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A new scheme of coded ultrasound using Golay codes*

Cheng JIN†1,2, Si-ping CHEN†‡1, Zheng-di QIN1, Tian-fu WANG1
(1School of Medicine, Shenzhen University, Shenzhen 518060, China)
(2School of Biomedical Engineering and Instrument Science, Zhejiang University, Hangzhou 310027, China)
†E-mail: jincheng@szu.edu.cn; chensiping@szu.edu.cn

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Abstract: Golay codes are the most practical code in coded ultrasound imaging systems. But the trade-off for perfect range sidelobe cancellation is the requirement for two firings, thus resulting in motion-dependent decoding errors. In view of this, we propose a new scheme using the simultaneous emission of code pairs. The code pair is allocated to different elements of an aperture and transmitted simultaneously. The process of separating the code pair from the echo received is based on the orthogonality of the code pair. At last the autocorrelation functions of the individual Golay codes are added together. The simultaneous emission of code pairs instead of two firings recovers the frame rate loss, and eliminates the motion-dependent decoding error. Our theoretical analysis and simulations show that the scheme can be used to eliminate the tissue motion effects.

Key words: Coded ultrasound, Golay codes, Tissue motion, Ultrasonic imaging
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1 Introduction

Coded signals, as a research topic of long history in medical ultrasound (Takeuchi, 1979; O′Donnell, 1992; Chiao and Hao, 2005; Cowell and Freear, 2008), are primarily used to either improve the signal-to-noise ratio (SNR) or increase the frame rate in ultrasound imaging (Misaridis and Jensen, 2005; Gran and Jensen, 2008).

Under extensive investigations for ultrasound imaging are the binary codes such as Barker codes (Welch and Fox, 1998; Zhao *et al.*, 2007) and m-sequences (Shen and Ebbini, 1996). However, the most promising codes seem to be the ones based on Golay pairs (Leavens *et al.*, 2007a; Lewandowski and Nowicki, 2008; Pezeshki *et al.*, 2008) which have been tried by many research groups around the world. Takeuchi (1979) proposed and described a system using Golay codes. A research group from General Electric Company has been investigating the use of such codes (Chiao and Thomas, 2000; Chiao and Hao, 2005).

The trade-off for perfect range sidelobe cancellation is the requirement for two firings, which leads to the motion-dependent decoding error and may reduce the frame rate by a factor of two. This frame rate loss can be recovered for pairs of focal zones by using orthogonal Golay codes (Chiao and Thomas, 2000) or other methods (Kim *et al.*, 2002; Leavens *et al.*, 2007b). However, the motion-dependent decoding error cannot be eliminated with the existence of motion between the emissions, resulting in less range sidelobe degradation. Some researchers (Cannon *et al.*, 2008) have presented a method of motion compensation for complementary coded signals, but the method cannot eliminate the motion-dependent decoding error. In this paper we propose a scheme to solve the problem of two firings. The simultaneous emission of a code pair instead of two firings is used to transmit the code pair, and thus the tissue motion effects are eliminated.

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‡ Corresponding author

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2 The proposed new scheme

2.1 Golay pair and tissue motion effects

The idea behind the use of Golay codes is illustrated in Fig. 1. Two binary codes, *A* and *B*, are necessary. Each of the binary codes has an autocorrelation function, whose side lobes are equal in magnitude and opposite in sign to the corresponding side lobes of the other code (Dokovic, 1998). Thus, the sum of the autocorrelation functions results in cancellation of the sidelobes. Under ideal conditions, temporal coding using Golay sequences would lead to ideal resolution and contrast.

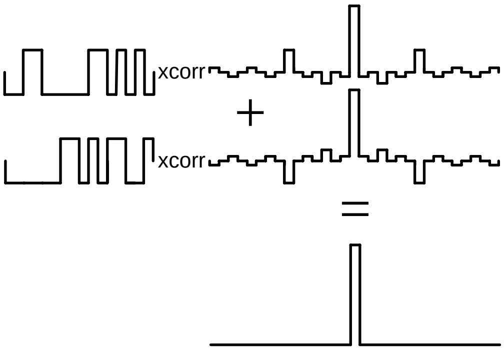

Fig. 1 Pulse compression using Golay codes

Golay codes assume stationarity between the two emissions for the complementarity condition to hold. With tissue motion, the range sidelobe cancellation is imperfect between the two transmits, and obvious range sidelobes appear. Thus the decoding will suffer from motion artifacts, rendering Golay coding sensitive to imaging moving objects.

We assume that the point spread function (PSF) of the ultrasound images does not change significantly for small changes in the axial position (for simplicity, only the changes in the axial position were considered):

$$p_0(x, y, z)=p_0(x, y, z+\Delta z), \qquad (1)$$

where p_0 is the PSF, and Δz is the change in the axial position. It means that the PSF at position $(x, y, z+\Delta z)$ is merely a translated version of the PSF at position (x, y, z).

Fig. 2 shows the simulated effect of axial motion on range sidelobes. Fig. 2a shows the positions of the scatterers and the PSFs of the images obtained at the first emission. Fig. 2b gives the positions of the scatterers and the PSFs of the images obtained at the second emission when there is motion between the emissions. Fig. 2c illustrates the resulting images obtained by two firings when scatterers are stationary between the two emissions of the Golay pair. The resulting images obtained by two firings are presented in Fig. 2d. Note that the motion-dependent decoding error is obvious.

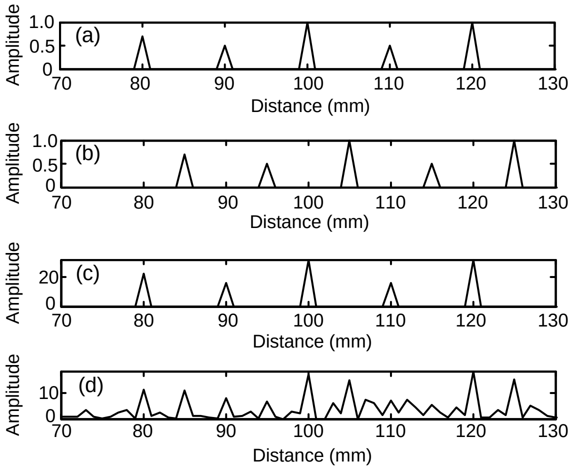

Fig. 2 Simulation results of tissue motion effects

Positions of the scatterers and point spread functions of the images at (a) the first emission and (b) the second emission. Resulting images obtained by two firings when (c) scatterers are stationary and (d) there is motion between the emissions

2.2 New scheme

The essential reason of the existence of the motion-dependent decoding error is the two firings of the Golay pair. If the simultaneous emission of the code pair can be implemented, the error will be eliminated. There are two key points in considering one firing of the Golay pair:

1. How to emit Golay pair simultaneously?

2. How to distinguish two codes from the echo received?

We propose a new scheme (Fig. 3): the code pair is allocated to different elements of an aperture and transmitted simultaneously. And the process of separating the code pair from the echo received is based on the orthogonality of the code pair.

Golay codes G_1 and G_2 are modulated on carrier waves with the same center frequency. Subsequently the modulated signals can be allocated to different elements of a transmit aperture and transmitted simultaneously. The signals received from the generated field can then be expressed as the convolution of the transmitted signals and a wave propagation operator.

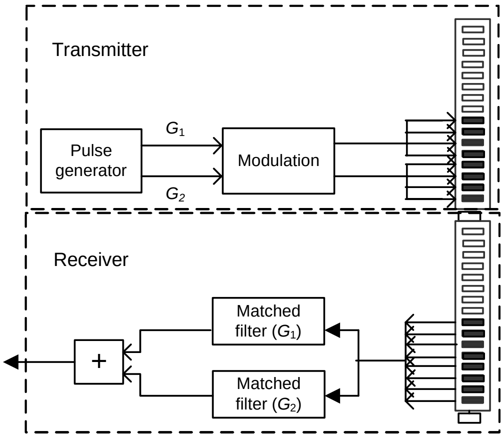

Fig. 3 Illustration of the proposed scheme

Next, we separate the two Golay codes from the echo received by using the orthogonality of the code pair. In the separation process we perform matched filtering for separated signals. The outputs of matched filters are finally summed, and thus the imaging result is obtained. In the following part we will make a theoretical analysis of the scheme.

2.3 Theoretical analysis of the scheme

A typical set of complementary Golay sequences is given by: G_1 ($g_1(n)$, n=1, 2, ..., N), G_2 ($g_2(n)$, n=1, 2, ..., N). The transmitting signals are denoted as

$$T_{x1}(t)=G_1(t)\cdot \mathrm{e}^{\mathrm{j}2\pi f_0 t},\ T_{x2}(t)=G_2(t)\cdot \mathrm{e}^{\mathrm{j}2\pi f_0 t}, \qquad (2)$$

where f_0 is a baseband pulse waveform. Let the medium at a given depth r be characterized by its impulse response, a scattering (or reflectivity) function $h(t, r)$. For instance, the impulse response of a single broadband (point) scatterer located at depth r_i is a delta function (Jensen, 1996):

$$h(t,r_i)=k(r_i)\cdot\delta(t-r_i/c), \qquad (3)$$

where k is generally a function of depth incorporating acoustic diffraction and attenuation, and c is the speed of sound. Assuming that the impulse responses are short, the medium's scattering function $h(t)$ will be the sum of the impulse responses for all depths. The returned signal $R_x(t)$ will then be the sum of the convolution of the scattering function $h(t)$ with the excitation signal, i.e.,

$$R_x(t)=T_{x1}(t)*h(t)+T_{x2}(t)*h(t), \qquad (4)$$

where '*' is the convolution operator. Substituting Eq. (3) in Eq. (4), the signal returned from a single scatterer is simply a scaled time-shifted version of the transmitted signal:

$$R_x(t)=(G_1(t-\tau_0)+G_2(t-\tau_0))\cdot k\cdot \mathrm{e}^{\mathrm{j}2\pi f_0(t-\tau_0)}, \qquad (5)$$

where $\tau_0=r_i/c$ is the time instant after the start of transmission (t=0), at which the signal is being received.

If frequency-dependent attenuation is considered and a downshift of frequency is f_d, we obtain

$$R_x(t)=(G_1(t-\tau_0)+G_2(t-\tau_0))k\,\mathrm{e}^{\mathrm{j}2\pi(f_0-f_d)(t-\tau_0)}. \qquad (6)$$

Subsequently matched filtering is performed. Let $R_x(t)$ be correlated by G_1 and G_2 respectively:

$$\begin{aligned} V_1(t) &= R_x(t)*G_1(t-\tau_0) \\ &= \left[\left(G_1(t-\tau_0)+G_2(t-\tau_0)\right)k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_d)(t-\tau_0)}\right]*G_1(t-\tau_0) \\ &= \left[G_1(t-\tau_0)*G_1(t-\tau_0)+G_2(t-\tau_0)*G_1(t-\tau_0)\right] \\ &\quad \cdot k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_d)(t-\tau_0)}. \qquad (7) \\ V_2(t) &= R_x(t)*G_2(t-\tau_0) \\ &= \left[\left(G_1(t-\tau_0)+G_2(t-\tau_0)\right)k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_d)(t-\tau_0)}\right]*G_2(t-\tau_0) \\ &= \left[G_1(t-\tau_0)*G_2(t-\tau_0)+G_2(t-\tau_0)*G_2(t-\tau_0)\right] \\ &\quad \cdot k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_d)(t-\tau_0)}. \qquad (8) \end{aligned}$$

If G_1 and G_2 are mutually orthogonal, the sum of cross-correlations between corresponding sequences is zero at every lag:

$$g_1(n)*g_2(-n)+g_2(n)*g_1(-n)=0. \qquad (9)$$

The real parts of Eqs. (7) and (8) are

$$\begin{aligned}&\mathrm{Re}\{V_1(t)\}\\&=\mathrm{Re}\left\{\left[G_1(t-\tau_0)*G_1(t-\tau_0)\right]k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_\mathrm{d})(t-\tau_0)}\right\}\\&=\left[G_1(t-\tau_0)*G_1(t-\tau_0)\right]\cdot\mathrm{Re}\left\{k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_\mathrm{d})(t-\tau_0)}\right\}.\end{aligned}\tag{10}$$

$$\begin{aligned}&\mathrm{Re}\{V_2(t)\}\\&=\mathrm{Re}\left\{\left[G_2(t-\tau_0)*G_2(t-\tau_0)\right]k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_\mathrm{d})(t-\tau_0)}\right\}\\&=\left[G_2(t-\tau_0)*G_2(t-\tau_0)\right]\cdot\mathrm{Re}\left\{k\mathrm{e}^{\mathrm{j}2\pi(f_0-f_\mathrm{d})(t-\tau_0)}\right\}.\end{aligned}\tag{11}$$

The received and sampled signals of G_1 and G_2 are separated, and the pulse compression of Golay codes is performed. Ideal compression in terms of axial resolution and contrast can be achieved according to the complementarity condition (Chiao and Hao, 2005): The pair of biphase sequences G_1 and G_2 of length N is a Golay code if, and only if, it satisfies

$$g_1(n)*g_2(-n)+g_2(n)*g_1(-n)=2N\delta(n). \tag{12}$$

For the details of pulse compression, the reader is referred to Chiao and Hao (2005).

In summary, it is very important to design the transmitting signal and separate the two Golay codes from the echo received.

3 Computer simulations

It is necessary to investigate the advantage of the new scheme when motion between the emissions is present.

3.1 Simulation conditions

The simulation program was implemented using Matlab instead of those complex and time-consuming ultrasound simulation programs. As a matter of fact, this program is sufficient for investigating tissue motion effects.

We chose orthogonal Golay sequences G_1 and G_2, modulated on the carrier wave of a 2-MHz center frequency.

Figs. 2a and 2b give the positions of the scatterers and the PSFs of the images obtained at the first and the second emission respectively, when motion between the emissions was present. The change in the axial position between two emissions was Δz=5. The relation between the PSF and the position is shown as Eq. (1).

3.2 Simulation results

Fig. 4 shows simulation results using the proposed scheme when motion between the emissions is present. Figs. 4a and 4b are the same as Figs. 2a and 2b, respectively. Results of the two emissions are given in Figs. 4c and 4d. Note that using the proposed scheme the motion-dependent decoding error is eliminated.

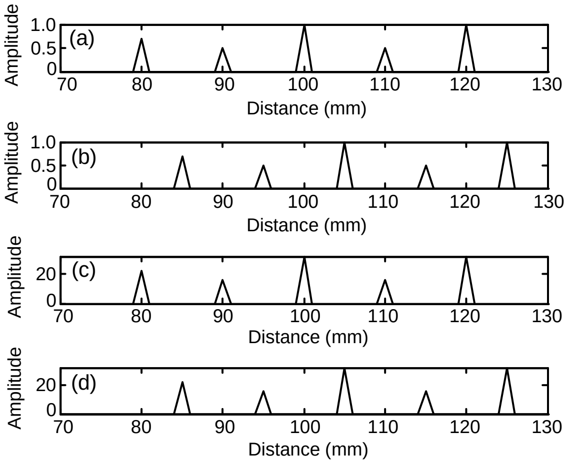

Fig. 4 Simulation results of the proposed coding scheme when motion between the emissions is present
Positions of the scatterers and point spread functions of the images at (a) the first emission and (b) the second emission. The resulting images obtained at (c) the first emission and (d) the second emission

3.3 Discussion

A number of related factors with the coded ultrasound imaging system, such as effect of the transducer, effect of noise, robustness of Doppler shift, were not investigated, as the effect of these factors on the scheme is the same as that on the conventional scheme of two firings.

4 Conclusions

The essential reason of the existence of the motion-dependent decoding error is the two firings of a Golay pair. The simultaneous emission of a code pair can be used to eliminate the error. In this paper we propose a scheme to implement the simultaneous emission of a code pair and separate the two Golay codes from the echo received. The scheme is very important as Golay codes are the most practical code in coded ultrasound imaging systems.

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