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Thermal strain response of saturated clays in 1D condition*#

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1 Introduction

In recent years the thermomechanical behavior of soils has attracted more attention due to an increasing number of geotechnical practices involving thermal effects (Bourne-Webb et al., 2013; Olgun et al., 2014; Loria et al., 2018; Peng et al., 2018; Pinyol et al., 2018; Kaddouri et al., 2019; Mu et al., 2019). Geomechanical applications for these problems require an understanding of the thermomechanical behavior of soils and their numerical modelling.

Based on experimental observations, various thermomechanical models have been developed to interpret the observed thermal-related yield mechanisms and reproduce the temperature-stress-strain relationships of soils. For example, Hueckel and Borsetto (1990) first proposed a critical state-based thermoplasticity model. Cui et al. (2000) developed a thermomechanical model with two yield surfaces: the loading yield surface and the thermal yield surface. They paid particular attention to the coupling and hardening phenomena under the combined effects of stress and temperature. Graham et al. (2001) developed a non-isothermal model, in which the yield loci do not change in shape but change in the slopes of unload-reload lines when undergoing variation in temperature. Similarly, Abuel-Naga et al. (2009) proposed an isotropic thermomechanical model with two yield surfaces to capture the volumetric plastic strain induced by both mechanical and thermal loading based on the results of Bangkok clay. Laloui and François (2009) developed an advanced thermomechanical model (ACMEG-T), which adopted multi-mechanism plasticity and bounding surface theory: the isotropic and deviatoric mechanism. Yao and Zhou (2013) presented a non-isothermal unified hardening model to interpret the thermo-elastic-plastic behavior of clays, especially overconsolidated clays. Hong et al. (2016) presented an advanced thermomechanical model in the framework of elasto-plasticity theory and based on the model proposed by Cui et al. (2000). In addition, some other non-isothermal thermomechanical models have also been developed (di Donna, 2014; Zhou et al., 2017; Bai et al., 2019; Zhu et al., 2020). Among these models, only a few of them can capture the accumulation plastic strain of soil under cyclic heating and cooling. The model developed by di Donna (2014) incorporates nesting surfaces in order to consider the cyclic effect. Multimechanism plasticity was adopted, making the model more complex. Zhou et al. (2017) proposed a new bounding surface model to simulate thermal cyclic behavior. The model is simpler and has fewer parameters. Although these models can reproduce the response of soils under cyclic heating and cooling, their mechanism analysis of thermoplasticity is insufficient.

The main purpose of this study is to interpret the thermoplastic volumetric response of saturated clay during heating and cooling based on thermoplasticity. A two-yield-surface model for describing the thermomechanical behavior of both normally consolidated and overconsolidated saturated clay is proposed. Compared with similar existing models, the novelty

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of the proposed model lies mainly in two aspects: (a) a new equation directly expressing the thermoplastic strain with one additional parameter is proposed which is related to the stress condition and temperature increment; (b) a newly defined coupling mechanism of thermal and mechanical surfaces is used which is more concise. The capabilities of the proposed models to describe the observed experimental behavior were analyzed by predicting the thermal deformation of illite clay and loess suffering thermomechanical loading. Specifically, the accumulated volumetric strains in 1D conditions after multiple heating and cooling cycles were simulated and discussed.

The theory of the proposed thermomechanical model is provided in Data S1, including all constitutive equations and explanations of variables. The performance of the model and its validations are presented and discussed.

2 Results and discussion

2.1 Temperature-induced volume change

For evaluating the performance of the proposed thermomechanical model, a set of synthetic tests was generated adopting a hypothetical clay.

The hypothetical clay was first consolidated to preconsolidation pressure at reference temperature T_0, $p_c(T_0)$ (point *A* in Fig. 1) and then unloaded to $p_c(T_0)/2$, $p_c(T_0)/4$, and $p_c(T_0)/8$ at T_0=20 °C, to generate samples with overconsolidated ratio, OCR=1, 2, 4, and 8, respectively. Then, ten heating and cooling cycles (20 °C–90 °C –20 °C) were applied to the four samples: (a) Thermal loading path *AE*. At point *A* (the normally consolidated state), the proposed thermal yield surface (TY) and mechanical yield surface (MY) coincides, and both are reached immediately. Upon heating from point *A*, $p_c(T_0)$ evolves as the development of thermoplasticity and mechanical plasticity. (b) Thermal loading paths *ABF* and *ACG*. At points *B* and *C*, the soil is intermediately consolidated (OCR=2 and 4). Upon heating from T_0, thermal expansion dominated before reaching the TY limits. Afterward, thermoplasticity deformation develops beyond the aforementioned transition temperature, and then $p_c(T_0)$ evolves. (c) Thermal loading path *ADH*. For heavily overconsolidated soil (OCR=8), thermal expansion only occurs upon heating within the concerned temperature range. As no plastic volumetric strain will be produced along this loading path, $p_c(T_0)$ will not harden.

The response of the hypothetical clay due to thermal cyclic loading was predicted quantitatively by the proposed model with the assumed parameters as: e_0=1.0, λ=0.1, κ=0.02, θ=0.15, $\alpha=1.0\times10^{-5}$ °C^{-1}, $\omega=1.5\times10^{-5}$ °C^{-2}, β_0=5.0. Here, e_0 is the initial value of void ratio e, λ and κ are the normal compression and recompression slopes in e-ln$p$ plane, where p is the pressure. θ is a thermal-related material parameter, α is the volumetric thermal elastic expansion coefficient in drained condition, ω is a thermal related parameter controlling the development of thermal volumetric plastic strain, and β_0 is the initial value of β, which is a material parameter expressing the state of thermal yielding surface. The predicted relationships between the volumetric strain and the temperature for the first thermal cycle are shown in Fig. 2. The predicted results agree with the observed thermal phenomenon reported in the literature (Demars and Charles, 1982; Baldi et al., 1991; Sultan et al., 2002).

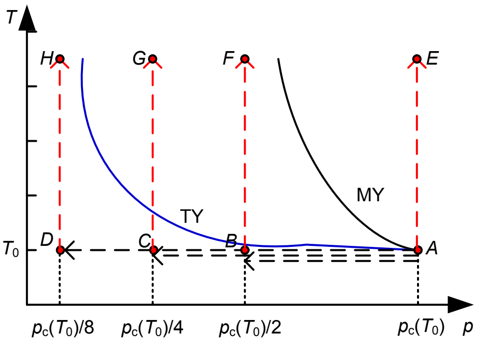


Fig. 1 Explanation of model responses subjected to heating and cooling at different OCR conditions

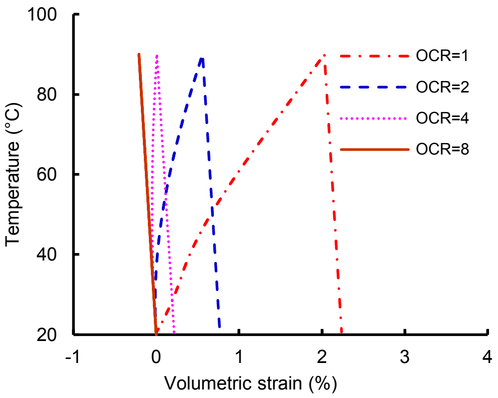


Fig. 2 Thermal volumetric deformation under one heating and cooling cycle at different OCRs

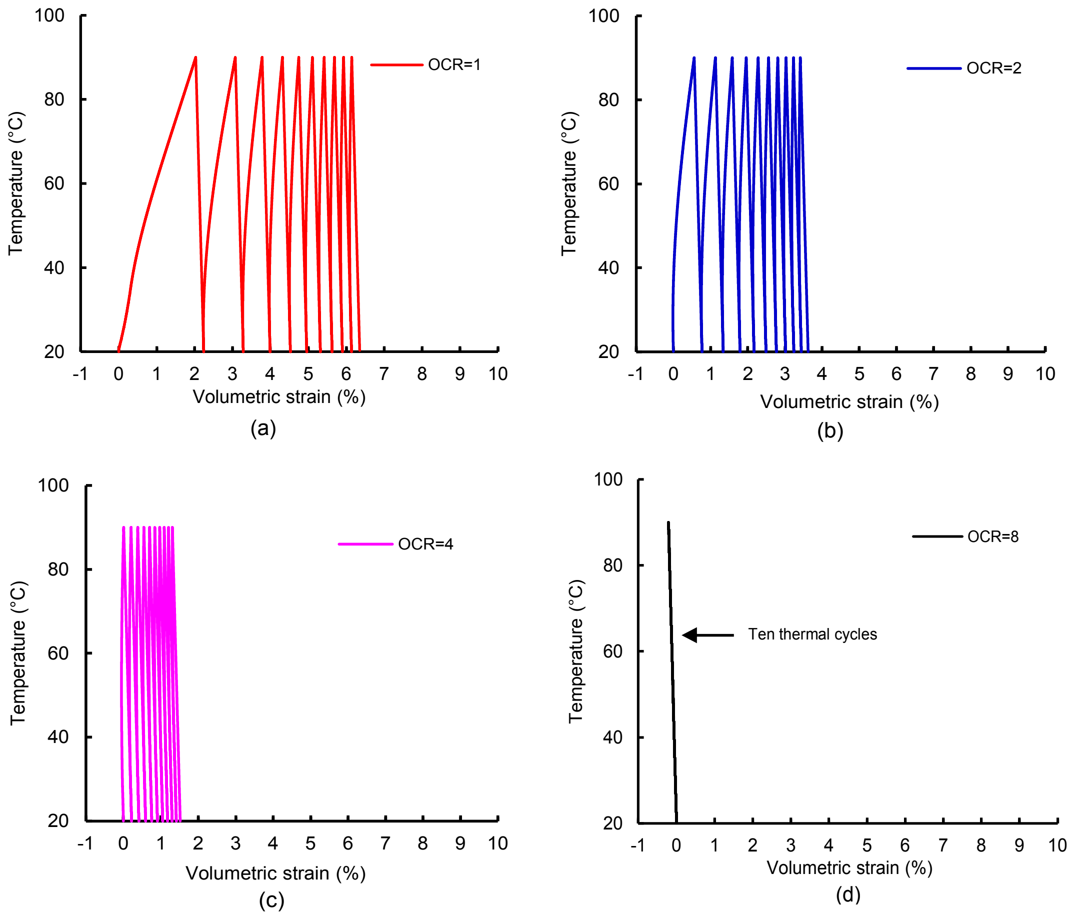
2.2 Influence of thermal cyclic loading

As introduced above, the simulated ten thermal cycles were applied to hypothetical clay under different OCR conditions. The results in terms of volumetric strain with temperature are presented in Fig. 3. For normally and intermediately consolidated states, a volumetric contraction corresponding to thermoplasticity occurs after the first heating and cooling phase. The responses of the three tests under OCR=1, 2, 4 are qualitatively similar, but show large quantitative differences. As the number of thermal cycles increases, the total thermoplastic deformation gradually increases, but the deformation for each cycle gradually decreases. The highly overconsolidated test (OCR=8) shows a thermoelastic response, with a reversible deformation which corresponds to the thermoelastic expansion and contraction of the solid skeleton. The model predictions are in accordance with the experimental results (di Donna, 2014).

2.3 Validation of the model

2.3.1 Illite clay

di Donna (2014) conducted a thermal cyclic test on a reconstituted illite clay. The initial void ratio e_0 of the sample consolidated at 60 kPa is equal to 1.43. For the test procedure, the tested sample was first saturated. Then, the sample was normally consolidated in the oedometric cells at target stresses and then subjected to drained thermal cycles under normal consolidation conditions. The experimental results in terms of volumetric strain during thermal cycles are shown in Fig. 4. The volumetric strain here denotes the vertical strain as the tests were conducted using an oedometer. Due to the thermal expansion and contraction of the oedometer ring, it is difficult to accurately simulate the response of horizontal stress and strain with temperature (Zhou et al., 2017; Zhou and Ng, 2018). For simplification, the horizontal strain is



(a)

(b)

(c)

(d)

Fig. 3 Thermal volumetric deformation under multiple heating and cooling cycles

(a) OCR=1; (b) OCR=2; (c) OCR=4; (d) OCR=8 assumed to be zero. For the prediction, the conventional parameters λ=0.12, κ=0.02, e_0=1.43, and $p_c(T_0)$=60 kPa are obtained from di Donna (2014). Due to the lack of sufficient tests to determine the model parameters, the first thermal cyclic test on the consolidated sample is used for calibrating the other model parameters as shown in Table 1. Thus, the simulations on the second heating-cooling phase are a blind-prediction. From the comparisons, the model prediction correctly reproduces the thermal cycles for multi-heating and cooling phases. In addition, the accommodation phenomenon is reproduced correctly by reducing the increment of irreversible deformation. The final plastic deformation is also satisfactorily predicted.

Table 1 Model parameters for the selected clays

Clay	θ	α ($\times10^{-5}$ °C^{-1})	ω ($\times10^{-5}$ °C^{-2})	β_0
Illite clay	0.10	2.0	0.36	5.0
Loess	0.17	2.5	0.23	6.0

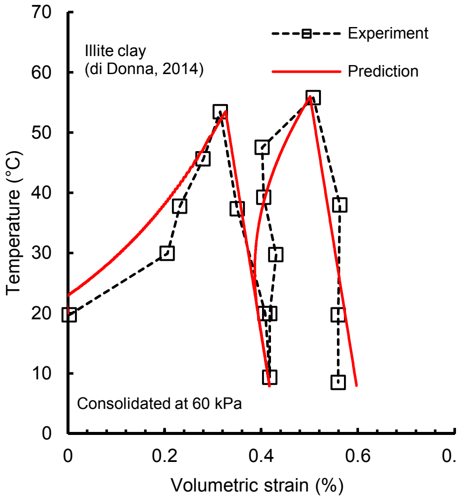


Fig. 4 Modelling the thermal volumetric behavior of illite clay under two thermal cycles: soil normally consolidated at 60 kPa

2.3.2 Loess

Ng et al. (2019) investigated the cyclic thermal behaviour of reconstituted, intact, and recompacted loess using a temperature-controlled oedometer. To avoid the influence of soil structure, only the tests on reconstituted loess were used for simulation. The reconstituted loess sample was first consolidated at an effective vertical stress of 50 kPa. Then, it was subjected to multi-cycles at temperatures ranging from 15 to 70 °C.

For the modelling, the values of parameters λ=0.09, κ=0.006, e_0=0.812, and $p_c(T_0)$=50 kPa were determined from the compression tests in (Ng et al., 2019). Table 1 shows the other parameters calibrated from the first thermal cyclic test in Fig. 5. The latter three thermal cyclic tests were then used for blind-predicting. Comparing the experiment and the predicted results, the calculated total thermal strain is consistent with the test observation. However, the thermal strains after the second and third thermal cycles are underestimated. This may be related to the simplified simulation method in the 1D stress condition or the parameter-determined method. Generally, the proposed model can be used for modelling the thermal strain of saturated loess.

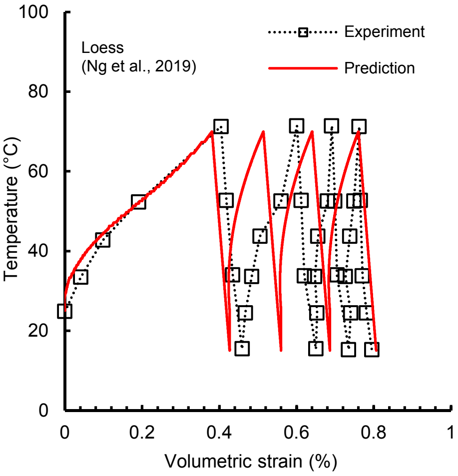


Fig. 5 Modelling the thermal strain of normally consolidated loess under multi-cycles

2.4 Discussion

The models developed by Cui et al. (2000) and Abuel-Naga et al. (2007) are well-known in the thermomechanical modeling of saturated clay. Comparing with them, the proposed model also used two separate yield surfaces to describe the yielding behavior in p-T plane. However, the proposed model presented different theoretical mechanisms to interpret thermoplasticity.

It is always a challenge to reproduce the response of saturated clay under thermal cyclic loading. The difficulty lies in the modelling of the accommodation phenomenon, i.e. the accumulation of plastic deformation after multithermal cycles performed between the maximum and minimum temperature. In particular, an increment of plastic deformation must be produced during the phase of reheating. The amount of plastic deformation should be smaller cycle after cycle, tending to zero after a certain number of cycles. In the present model, this is introduced by allowing the mobilization of thermoplasticity of the thermal yield mechanism evolving during cooling. This can be described as a re-initialization of the yield locus (controlled by β) at which thermoplasticity will regenerate along with the next heating process.

The comparisons between the predicted and experimental thermal strains on illite clay and loess under cyclic heating and cooling show that the proposed model overestimates/underestimates the thermal strain induced by the second and subsequent thermal cycles. This may be due to two reasons. One is that there are fewer cyclic thermal tests conducted systematically, and the dispersion of test results is difficult to fit the reasonable model parameters. As a result, the calibration of model parameters may deviate from reality. The other is that it attributes to parameter β, the key parameter that determines the thermoplastic triggering. The evolution of β needs to satisfy the consistency equation, which limits the flexibility of the model.

3 Conclusions

A two-surface-based thermomechanical constitutive model is proposed under the classic elastoplastic theory framework for describing the volumetric thermo-plasticity of saturated soils in a 1D condition. The model has six materials and two state variables, which have clear physical interpretations and can be easily determined based on two compression tests under different temperatures, and two drained volumetric heating/cooling tests under different OCR conditions.

The performance of the model in reproducing the main features of clay thermomechanical volumetric behavior was studied by a set of synthetic tests adopting a hypothetical clay. The capabilities of the model were also examined by comparing with the selected temperature-controlled experimental results on illite clay and loess. The main features include thermal volumetric deformation of the soil at different OCR values in multi-heating-cooling cycles. All comparisons between the experimental results and the numerical simulations demonstrate that the proposed model is capable of reproducing the thermal volumetric behavior of clays in different consolidated states and heating-cooling cycles.

Contributors

Qi-yin ZHU processed the corresponding data. Tian-yu ZHAO wrote the first draft of the manuscript. Pei-zhi ZHUANG revised and edited the final version.

Conflict of interest

Qi-yin ZHU, Tian-yu ZHAO, and Pei-zhi ZHUANG declare that they have no conflict of interest.

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List of electronic supplementary materials

Data S1 Theory