, SWCC-w, or in the form of the volumetric water content, SWCC-θ, or in the form of the degree of saturation, SWCC-S); (9) temperature variation; (10) different initial saturated water contents or different initial void ratios; (11) different confining pressures. Both the natural variation in soil textures and measurement error are common in engineering practice and they cannot be avoided in experimental measurement. In recent years different technologies or apparatuses have been developed for the determination of the SWCC. However, the results from one technique may deviate from those of others. The experience of the operator is mainly dependent on the soil preparation and the determination of the equilibrium condition. If the soil is disturbed or the determination of the equilibrium condition is incorrect, then results can be significantly different from those obtained following the standard procedure. As the measurements from the laboratory can only generate limited discrete data points while the SWCC is commonly represented by a continuous mathematical equation, a best fit equation is commonly used to fit these points. The fitting parameters in the equation are normally determined via regression analysis. Therefore, the solutions for those fitting parameters are not unique, and are dependent on the number of data points, the range of suction covered in the measurement, and the best fit equations selected for the regression analysis. Zhai and Rahardjo (2013) proposed a method to quantify the variability of SWCC from the residual error by using the first-order error method. Fredlund (2006) indicated that different SWCCs could be obtained for the same soil if the SWCC was expressed in different forms such as SWCC-w, SWCC-θ, and SWCC-S. Different temperatures result in different viscosities and surface tensions, which in turn lead to different SWCCs. Different initial water contents or initial void ratios lead to different pore-size distribution functions of soil. As a result, different SWCCs can be obtained from the soil with different initial water contents or void ratios. During the measurement, different confining pressures cause different densities or different void ratios of soil. Therefore, the confining pressure changes the pore-size distribution functions of soil and different SWCCs can be obtained under different confining pressures. In this section, the variability of SWCC due to different factors which were reported by different researchers is reported and demonstrated.

2.1 Variability of SWCC due to different measurement techniques

Agus et al. (2001), Fu et al. (2011), Fredlund et al. (2012), Leong and Wijaya (2015), and Satyanaga et al. (2019) showed the variability of SWCC from different apparatuses such as the Tempe cell, pressure plate, and salt solution method. Patrick et al. (2007) found the uncertainty in SWCC between the measurement techniques such as chilled-mirror and filter paper measurement. Nam et al. (2010) reported the variability of SWCC from six testing methods such as filter paper, Tempe cell, pressure plate, dew point potentiometer, vapor equilibrium, and osmotic method. Rahardjo et al. (2019) compared SWCC measurements results for the residual soil from Bukit Timah Granite from six measurement techniques such as Tempe cell, pressure plate (with the maximum air pressure of 500 kPa), pressure plate (with the maximum air pressure of 1500 kPa), salt solution test, small-scale centrifuge, and dew point chilled-mirror test. The variability in the measured SWCC for the residual soil is illustrated in Fig. 1.

As shown in Fig. 1, the measured data can be more scattered if the SWCC data are combined from different measurement techniques. The scattered data points cause the uncertainty in the determination of the fitting parameters in the best fit equation.

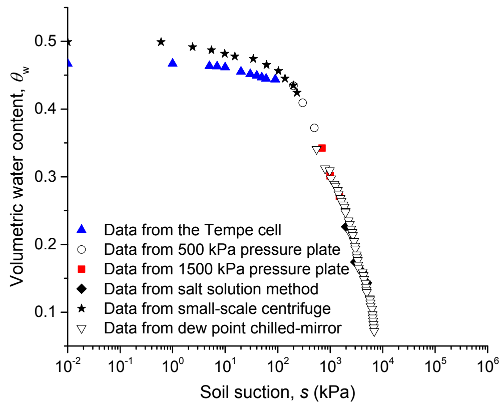

Fig. 1 Illustration of the measured SWCC for the residual soil from Bukit Timah Granite from six different measurement techniques (modified from (Rahardjo et al., 2019))

2.2 Variability of SWCC due to the number of data points and suction ranges covered in the measurement

Zapata (1999) and Zapata et al. (2000) discussed the effects of the number of data points, the suction range, and the best fit equations selected on the variability of the determined SWCC. As Fredlund and Xing (1994)'s equation and van Genuchten (1980)'s equation are popularly used by researchers, Fredlund and Xing (1994)'s equation is selected throughout this study. As a result, the variability of SWCC due to different best fit equations can be eliminated because there is only one best fit equation adopted. Zhai and Rahardjo (2013) proposed the use of the upper and lower bounds of the determined SWCC to quantify the variability of SWCC. The upper and lower bounds of the determined SWCC from Zhai and Rahardjo (2013) were computed from the variance of the fitting parameters. This is calculated from the residual error using the first-order error method. Zhai and Rahardjo (2013) illustrated the variability in the determined SWCC due to different numbers of data points with different suction ranges using regression analyses. The experimental data and best fitted SWCC are illustrated in Fig. 2a. The SWCC best fitted line generated from the data points 1, 3, 5, 7, 9, 11, 13, 15, 17, and 20 (a total of 10 data points) is defined as SWCC1. The SWCC best fitted line generated from the data points 1 to 10 (also 10 data points but smaller suction range) is defined as SWCC2. The SWCC best fitted line generated from data points 1, 5, 9, 13, 17, and 20 (a total six data points) is defined as SWCC3. The SWCC best fitted line generated from the data points 1 to 6 (also six data points but smaller suction range) is defined as SWCC4. The comparisons of the best fitted SWCC line produced from all the data points and SWCC1 to SWCC4 are illustrated in Fig. 2b.

As shown in Fig. 2, it is possible to have different SWCCs based on different collected experimental data as illustrated in the cases of 1 to 5 (i.e. determined SWCC1 to SWCC4 and best fitted curve). Zhai and Rahardjo (2013) recommended obtaining a minimum of five data points with the suction range to be beyond the second bending point (or residual condition) for the determination of an acceptable SWCC.

2.3 Variability of SWCC due to variation in temperature

Romero et al. (2001), Ye et al. (2009), Zhou and Ng (2014), Gao and Shao (2015), and Roshani and Sedano (2016) indicated that the variation in temperature altered the SWCC. Rahardjo (1990) conducted an unsaturated consolidation test and observed that the results fluctuated because of the variation in daily temperature. Tang and Cui (2005) indicated that the surface tension decreased with the increase in temperature, and this led to a decrease in the water retention capacity. Wan et al. (2015) illustrated the variability of SWCC at different temperatures as shown in Fig. 3.

Fig. 3 shows that the measured SWCCs can be affected by the temperature of the environment. In certain areas, the variation of daily temperature is significant, and it may result in different SWCCs of the same soil. Different SWCCs illustrated in Fig. 3 can result in different estimated unsaturated shear strengths.

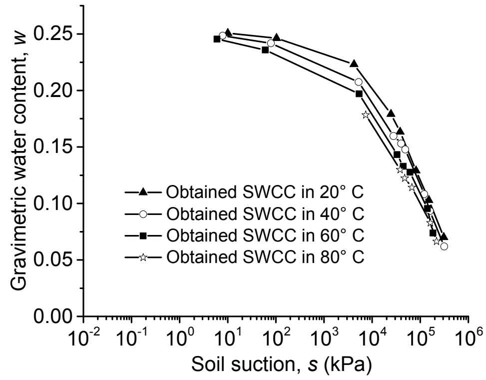

Fig. 3 Illustration of variability of SWCC due to the variation in temperature (modified from (Wan et al., 2015))

2.4 Variability of SWCC due to the initial water content or the initial void ratios

Fredlund (1964) reported different SWCCs for Regina clay with different initial saturated water contents. Birle et al. (2008) and Mendes and Toll (2016) stated that different SWCCs could be obtained from the soil with different initial water contents for compacted clay and sandy clay. Fredlund and Houston (2013) reported the variability of SWCC for oil sands tailing with different initial water contents. When the soil is fully saturated, the different initial saturated water contents represent different initial void ratios. Zhou et al. (2012) proposed a method to model the SWCCs with different initial void ratios. Gao et al. (2019) conducted SWCC measurements for the compacted Pearl clay with different void ratios as shown in Fig. 4.

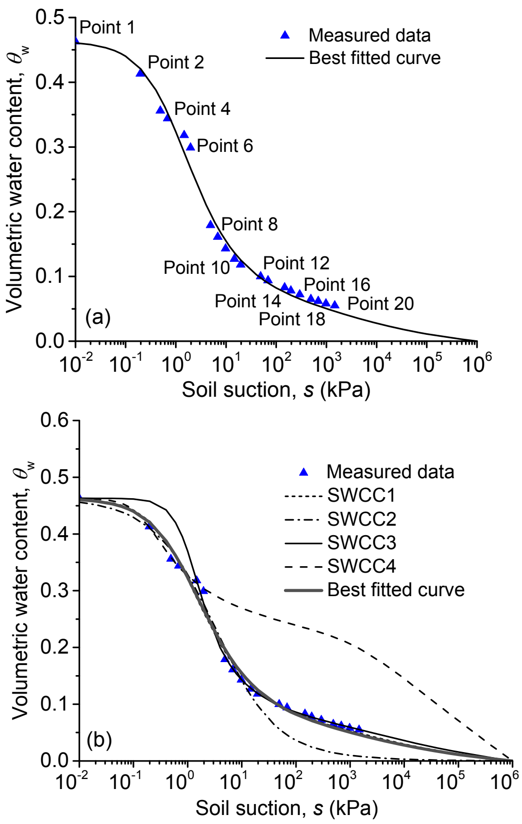

Fig. 2 Illustration of variability of SWCC due to different numbers of data points with different suction ranges
(a) Experimental data points and best fitted SWCC; (b) Obtained SWCCs from different numbers of data points with different suction ranges

Rahardjo et al. (2012) and Zhai et al. (2016) indicated that the void ratios of residual soil in Singapore varied with depth and region. Therefore, different SWCCs for the same soil may be obtained due to the variation of the initial void ratios.

2.5 Variability of SWCC due to different confining pressures

Unsaturated triaxial equipment has become more popular for the measurement of SWCC because of the different confining pressures that can be applied to the soil specimen. However, different confining pressures may change the pore-size distribution of soil during the measurement, which in turn alters the SWCC of soil. Ng and Pang (2000), Thu et al. (2007), Dastjerdi et al. (2014), and Roy and Rajesh (2018) reported that the SWCC of soil can be altered by different confining pressures. Lee et al. (2005) reported different SWCCs for the same soil sample such as the weathered granite under different confining pressures as shown in Fig. 5.

Different confining pressures represent different stress states in soil. As a result, for the same type of soil, the SWCC of the soil located 20 m below the ground surface may be different from that of the soil near the ground surface. It is noted that the slip surface for slope stability analysis can cut through the slope at different depths. The bases of different slices will have different stress states and different SWCCs. Therefore, it is important to use a correct SWCC for the estimation of unsaturated shear strength of soil with different stress levels.

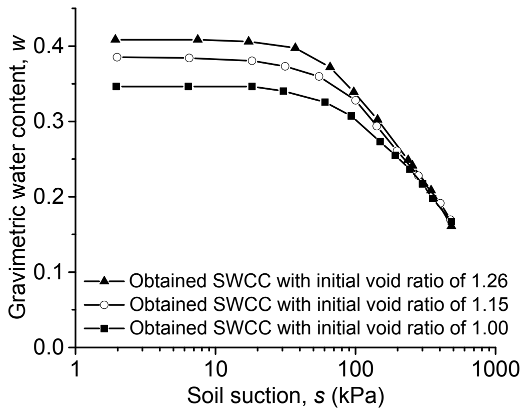

Fig. 4 Illustration of variability of SWCC due to different initial void ratios (modified from (Gao et al., 2019))

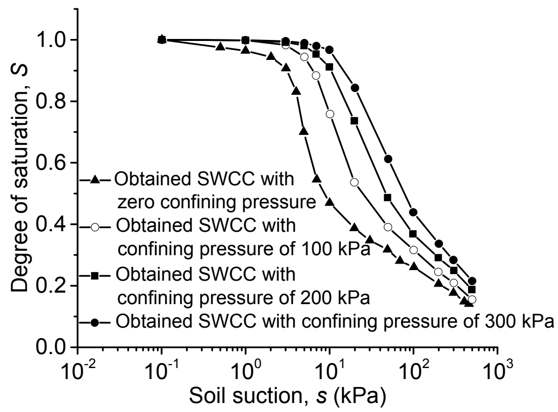

Fig. 5 Illustration of variability of SWCC due to different confining pressures (modified from (Lee et al., 2005))

3 Estimation of the unsaturated shear strength from soil-water characteristic curve

The Mohr-Coulomb (MC) failure criterion, which considers that the shear strength of soil is mainly affected by the cohesion and friction angle, is commonly used to describe the shear strength using the effective stress variable from Terzaghi (1936), as illustrated by

$$\tau_{\mathrm{ff}} = c' + (\sigma_{\mathrm{f}} - u_{\mathrm{w}})_{\mathrm{f}} \tan\phi', \tag{1}$$

where τ_{ff} is the shear stress on the failure plane at failure, c' is the effective cohesion, $(\sigma_{\mathrm{f}}-u_{\mathrm{w}})_{\mathrm{f}}$ is the effective normal stress on the failure plane at failure, σ_{f} is the normal stress on the failure plane, u_{w} is the pore-water pressure at failure, and ϕ' is the effective angle of internal friction. The concepts of shear strength based on the MC failure criterion are straight forward, and depend on the effective cohesion and effective angle of internal friction of the soil. Therefore, the MC failure criterion is extended for the explanation of the shear strength of the unsaturated soil. Fredlund et al. (1978) proposed a linear form for the unsaturated shear strength, which is given by

$$\tau_{\mathrm{ff}} = c' + (\sigma_{\mathrm{f}} - u_{\mathrm{a}})_{\mathrm{f}} \tan\phi' + (u_{\mathrm{a}} - u_{\mathrm{w}})_{\mathrm{f}} \tan\phi^{\mathrm{b}}, \tag{2}$$

where ϕ^{b} is the angle indicating the rate of increase in shear strength with respect to a change in matric suction, $(u_{\mathrm{a}}-u_{\mathrm{w}})_{\mathrm{f}}$ is the matric suction at failure, and u_{a} is the pore air pressure. ϕ^{b} is commonly obtained from the best fitting Eq. (2) with the measured experimental data.

In order to predict the unsaturated shear strength from SWCC, Vanapalli et al. (1996) extended Fredlund et al. (1978)'s equation by defining $\tan\phi^{\mathrm{b}}=(\theta-\theta_{\mathrm{r}})/(\theta_{\mathrm{s}}-\theta_{\mathrm{r}})\tan\phi'$ as follows:

$$\tau_{\mathrm{ff}} = c' + \left[(\sigma_{\mathrm{f}} - u_{\mathrm{a}})_{\mathrm{f}} + \left(\frac{\theta_{\mathrm{w}} - \theta_{\mathrm{r}}}{\theta_{\mathrm{s}} - \theta_{\mathrm{r}}} \right) (u_{\mathrm{a}} - u_{\mathrm{w}})_{\mathrm{f}} \right] \tan\phi', \tag{3}$$

where θ_{s} is the saturated volumetric water content, θ_{r} is the residual volumetric water content, and θ_{w} is the volumetric water content in soil with suction of $(u_{\mathrm{a}}-u_{\mathrm{w}})_{\mathrm{f}}$.

Zhai et al. (2019a) conducted stress analyses on a representative element and observed that immobile water might not contribute to the shear strength of unsaturated soil. Zhai et al. (2019a) recommended modifying the conventional SWCC by removing immobile water for the estimation of the unsaturated shear strength by adopting the concept of pore size distribution function (Zhai et al., 2019b, 2019c).

Consequently, Zhai et al. (2019a) proposed Eq. (4) for the estimation of the unsaturated shear strength.

$$\tau = c' + (\sigma - u_a)\tan\phi' + \frac{S-S'}{1-S'}(u_a - u_w)\tan\phi' + \sum_{i=m}^{N}\left\{\frac{1}{\pi}\left[\left(\frac{\psi_i}{\psi_m}\right)^2\alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m}\right)^2 - 1}\right] \times (u_a - u_w)\left[S(\psi_{i+1}) - S(\psi_i)\right]\right\}, \tag{4}$$

where σ is the normal stress, ψ_m is the applied suction, ψ_i is the equivalent suction corresponding to the pore radius of r_i, α_i is the angle defines the relationship between r and R, $\sin\alpha = r/R$, R is the radius of the meniscus and r is the radius of the capillary tube, S is the degree of saturation, $S(\psi_i)$ is the degree of saturation corresponding to suction of ψ_i, and S' is the degree of saturation contributed from immobile water.

Substituting Fredlund and Xing (1994)'s equation into Eq. (4) results in Eq. (5) at the bottom of this page, where ψ is the soil suction, and ψ'=3100 kPa as recommended by Zhai et al. (2019a).

As shown in Eq. (5), the unsaturated shear strength can be calculated directly from the fitting parameters (i.e. a_f, n_f, and m_f) in Fredlund and Xing (1994)'s equation, where a_f, n_f, and m_f are the fitting parameters in Fredlund and Xing (1994)'s equation, and C_r is the input value. Similarly, if van Genuchten (1980)'s equation is considered for Eq. (4), the unsaturated shear strength can be also calculated directly from the fitting parameters in van Genuchten (1980)'s equation.

4 Effects of variability of soil-water characteristic curve on the estimated unsaturated shear strength

Zhai and Rahardjo (2013) indicated that variability of SWCC can be quantified from different combinations of fitting parameters (a_{fmax}, a_{fmin}, n_{fmax}, n_{fmin}, m_{fmax}, and m_{fmin}), where a_{fmax}, a_{fmin}, n_{fmax}, n_{fmin}, m_{fmax}, and m_{fmin} are estimated from the residual error using the first-order error method. The computation of these parameters can be referred to Zhai and Rahardjo (2013). The combination of (a_{fmax}, n_{fmax}, m_{fmin}) (when $\psi < a_{fmax}$) or the combination of (a_{fmax}, n_{fmin}, m_{fmin}) (when $\psi > a_{fmax}$) defines the upper bound of the determined SWCC, and the combination of (a_{fmin}, n_{fmin}, m_{fmax}) (when $\psi < a_{fmin}$) or the combination of (a_{fmin}, n_{fmax}, m_{fmax}) (when $\psi > a_{fmin}$) defines the lower bound of the determined SWCC. Subsequently, for the best fitted SWCC, the upper bound of the determined SWCC and the lower bound of the determined SWCC (with confidence level of 95%) are used to estimate the unsaturated shear strength using Eq. (5). It is observed that the upper bound of the determined SWCC defines the lower bound of the estimated shear strength while the lower bound of the determined SWCC defines the upper bound of the estimated shear strength, as illustrated in Fig. 6. In this demonstration, the effective internal friction angle of soil is assumed to be 35°.

The method from Zhai and Rahardjo (2013) is mainly to quantify the variability of SWCC. This is computed from the collected experimental data and selected best fit equations. For the soil with different

$$\tau = c' + (\sigma - u_a)\tan\phi' + \frac{\left[1 - \frac{\ln(1+\psi/C_r)}{\ln(1+10^6/C_r)}\right]\left\{\frac{1}{\left\{\ln\left[e + (\psi/a_f)^{n_f}\right]\right\}^{m_f}} - \frac{1}{\left\{\ln\left[e + (\psi'/a_f)^{n_f}\right]\right\}^{m_f}}\right\}}{1 - \left[1 - \frac{\ln(1+\psi/C_r)}{\ln(1+10^6/C_r)}\right]\frac{1}{\left\{\ln\left[e + (\psi'/a_f)^{n_f}\right]\right\}^{m_f}}}\psi\tan\phi' + \sum_{i=m}^{N}\frac{\psi}{\pi}\left[\left(\frac{\psi_i}{\psi_m}\right)^2\alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m}\right)^2 - 1}\right]\left[\frac{1 - \frac{\ln(1+\psi_{i+1}/C_r)}{\ln(1+10^6/C_r)}}{\left\{\ln\left[e + (\psi_{i+1}/a_f)^{n_f}\right]\right\}^{m_f}} - \frac{1 - \frac{\ln(1+\psi_i/C_r)}{\ln(1+10^6/C_r)}}{\left\{\ln\left[e + (\psi_i/a_f)^{n_f}\right]\right\}^{m_f}}\right]. \tag{5}$$

initial void ratios and different confining pressures, the pore-size distribution function of soil changes with the variation in the void ratios. As a result, Zhai and Rahardjo (2013)'s method is inapplicable for simulating changes in the pore-size distribution function of soil. The SWCC for a denser soil can be estimated from that for the same soil in a relative looser condition, and vice-versa. Consequently, in the estimation of the unsaturated shear strength of soil with different void ratios, SWCC1 obtained from a direct measurement with zero confining pressure (i.e. using the conventional Tempe cell and pressure plate) needs to be modified to SWCC2 (say 50 kPa confining pressure, σ_3-u_a) and SWCC3 (say 300 kPa confining pressure, σ_3-u_a), as shown in Fig. 7a. As a result, those modified SWCCs will have higher air-entry values (AEVs) to represent more reasonable pore-size distribution functions under higher confining pressures. The estimated unsaturated shear strengths from those SWCC1, SWCC2, and SWCC3 are illustrated in Fig. 7b by assuming the effective angle of internal friction of soil to be 35°.

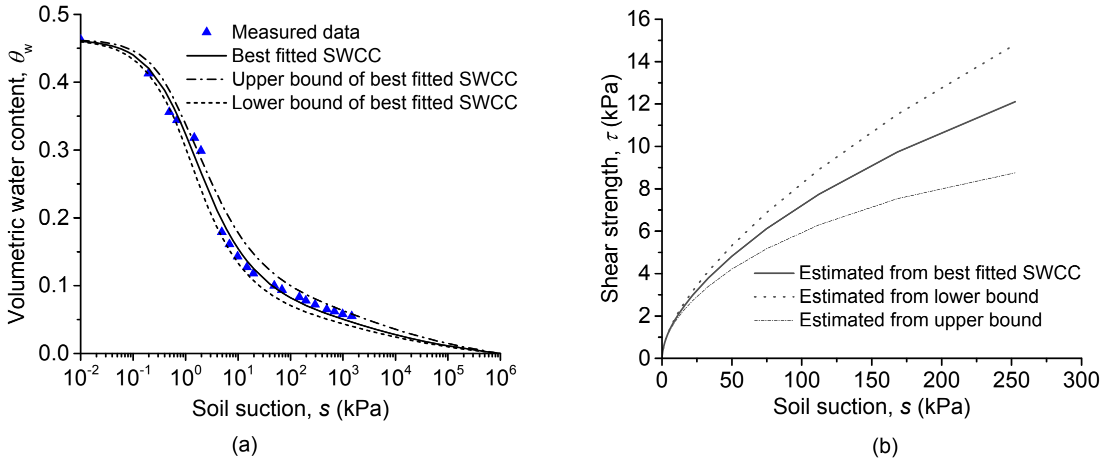

Fig. 6 Illustration of the estimation of unsaturated shear strength incorporating the variability of SWCC
(a) Variability of SWCC; (b) Variability of estimated shear strength for the unsaturated soil

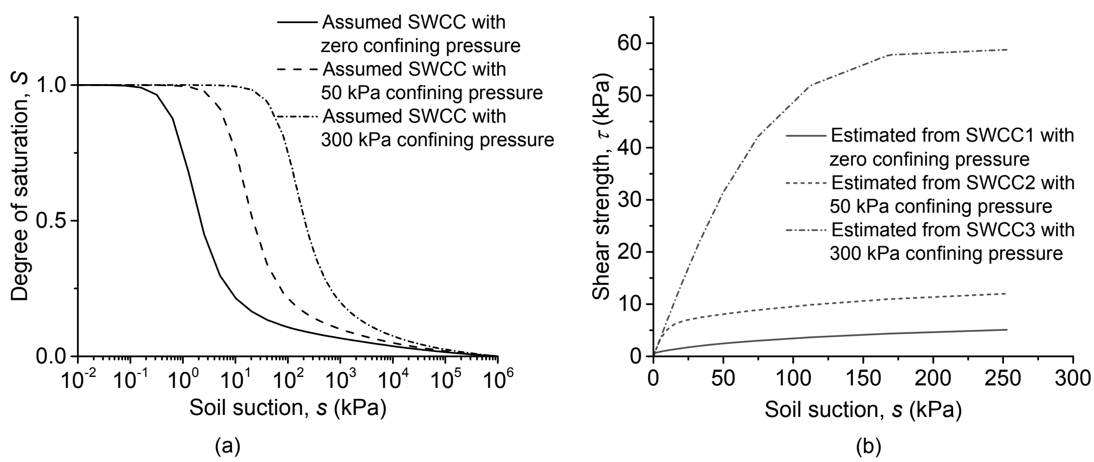

Fig. 7 Illustration of the estimation of unsaturated shear strength incorporating different void ratios
(a) Assumed SWCCs with different confining pressures; (b) Estimated shear strength from SWCCs with different confining pressures

5 Verification with experimental data

Three types of soil including the compacted residual soil from Bukit Timah Granite from Rahardjo et al. (2004), the weathered granite from Lee et al. (2005), and Indian Head Hill from Vanapalli et al. (1996) were selected for the verification of the method proposed in this study. The SWCCs and measured unsaturated shear strengths for these three types of soil are illustrated in Figs. 8 and 9. The SWCCs for the compacted residual soil from Bukit Timah Granite were obtained using two measurement

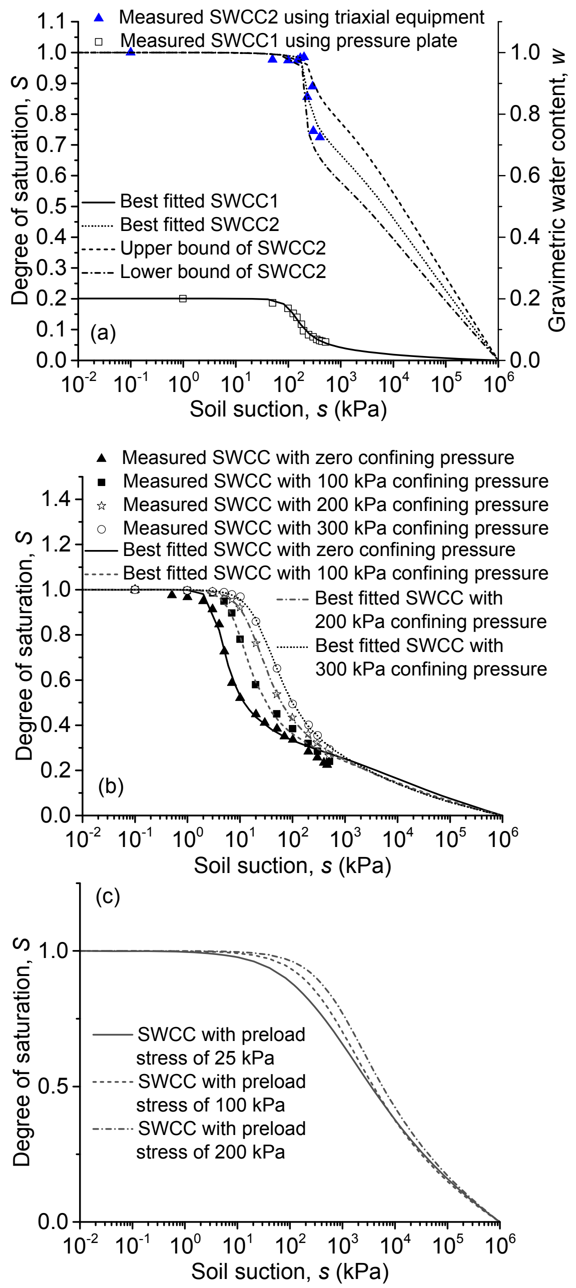

Fig. 8 Soil-water characteristic curves for these three types of soil

(a) Compacted residual soil from Bukit Timah Granite from Rahardjo et al. (2004); (b) Weathered granite from Lee et al. (2005); (c) Indian Head Hill from Vanapalli et al. (1996)

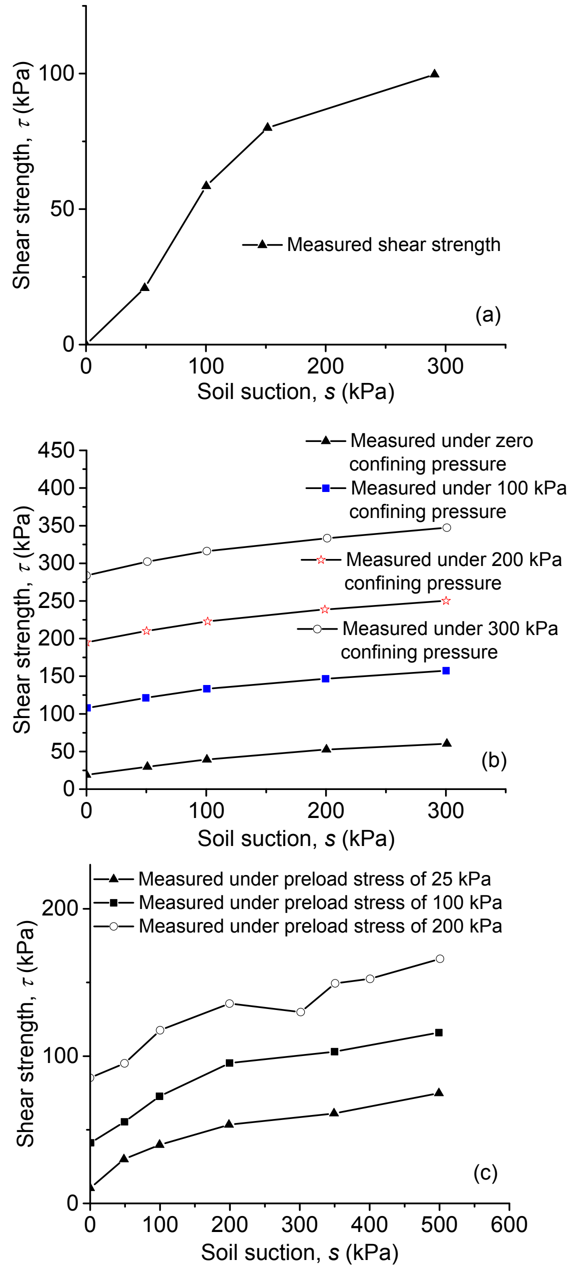

Fig. 9 Results of unsaturated shear strength for these three types of soil

(a) Compacted residual soil from Bukit Timah Granite from Rahardjo et al. (2004); (b) Weathered granite from Lee et al. (2005); (c) Indian Head Hill from Vanapalli et al. (1996)

techniques: (1) pressure plate which yields the SWCC in the form of gravimetric water content, (2) triaxial equipment which yields the SWCC in the form of degree of saturation by considering the soil volume change during the test. Both SWCCs for the compacted residual soil from Bukit Timah Granbite as obtained from the pressure plate, SWCC1, and triaxial test, SWCC2, are illustrated in Fig. 8a. The upper bound and lower bound of the determined SWCC2 are also computed using the equations from Zhai and Rahardjo (2013) as illustrated in Fig. 8a. As there are only the fitting parameters rather than the experimental data reported for the Indian Head Hill, the SWCCs for the Indian Head Hill are calculated from these fitting parameters as reported by Vanapalli et al. (1996) as illustrated in Fig. 8c. The fitting parameters in Fredlund and Xing (1994)'s equation are summarized in Table 1.

The effective angles of internal friction for the compacted residual soil from Bukit Timah Granite, weather granite, and Indian Head Hill were reported as 31.5°, 41.4°, and 23.0°, respectively, as shown in Table 1. The fitting parameters in Table 1 were subsequently used to estimate the unsaturated shear strengths for those soils using Eq. (5). As there are two methods adopted to obtain the SWCC for the compacted residual soil from Bukit Timah Granite, both SWCC1 (which is obtained from the pressure plate) and SWCC2 (which is obtained from the unsaturated triaxial equipment) were used to estimate the unsaturated shear strength. The comparison between the estimated unsaturated shear strengths and the measured data for the compacted residual soil from Bukit Timah Granite is illustrated in Fig. 10a. The comparisons between the estimated unsaturated shear strength and experimental data for the weathered granite under different confining pressures and Indian Head Hill with different preload stresses are illustrated in Figs. 10b and 10c, respectively. The coefficient of determination, R^2, which quantifies the agreement between the estimated results and measurement data, is also shown in Fig. 10.

Fig. 10 indicates that the estimated unsaturated shear strengths agree well with the experimental data for these three types of soil, and coefficients of determination, R^2, are relatively high for all the estimations. Fig. 10a indicated that most of the experimental data were observed within the band of the confidence limits (with confidence level of 95%). The upper and lower confidence limits are governed by the confidence level and all the experimental data would be within the band if a lower confidence level was adopted. It is interesting to observe in Fig. 10a that the estimated results from the SWCC2 agree well with the measured data while the estimated results from the SWCC1 underestimate the unsaturated shear strength for the compacted residual soil from Bukit Timah Granite. It is known that most SWCC data were collected using the conventional methods of Tempe cell and pressure plate with zero confining pressure. However, those measured SWCCs with zero confining pressure are commonly used to estimate the unsaturated shear strength of soil at different stress levels. Fig. 10a illustrates that it is risky to estimate the unsaturated shear strength with high stress level using the SWCC results obtained from the pressure plate test. In order to understand the errors associated with the estimated unsaturated shear strength under high stress levels using the SWCC with zero confining pressure, the unsaturated shear strength for the

Table 1 Fitting parameters of the SWCCs and effective friction angles of the soils

Soil	a_f (kPa)	n_f	m_f	C_r (kPa)	ϕ' (°)
Compacted residual soil from Bukit Timah Granite SWCC1 using pressure plate	108.48	3.66	0.71	1500	31.5
Compacted residual soil from Bukit Timah Granite SWCC2 using triaxial equipment	204.30	30.00	0.08	1500	31.5
Weathered granite with confining pressure of 0 kPa	3.26	4.12	0.41	1500	41.4
Weathered granite with confining pressure of 100 kPa	7.67	2.60	0.53	1500	41.4
Weathered granite with confining pressure of 200 kPa	14.72	2.07	0.61	1500	41.4
Weathered granite with confining pressure of 300 kPa	23.85	1.89	0.65	1500	41.4
Indian Head Hill with preload stress of 25 kPa	34.10	0.80	0.57	3000	23.0
Indian Head Hill with preload stress of 100 kPa	71.40	0.66	0.54	3000	23.0
Indian Head Hill with preload stress of 200 kPa	125.20	0.81	0.45	3000	23.0

weathered granite and Indian Head Hill are recalculated using a single SWCC (which corresponds to the lowest confining pressure or preload stress) and the results are illustrated in Fig. 11.

Fig. 11 shows that R^2 decreases significantly for both soils if only the SWCC with a low confining pressure or preload stress is used for the estimation. Fig. 11 indicates that the errors in the estimated unsaturated shear strength increase with the increase in suction level and stress level. As a result, the SWCC obtained from the conventional measurement methods like the Tempe cell and pressure plate should be corrected by considering different confining pressures or different initial void ratios before being used for the estimation of the unsaturated shear strength.

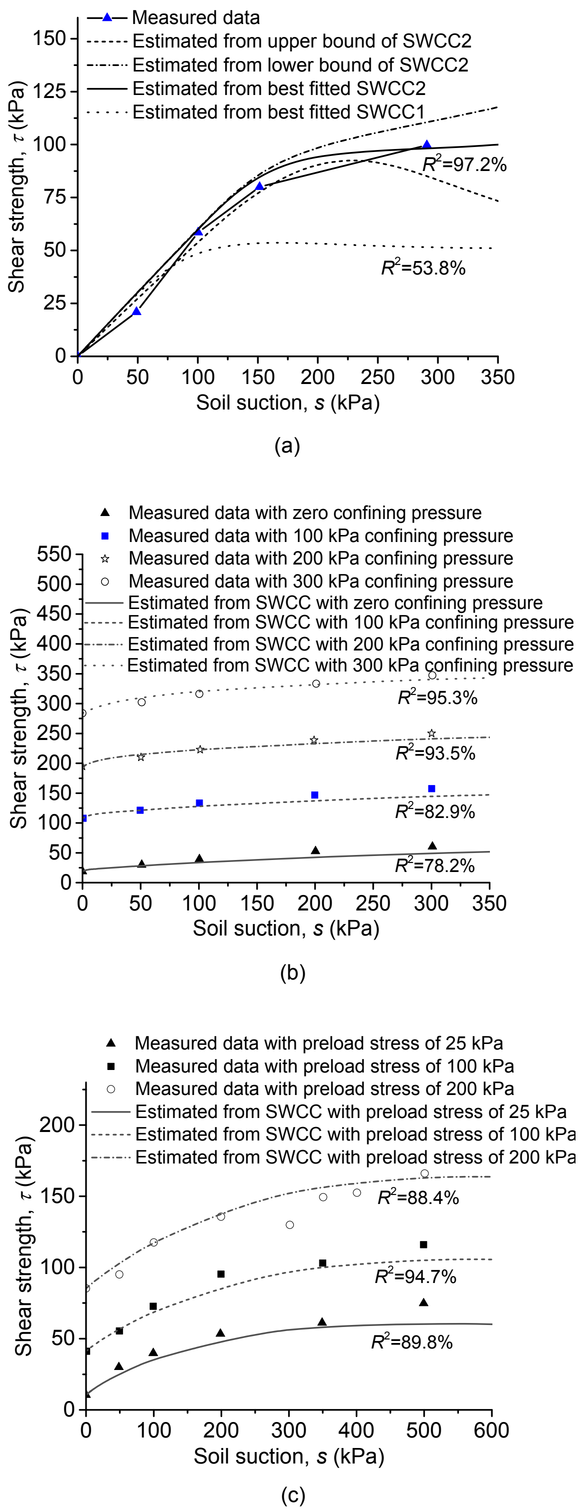

Fig. 10 Comparison between estimated and measured unsaturated shear strengths for these three types of soil
(a) Compacted residual soil from Bukit Timah Granite; (b) Weathered granite; (c) Indian Head Hill

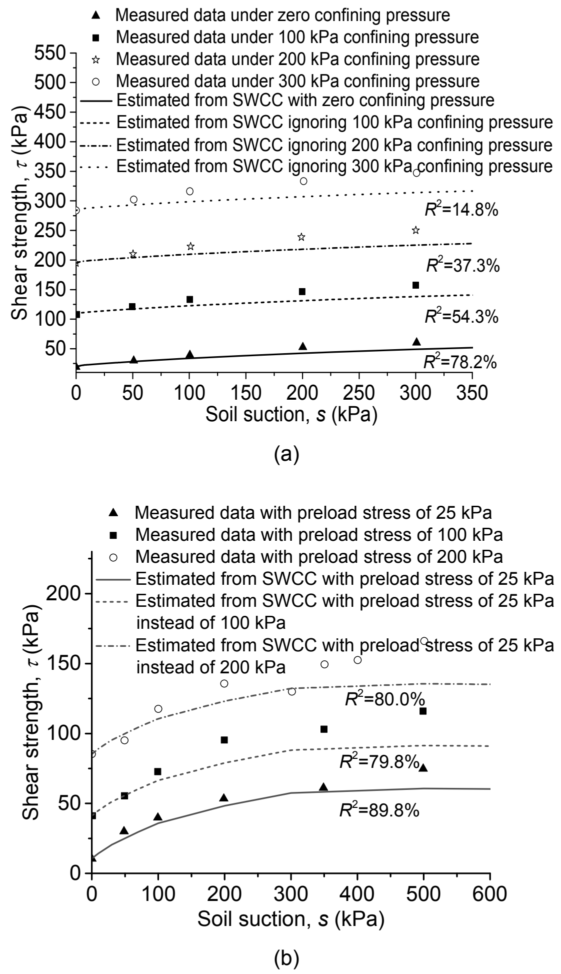

Fig. 11 Estimated shear strength for unsaturated soil from SWCC ignoring confining pressure or with a low preload stress
(a) Weathered granite; (b) Indian Head Hill

The variability of the estimated shear strength of unsaturated soil can also be affected by different mathematical equations (or models) used in the estimation of the shear strength. In this study, only one estimation equation (i.e. Zhai et al. (2019a)'s equation) was adopted in the analysis and the uncertainties in the estimated results due to different mathematical equations (or models) are eliminated. In addition, Vanapalli et al. (1996) discussed the effects of residual suction on the estimated shear strength of the unsaturated soils. In this study, the immobile water was estimated from the degree of saturation with respect to suction of 3100 kPa. Therefore, the residual suction was fixed at 3100 kPa in this study. As a result, the effect of the variation of the residual suction on the estimated unsaturated shear strength was not considered in this study.

6 Conclusions and recommendations

The variability of SWCC due to different factors such as natural spatial variability, measurement errors, measurement techniques, operators, temperature, initial water content, initial void ratio, and different confining pressures were introduced. The methods for the quantification of the estimated unsaturated shear strength due to the variation of the SWCC were proposed in this study. The proposed methods were verified with experimental data from previous studies. It is recommended that SWCC obtained from the conventional Tempe cell or pressure plate should be corrected by considering the different void ratios or different confining pressures for the estimation of the unsaturated shear strength. It is observed that the accuracy in the estimated unsaturated shear strength for a soil specimen under a high confining pressure can decrease significantly if the SWCC with a low confining pressure is adopted for the estimation.

Contributors

Qian ZHAI wrote the first draft of the manuscript and edited the final version, Harianto RAHARDJO and Alfrendo SATYANAGA helped to organize the manuscript, and Guo-liang DAI and Yan-jun DU revised and edited the final version.

Conflict of interest

Qian ZHAI, Harianto RAHARDJO, Alfrendo SATYANAGA, Guo-liang DAI, and Yan-jun DU declare that they have no conflict of interest.

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中文概要

题　目：水-土特征曲线的不确定性对估算非饱和土抗剪强度的影响

目　的：系统讨论引起水-土特征曲线不确定性的各种可能因素，深入探讨水-土特征曲线的不确定性对

非饱和土体抗剪强度的影响，并总结采用水-土特征曲线估算非饱和土体抗剪强度所需要考虑的关键因素。

创新点： 在采用水-土特征曲线计算非饱和土抗剪强度时，综合考虑水-土特征曲线的不确定性。

方　法： 1. 查阅文献，对比试验数据，总结造成水-土特征曲线不确定性的主要因素；2. 依据现有估算公式，采用不同水-土特征曲线估算非饱和土抗剪强度；3. 对比估算结果和试验测量值，讨论估算过程需要注意的关键事项。

结　论： 1. 同一土样在不同测试方法、不同试验环境及不同初始空隙比的情况下，所获得的水-土特征曲线可能表现各异；2. 在估算非饱和土抗剪强度时，必须考虑土体的应力状态（或者空隙比），并对现有水-土特征曲线做必要修正，以保证在估算过程中所采用的水-土特征曲线能够真实地反映剪切土样的孔径分布。

关键词： 非饱和土抗剪强度；孔径分布函数；不确定性；水-土特征曲线；置信度