Journal of Zhejiang University-SCIENCE C (Computers & Electronics)
ISSN 1869-1951 (Print); ISSN 1869-196X (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Centralized and distributed resource allocation in OFDM based multi-relay system*

Rui YIN†1, Yu ZHANG[1], Guan-ding YU†‡1, Zhao-yang ZHANG†1, Jie-tao ZHANG[2]

([1]Department of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, China)

([2]Wireless Research Department, Huawei Technologies Co., Ltd., Shenzhen 518129, China)

†E-mail: {yinrui, yuguanding, ning_ming}@zju.edu.cn

Received July 5, 2009; Revision accepted Nov. 4, 2009; Crosschecked May 4, 2010

Abstract: In the presence of multiple non-regenerative relays, we derived optimal joint power allocation, relay selection, and subchannel pairing schemes in orthogonal frequency division multiplexing (OFDM) based wireless networks. The Lagrange dual method was employed to design the optimal algorithm. First, the optimization problem was formulated for the single-relay system and the optimal centralized algorithm was presented by resolving the dual problem. Next, the optimal algorithm for a multi-relay system was proposed in a similar way. Compared with the exhaustive search method, the computational complexity of the proposed optimal algorithms was reduced from non-polynomial to polynomial time. Finally, the centralized algorithm was extended to the distributed algorithm, which was more feasible for the practical system. Simulation results verify our analysis.

Key words: Wireless relay network, Orthogonal frequency division multiplexing (OFDM), Relay selection, Convex optimization, Dual method

doi:10.1631/jzus.C0910405 **Document code:** A **CLC number:** TN914.51

1 Introduction

Wireless communication systems are expected to provide a high data rate, low bit error rate, and large coverage service. To achieve this, high bandwidth utilization is inevitably required. The favorable modulation technique is orthogonal frequency division multiplexing (OFDM) since it is better able to deal with the multi-path fading brought by high bandwidth (Chow *et al*., 1995) and brings flexibility to resource allocation. Subchannels in OFDM can be adaptively allocated to users to exploit both the frequency and multiuser diversity. Furthermore, power allocation over subchannels used in conjunction with adaptive bit-loading has also shown great potential in improving the system performance (Wong *et al*., 1999; Kivance and Liu, 2000).

Wireless coverage is another issue that network operators face during actual deployment of the system. Base stations (BSs) can support only services in their areas so that the dead spots or coverage holes form due to high path-loss and shadowing. Deploying additional BSs could solve this problem; however, this solution is too expensive to implement in practice. As an alternative, relay stations are a cost-effective approach (Zhao *et al*., 2007). The cooperation between OFDM access (OFDMA) and relaying techniques, therefore, provides a promising technology to achieve high quality wireless services.

There are, however, a number of challenges. Joint resource allocation algorithms have been extensively studied in OFDMA based one-hop networks (Wong *et al*., 1999), but these schemes cannot be directly extended to a system with relays. The means by which to jointly perform resource allocation in OFDMA based multi-relay systems is complicated.

‡ Corresponding author

* Project supported by the National Natural Science Foundation of China (No. 60802012), the National Basic Research Program (973) of China (No. 2009CB320405), the Educational Commission of Zhejiang Province, China (No. 20070234), and Huawei Collaborative Research Funding (No. YBWL2008046)

Generally speaking, three important issues should be considered in this situation: (1) Which relay should be chosen to serve the user? (2) How to allocate and pair the subchannels at the first and second hops? (3) How to allocate the transmitting power at the source and the relay? In OFDM based multi-relay systems, for each subchannel of the source at the first hop, a particular subchannel is selected at the relay, which helps source accomplish its transmission in the second hop. This method is called subchannel pairing.

Recently, some of these three issues have been studied (Hammerstroem and Wittneben, 2006), wherein, for example, a sub-optimal power allocation is provided by the iterative method for the two-hop OFDM based amplified-forward (AF) single-relay link on the condition of either the individual power constraint at the source and the relay node or the total power constraint during a time frame. However, subchannel pairing is not considered in this scenario. Others (Shi *et al*., 2005) have considered both subchannel pairing and power allocation at once, with a sub-optimal algorithm proposed. Similar to Hammerstroem and Wittneben (2006), Shi *et al* (2005) considered only the system with a single pair of source-destination nodes in the presence of a single-relay. Li *et al*. (2008) considered a joint subchannel matching and power allocation scheme to maximize the channel capacity instead. By separating subchannel matching and power allocation, the sub-optimal algorithm was then proposed. Yu *et al*. (2005) considered the power allocation algorithm for three-node relay systems with different relay strategies: AF and decode-and-forward (DF). They used the effective capacity for resource allocation. Still a sub-optimal algorithm was proposed. Müller *et al*. (2007) proposed a dynamic resource allocation method to minimize the total transmission power. In this method, the total power is divided into two parts according to the size of sub-frames, and then each part of the transmission power is allocated to each subcarrier at both the source and the relay. Mesbah and Davidson (2008) considered joint power allocation and channel allocation in the time domain for a two-user orthogonal AF cooperation scheme. As far as the fairness requirement is concerned, the tradeoff between the transmission rate and subchannel occupation fairness among multiple users has been examined by Li and Liu (2006). Power allocation, however, is not addressed in their work. Li *et al.* (2009) considered joint power allocation, subchannel pairing, and maximin fairness. Again, a sub-optimal algorithm was proposed.

To the best of our knowledge, our study is the first to derive the optimal algorithm of joint subchannel matching, relay selection, and power allocation in both single- and multi-relay wireless networks (or called diamond wireless networks) in both centralized and distributed ways. To derive the optimal algorithm in single-relay systems, the Lagrange dual decomposition method was employed. According to Yu and Lui (2005), as the number of subchannels approaches infinity, the frequency sharing condition is satisfied and the dual gap is zero. Therefore, the optimal algorithm of the original problem can be found by solving the dual problem. Then, the same technique is extended to systems with multiple relays and the optimal algorithm is also derived. Next, for computational complexity analysis, the exhaustive search method is designed. The proposed optimal algorithms have much lower computation complexity, and further analysis is performed on the dual gap when the frequency sharing condition is not satisfied. This approach shows that under certain conditions, the dual gap is zero even though the number of subchannels is small. Finally, we extend the centralized scheme to a distributed one because of the characteristics of the Lagrange dual decomposition method.

In this study, we use $(x)^+$ to denote $\max(x, 0)$. $E(x)$ is the expectation value of variable x. The bold symbol denotes a vector or a matrix. p_S^n is the source power allocation on subchannel n in the first hop, and $p_k^{n'}$ is the power allocation of relay k on subchannel n'. $I_{n,n'}^k$ is the indicator function. If subchannel n in the first hop is paired with subchannel n' in the second hop at relay k, its value is one, otherwise zero.

2 System model

We consider a dual-hop scenario where a single pair of source and destination communicates through a total number of K relays (Fig. 1). Since OFDM modulation is employed, the whole system bandwidth is divided uniformly into N subchannels and each subchannel is considered to be frequency-flat. Each transmission includes two time slots. The source transmits over all the subchannels in the first time slot. Then in the second slot the relay nodes forward the received signal to the destination according to the amplified-and-forward (AF) policy.

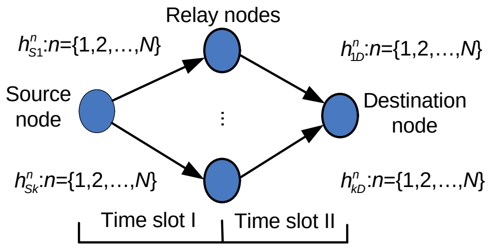

Fig. 1 OFDM based multi-relay system

The received signal in a link between two nodes A and B on subchannel n is given by $y_B^n = h_{AB}^n \cdot x_A^n + N_B^n$, where x_A^n is the signal transmitted at node A, $h_{AB}^n \sim \mathrm{CN}(0, \Omega_{AB}^n)$ is the channel gain between the link A-B on subchannel n, and $N_B^n \sim \mathrm{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at node B. For each link, let $H_{AB}^n \triangleq \left|h_{AB}^n\right|^2$ be the instantaneous squared channel strength between nodes A and B on subchannel n, obeying an exponential distribution with $E\{H_{AB}^n\} \triangleq \Omega_{AB}^n$. A can be a source node or any relay node and B can be any relay node or destination node. The total transmission power of the source and relays are constricted to P, $\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^k \cdot (p_S^n + p_k^{n'}) = P$. If subchannel n in the first hop is paired with subchannel n' in the second hop at relay k, the spectrum efficiency is given by (Hammerstroem and Wittneben, 2006)

$$R(p_S^n, p_k^{n'}) = \frac{1}{2}\log_2\left(1 + \frac{p_S^n p_k^{n'} H_{Sk}^n H_{kD}^{n'}}{p_S^n H_{Sk}^n + p_k^{n'} H_{kD}^{n'} + N_0}\right). \quad (1)$$

To derive the optimal joint resource allocation scheme, we make the following assumptions:

Assumption 1 The channel gains for all links on each relay node are statistically independent, and all terminals have the same noise variance N_0. Here we consider the block fading channel model.

Assumption 2 Each node in the network knows the overall instantaneous channel state information (CSI) between source and relay (h_{Sk}^n) and between relay and destination (h_{kD}^n). The full CSI can be achieved in the following ways:

1. The destination node feeds back the CSI h_{kD}^n to each relay k.

2. After collecting the CSI h_{kD}^n from the destination node, each relay node k feeds back both h_{Sk}^n and h_{kD}^n on each subchannel n to the source node.

This assumption is more theoretical than practical because it will bring a high overhead. This assumption was not valid when we derived the distributed algorithm.

Assumption 3 The direct transmission between source and destination is not considered due to its highly unreliable channel condition.

3 Optimal algorithm in single-relay systems

Firstly, we consider the power allocation and subchannel pairing algorithm in a single-relay network. Only subchannel pairing needs to be considered because there is only one relay in the system. The optimization problem can be formulated as follows:

$$\max_{I_{n,n'},\, p_S^n,\, p_R^{n'}} \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} R(p_S^n, p_R^{n'}) \quad (2)$$

s.t.

C1: $\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} p_S^n + \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} p_R^{n'} \le P,\ p_S^n \ge 0,\ p_R^{n'} \ge 0;$

C2: $\sum_{n=1}^{N} I_{n,n'} = 1,\ \forall n' = 1, 2, \cdots, N;$

C3: $\sum_{n'=1}^{N} I_{n,n'} = 1,\ \forall n = 1, 2, \cdots, N;$

C4: $I_{n,n'} \in \{0,1\},\ \forall n, n' = 1, 2, \cdots, N.$

The objective is to maximize the system spectrum efficiency. The constraint C1 means that the overall transmit power of the source and the relay is limited. C2 and C3 represent that each subchannel at hop 1 is matched only with one subchannel at hop 2, and vice versa. Since the problem Eq. (2) is an

NP-hard non-convex problem, the Lagrange decomposition method is used in the dual domain. With the Lagrange multiplier λ related to the constraint C1, the Lagrange dual function of the problem Eq. (2) is given by

$$\begin{aligned} g(\lambda) &= \max_{\boldsymbol{I},\boldsymbol{p}_S,\boldsymbol{p}_R} J(I,p_S,p_R,\lambda) \\ &= \max_{\boldsymbol{I},\boldsymbol{p}_S,\boldsymbol{p}_R} \left\{ \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} R(p_S^n, p_R^{n'}) \right. \\ &\quad \left. + \lambda\left(P - \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} p_S^n - \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} p_R^{n'} \right) \right\} \\ &= \max_{\boldsymbol{I},\boldsymbol{p}_S,\boldsymbol{p}_R} \left\{ \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} [R(p_S^n, p_R^{n'}) - \lambda p_S^n - \lambda p_R^{n'}] + \lambda P \right\}, \end{aligned} \tag{3}$$

where $\boldsymbol{p}_S = [p_S^1, \cdots, p_S^n, \cdots, p_S^N]$, $\boldsymbol{p}_R = [p_R^1, \cdots, p_R^n, \cdots, p_R^N]$ and $\boldsymbol{I}=(I_{i,j})_{N\times N}$. The dual optimization problem is then formulated as

$$\min_{\lambda \ge 0} g(\lambda). \tag{4}$$

This dual problem is convex since $g(\lambda)$ is linear in λ (Boyd and Vandenberghe, 2004). To solve the dual problem, we need to calculate $\max_{\boldsymbol{I},\boldsymbol{p}_S,\boldsymbol{p}_R} J(I,p_S,p_R,\lambda)$. Here, we use $\pi(n)$ as the subchannel pairing strategy. Assume subchannel n in hop 1 is matched with subchannel $\pi(n)$ in hop 2, i.e., $I_{n,\pi(n)}=1$. Then the dual function can be decomposed into N independent optimization problems:

$$g(\lambda) = \max_{\pi(n)} \sum_{n=1}^{N} g_{n,\pi(n)}(\lambda) + \lambda P, \tag{5}$$

where

$$g_{n,\pi(n)}(\lambda) = \max_{p_S^n \ge 0, p_R^{\pi(n)} \ge 0} \{R(p_S^n, p_R^{\pi(n)}) - \lambda(p_S^n + p_R^{\pi(n)})\}. \tag{6}$$

Eq. (6) is an optimal power allocation problem for maximizing the system spectrum efficiency after subchannel n is paired with subchannel $\pi(n)$. Unfortunately, $R(p_S^n, p_R^{\pi(n)})$ is not a convex function because its Hessian is not positive semi-definite.

Proposition 1 $R(p_S^n, p_R^{\pi(n)})$ is not convex since its Hessian is not positive semi-definite.

Proof See Appendix A.

From Proposition 1, we observe that the Hessian of $R(p_S^n, p_R^{\pi(n)})$ is not semi-definite because $H_S^n p_S^n \cdot H_R^{\pi(n)} p_R^{\pi(n)}$ might be less than $N_0/2$. But, the reason why we use a relay is that the signal-to-noise ratio (SNR) of a direct path between source and destination is not good enough for transmitting the data correctly. Furthermore, it is well known that the relay strategy is chosen when both the SNR between source and relay and the SNR between relay and destination are better than the SNR between source and destination, i.e., $H_{SR}^n / N_0 \gg H_{SD}^n / N_0$, $H_{RD}^{\pi(n)} / N_0 \gg H_{SD}^{\pi(n)} / N_0$. Hence, it is reasonable to assume that $H_{SR}^n p_S^n H_{RD}^{\pi(n)} p_R^{\pi(n)} \ge N_0/2$ when transmission with the help of relay happens. We use the following equation to approximate the original $R(p_S^n, p_R^{\pi(n)})$ (Han *et al.*, 2005):

$$R(p_S^n, p_R^{\pi(n)}) \triangleq \frac{1}{2}\log_2\left(1 + \frac{p_S^n p_R^{\pi(n)} H_{SR}^n H_{RD}^{\pi(n)}}{N_0(p_R^{\pi(n)} H_{RD}^{\pi(n)} + p_S^n H_{SR}^n)}\right). \tag{7}$$

With Eq. (7), by the derivation of Eq. (6) with respect to p_S^n and $p_R^{\pi(n)}$, the optimal solution of Eq. (6) can be derived as follows:

$$\frac{\partial R}{\partial p_S^n} = \lambda, \quad \frac{\partial R}{\partial p_R^{\pi(n)}} = \lambda. \tag{8}$$

Let

$$p_S^n = x_n,\ p_R^{\pi(n)} = y_{\pi(n)},\ H_{SR}^n = a_n / N_0,\ H_{RD}^{\pi(n)} = b_{\pi(n)} / N_0.$$

From Eq. (8), we have

$$a_n x_n^2 = b_{\pi(n)} y_{\pi(n)}^2. \tag{9}$$

From Eqs. (8) and (9), we obtain

$$x_n = \left(\sqrt{\frac{b_{\pi(n)}}{a_n}} \cdot y_{\pi(n)} \right)^+, \tag{10}$$

$$y_{\pi(n)} = \left(\frac{a_n b_{\pi(n)} - \left(a_n + 2\sqrt{a_n b_{\pi(n)}} + b_{\pi(n)}\right)\lambda'}{\left(a_n b_{\pi(n)} + b_{\pi(n)}\sqrt{a_n b_{\pi(n)}}\right)\lambda'} \right)^+, \tag{11}$$

where $\lambda'=\lambda\cdot\ln 2$. Putting Eqs. (10) and (11) back into Eq. (6), we obtain the maximum value of $g_{n,\pi(n)}(\lambda)$:

$$g_{n,\pi(n)}(\lambda)=\frac{1}{2}\log_2\left[1+\frac{x_n y_{\pi(n)} H_{SR}^n H_{RD}^{\pi(n)}}{N_0(x_n H_{SR}^n + y_{\pi(n)} H_{RD}^{\pi(n)})}\right] - \lambda(x_n + y_{\pi(n)}). \quad (12)$$

Since the paired subchannel $\pi(n)$ in hop 2 with subchannel n in hop 1 can be any one in the set $\pi(n)\in\{1, 2, \ldots, N\}$, $\max_{\pi(n)} \sum_{n=1}^{N} g_{n,\pi(n)}(\lambda)$ is transformed to the following optimization problem:

$$\max_{I} \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'} g_{n,n'}(\lambda) \quad (13)$$

s.t.

C1: $\sum_{n=1}^{N} I_{n,n'}=1, \forall n'=1, 2, \cdots, N;$

C2: $\sum_{n'=1}^{N} I_{n,n'}=1, \forall n=1, 2, \cdots, N;$

C3: $I_{n,n'}\in\{0,1\}, \forall n, n'=1, 2, \cdots, N.$

Herein,

$$g_{n,n'}(\lambda)=\frac{1}{2}\log_2\left[1+\frac{x_n y_{n'} H_{SR}^n H_{RD}^{n'}}{N_0(x_n H_{SR}^n + y_{n'} H_{RD}^{n'})}\right] - \lambda(x_n + y_{n'}). \quad (14)$$

It is an assignment problem, and therefore the Hungarian algorithm can be used to solve it with computational complexity $O(N^3)$. After finding the maximum value of $J(p_S^n, p_R^{n'}, \lambda)$, the dual problem of Eq. (4) is related only to the parameter λ. Since λ is a scalar, we use the bisection search method to achieve the optimal λ.

Proposition 2 For the dual problem Eq. (4), with the primal problem defined in Eq. (2), the following is a subgradient of $g(\lambda)$:

$$d = P - \sum_{n=1}^{N}(p_S^{n^*} + p_R^{\pi(n)^*}),$$

where $p_S^{n^*}$ and $p_R^{\pi(n)^*}$ are optimal power allocation at subchannel n at hop 1 and subchannel $\pi(n)$ at hop 2 to maximize the Lagrange $J(I, p_S, p_R, \lambda)$ in Eq. (3).

Proof See Appendix B.

In the bisection search method, the search direction is defined by Proposition 2, and the step size in t will be half of the step size in t–1. Hence, the optimal algorithm for the single-relay system (called OAS) can be described as follows:

Step 1: Set $\lambda_{\min}$=0, $\lambda_{\max}$ to some big value.

Step 2: Let $\lambda=(\lambda_{\max}+\lambda_{\min})/2$. For each possible subchannel pair $n\in\{1, 2, \ldots, N\}$ at hop 1 and $n'\in\{1, 2, \ldots, N\}$ at hop 2, calculate $g_{n,n'}(\lambda)$ using Eq. (14). Then the transmission rate matrix is generated, such as

$$\begin{bmatrix} g_{1,1}(\lambda) & \cdots & g_{1,n'}(\lambda) & \cdots & g_{1,N}(\lambda) \\ \vdots & & \vdots & & \vdots \\ g_{n,1}(\lambda) & \cdots & g_{n,n'}(\lambda) & \cdots & g_{n,N}(\lambda) \\ \vdots & & \vdots & & \vdots \\ g_{N,1}(\lambda) & \cdots & g_{N,n'}(\lambda) & \cdots & g_{N,N}(\lambda) \end{bmatrix}. \quad (15)$$

Then by using the Hungarian algorithm, the optimal subchannel pairing is found under current λ.

Step 3: If $\sum_{n=1}^{N}(p_S^n + p_R^{\pi(n)})=P$, then the optimal value of λ is found. If $\sum_{n=1}^{N}(p_S^n + p_R^{\pi(n)})<P$, updating $\lambda_{\max}=\lambda$. Otherwise, $\lambda_{\min}=\lambda$. Then return to Step 2.

4 Optimal algorithm in multi-relay systems

In a multi-relay system, the same technique developed in single-relay systems can be modified to find the optimal joint resource allocation algorithm. The optimization problem with multiple relays is formulated as follows:

$$\max_{I_{n,n'}^k, p_S^n, p_k^{n'}} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^k R(p_S^n, p_k^{n'}) \quad (16)$$

s.t.

C1: $\sum_{k=1}^{K}\sum_{n=1}^{N} I_{n,n'}^k=1, \forall n'=1, 2, \cdots, N;$

C2: $\sum_{k=1}^{K}\sum_{n'=1}^{N} I_{n,n'}^k=1, \forall n=1, 2, \cdots, N;$

C3: $0\le\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^k\le N, \forall k=1, 2, \cdots, K;$

C4: $\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^k (p_S^n + p_k^{n'})\le P, p_S^n\ge 0, p_R^{n'}\ge 0;$

C5: $I_{n,n'}^{k} \in \{0, 1\}, \forall n, n' = 1, 2, \cdots, N, \forall k = 1, 2, \cdots, K.$

The extra constraint C3 represents that each relay k can have multiple subchannel pairs. The dual function of problem Eq. (16) with respect to the total power constraint is expressed as

$$L(\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K, \lambda) = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k} R(p_S^n, p_k^{n'}) + \lambda\left\{P - \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k}(p_S^n + p_k^{n'})\right\}. \tag{17}$$

Then the Lagrange dual objective function $g(\lambda)$ is

$$g(\lambda) = \max_{\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K} L(\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K, \lambda), \tag{18}$$

where λ is the Lagrange multiplier. To find the maximum value of $L(\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K, \lambda)$, assuming subchannel n is coupled with subchannel $\pi(n)$ at relay $k(n)$, i.e., $I_{n,\pi(n)}^{k(n)} = 1$, the dual function Eq. (17) can be decomposed into N independent optimization problems:

$$\begin{aligned} g(\lambda) &= \max_{\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k} R(p_S^n, p_k^{n'}) \\ &\quad + \lambda\left[P - \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k}(p_S^n + p_k^{n'})\right] \\ &= \max_{\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k}\{R(p_S^n, p_k^{n'}) - \lambda(p_S^n + p_k^{n'})\} + \lambda P \\ &= \max_{\pi(n), k(n)} \sum_{n=1}^{N} L_{n,\pi(n)}^{k(n)}(\lambda) + \lambda P, \end{aligned} \tag{19}$$

where

$$L_{n,\pi(n)}^{k(n)}(\lambda) = \max_{p_S^n \geq 0, p_{k(n)}^{\pi(n)} \geq 0} \{R(p_S^n, p_{k(n)}^{\pi(n)}) - \lambda(p_S^n + p_{k(n)}^{\pi(n)})\}. \tag{20}$$

Similar to Eq. (6), the optimal power allocation is presented in Eqs. (10) and (11) under certain subchannel pairing $\pi(n)$ and relay selection scheme $k(n)$. Each subchannel n at hop 1 can be assigned to any relay k and paired with any subchannel n' at hop 2. Hence, to achieve the maximum value of $L(\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K, \lambda)$, we have to derive the optimal relay selection and subchannel pairing. With the optimal power allocation p_S^n at hop 1 and $p_{k(n)}^{\pi(n)}$ at hop 2, the optimal relay selection and subchannel pairing problem is concluded as

$$\max_{I_{n,n'}^{k}} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k} g_{n,n'}^{k}(\lambda) \tag{21}$$

s.t.

C1: $\sum_{k=1}^{K}\sum_{n'=1}^{N} I_{n,n'}^{k} = 1, \forall n = 1, 2, \cdots, N;$

C2: $\sum_{k=1}^{K}\sum_{n=1}^{N} I_{n,n'}^{k} = 1, \forall n' = 1, 2, \cdots, N;$

C3: $0 \leq \sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^{k} \leq N, \forall k = 1, 2, \cdots, K;$

C4: $I_{n,n'}^{k} \in \{0,1\}, \forall n, n' = 1, 2, \cdots, N, \forall k = 1, 2, \cdots, K.$

Herein,

$$g_{n,n'}^{k}(\lambda) = R(p_S^{n^*}, p_k^{n'^*}) - \lambda(p_S^{n^*} + p_k^{n'^*}). \tag{22}$$

$p_S^{n^*}$ and $p_k^{n'^*}$ are the optimal source and relay power allocation presented in Eqs. (10) and (11). With the optimal power allocation, the optimization problem Eq. (21) is an integer programming problem, which can be concluded as a maximum-weighted matching problem in the bipartite graph and can be solved using the Hungarian method.

After the maximum value $L(\boldsymbol{I}, \boldsymbol{p}_S, \boldsymbol{p}_K, \lambda)$ is calculated, the dual optimization problem is formulated as

$$\min_{\lambda \geq 0} g(\lambda). \tag{23}$$

As in the OAS algorithm, we employ the bisection search method to achieve the optimal λ. Finally, the optimal algorithm for a multi-relay system (OAM) is concluded in the following steps:

Step 1: Set $\lambda_{\min}$=0, $\lambda_{\max}$ to some big value.

Step 2: Letting $\lambda=(\lambda_{\max}+\lambda_{\min})/2$, each relay node k calculates the value $g_{n,n'}^{k}(\lambda)$ for all possible subchannel pairs (n, n') at hop 1 and hop 2 according to Eq. (22). Then each relay k formulates a bipartite graph $G(N_1, N_2, C)$ (Fig. 2), in which N_1 represents subchannels in hop 1, N_2 represents subchannels in hop 2, and the edges of the graph C serves as an abstraction of subchannel pairing. The weights on the edges, $w(e^k_{n,n'})=g^k_{n,n'}(\lambda)$, denote the achievable capacity of the subchannel pair n at hop 1 and n' at hop 2 with current λ. Every edge in the graph belongs to the same relay node and since there are K relays in the system, K such bipartite graphs will be generated. Among these K bipartite graphs, we pick up the edge on each subchannel pair n and n' with the maximum weight $w(e^{k^*}_{n,n'})$ and $k^*=\arg\max_k w(e^k_{n,n'})$ to formulate a new bipartite graph. The problem in Eq. (21) is converted to a maximum-weighted matching problem in this bipartite graph. Still the Hungarian method is applied to obtain the globally optimal solution in computational complexity $O(N^3)$.

Step 3: If $\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I^k_{n,n'}(p^n_S+p^{\pi(n)}_k)=P$, then the optimal value of λ is found. If $\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I^k_{n,n'}(p^n_S+p^{\pi(n)}_k)<P$, updating $\lambda_{\max}=\lambda$; otherwise, $\lambda_{\min}=\lambda$. Then go back to Step 2.

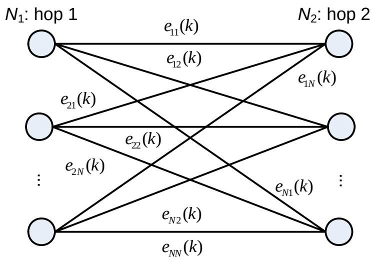

Fig. 2 Relay *k*'s bipartite graph

The optimal value achieved using the dual method is not equal to the value in the original optimization problem since there is a dual gap. Fortunately, according to Yu and Lui (2005) and Cendrillon *et al.* (2006), as the number of subchannels (N) goes to infinity, the frequency sharing condition is satisfied. Then, the dual gap of the optimization problem is always zero, regardless of the convexity of the objective function. In addition, simulation results (Yu and Lui, 2005; Cendrillon *et al.*, 2006) show that the above conclusion still holds even when the number of subchannels is not large, e.g., 8. It is well known that the subchannel number in a practical OFDMA system is usually very large. Therefore, it is reasonable to assume that the studied system satisfies the frequency-sharing condition.

Theorem 1 If the number of iterations, t, is large enough, the proposed OAM algorithm will converge to the optimal primary objective value $R(\lambda^*)=g(\lambda^*)=\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I^{k^*}_{n,n'}R(p^{n^*}_S,p^{n'^*}_k)$, where $(I^{k^*}_{n,n'},p^{n^*}_S,p^{n'^*}_k)$ is the optimal solution.

Proof See Appendix C.

5 Computational complexity and dual gap analysis

5.1 Computational complexity

Firstly, for comparison, the exhaustive search method (ESM) is introduced. In the exhaustive search algorithm, any possible combination of subchannel pairing and relay selection is considered, and we use the following method to allocate the transmit power. Let the subchannel n in hop 1 be paired with subchannel $\pi(n)$ in hop 2 at relay $k(n)$. Then the optimal power allocation problem is formulated as

$$\max_{p^n_S,p^{\pi(n)}_{k(n)}}\sum_{n=1}^{N}R(p^n_S,p^{\pi(n)}_{k(n)})\ \text{s.t.}\ \sum_{n=1}^{N}(p^n_S+p^{\pi(n)}_{k(n)})\le P.\quad(24)$$

Because it is a convex optimization problem, the dual gap is zero. The Lagrange dual method is still applied to this problem and the objective function of the dual problem is described as follows:

$$\begin{aligned}g(\lambda)&=\max_{p^n_S,p^{\pi(n)}_{k(n)}}\left\{\sum_{n=1}^{N}(p^n_S,p^{\pi(n)}_{k(n)})+\lambda\left[P-\sum_{n=1}^{N}(p^n_S+p^{\pi(n)}_{k(n)})\right]\right\}\\&=\sum_{n=1}^{N}\max_{p^n_S,p^{\pi(n)}_{k(n)}}\{R(p^n_S,p^{\pi(n)}_{k(n)})-\lambda(p^n_S+p^{\pi(n)}_{k(n)})\}+\lambda P.\end{aligned}\quad(25)$$

Hence, the dual problem is decomposed into N independent sub-problems:

$$g(\lambda)=\sum_{n=1}^{N}g_n(p^n_S,p^{\pi(n)}_{k(n)})+\lambda P,\quad(26)$$

where

$$g_n(p^n_S,p^{\pi(n)}_{k(n)})=\max_{p^n_S,p^{\pi(n)}_{k(n)}}\{R(p^n_S,p^{\pi(n)}_{k(n)})-\lambda(p^n_S+p^{\pi(n)}_{k(n)})\}.\quad(27)$$

Similar to Eq. (6), the optimal p_S^n and $p_{k(n)}^{\pi(n)}$ are presented in Eqs. (10) and (11). And the dual optimization problem is

$$\min_{\lambda\geq 0} g(\lambda). \tag{28}$$

The optimal λ can be calculated by the bisection search method. Then the optimal power allocation scheme (OPA) for some pre-defined relay selection and subchannel pairing scheme is stated as follows:

Step 1: Set $\lambda_{\min}=0$, $\lambda_{\max}$ to some big value.

Step 2: Let $\lambda=(\lambda_{\max}+\lambda_{\min})/2$. Calculate the transmission power allocated to each paired subchannel n in hop 1 and to subchannel $\pi(n)$ in hop 2 according to Eqs. (10) and (11).

Step 3: If $\sum_{n=1}^{N}(p_S^n+p_{k(n)}^{\pi(n)})=P$, then the optimal value of λ is found. If $\sum_{n=1}^{N}(p_S^n+p_{k(n)}^{\pi(n)})<P$, updating $\lambda_{\max}=\lambda$; otherwise, $\lambda_{\min}=\lambda$. Then go back to Step 2.

Then, the exhaustive search method (ESM) can be presented as follows:

Step 1: Find all possible subchannel pairing and relay selection schemes.

Step 2: For each scheme, according to the OPA algorithm, the optimal power allocation is derived.

Step 3: Calculate the spectrum efficiency according to the results achieved in Step 2 for each scheme and pick up the largest one. The corresponding joint resource allocation scheme is the optimal.

The only difference between the optimal algorithms (OAS or OAM) and the ESM is that, in the proposed optimal algorithm the resource allocation is decided after the estimation of parameter λ in each loop. This minor change brings significant computational complexity reduction.

Proposition 3 When there is only one relay in the system, the computational complexity is $O(N!)$ for the ESM and $O(mN^3)$ for the OAS algorithm. When there are K relays in the system, the computational complexity is $O(N!K^N)$ for the ESM and $O(m(KN^2+N^3))$ for the OAM algorithm. m is the number of iterations. An accuracy of ξ_λ requires $m=-\log_2\xi_\lambda$ iterations for optimizing λ in the bisection search method.

Hence, using the proposed optimal algorithms, the computational complexity is changed from non-polynomial time to polynomial time.

5.2 Dual gap analysis

The optimal value achieved using the Lagrange dual method is not equal to the value in the original optimization problem since there is a dual gap. However, as mentioned in Section 4, if the number of subchannels (N) approaches infinity, the frequency sharing condition is satisfied, and the dual gap of the optimization problem tends to zero (Yu and Lui, 2005; Cendrillon *et al.*, 2006).

Furthermore, with the total power constraint, we can derive the conditions under which the dual gap is zero. First, we analyze what creates the dual gap. It is known that if the original problem is convex, the dual gap will be zero (Boyd and Vandenberghe, 2004). In ESM, if subchannel pairing and relay selection are pre-defined, the problem is convex as given in Eq. (24) so that the dual gap is zero between Eqs. (24) and (28). Mathematically, we express the exhaustive method as $\max_{\{H,p\}}\{\min_{\lambda} g(H,\lambda)\}$. The difference between our proposed algorithm and the exhaustive search algorithm is that in the former algorithm, the resource allocation is decided after the estimation of λ in each loop. Mathematically, we write it as $\min_{\lambda}\{\max_{\{H,p\}} g(H,\lambda)\}$.

As above analyzed, for ESM, there are $N!K^N$ possible Lagrange dual functions $g(H, \lambda)$, each of which corresponds to one possible subchannel pairing and relay selection scheme H. Assume two Lagrange functions $g_i(H_i, \lambda)$ and $g_j(H_j, \lambda)$ (Fig. 3). ESM first calculates the optimal values $g_i^*(H_i,\lambda_i^*)$ and $g_j^*(H_j,\lambda_j^*)$ for each Lagrange function, and then compares their values, and picks up the global optimal value, which is $g_j^*(H_j,\lambda_j^*)$ in Fig. 3. In contrast, in our proposed algorithm, since λ is estimated before resource allocation, it searches for the optimal value $g^*(H, \lambda)$ and λ^* is searched for on the square line, which is the intersection of $g_i(H_i, \lambda)$ and $g_j(H_j, \lambda)$ (Fig. 3). If λ_j^* is not included in the intersection, there is a dual gap, $\max_{\{H,p\}}\{\min_{\lambda} g(H,\lambda)\}\neq\min_{\lambda}\{\max_{\{H,p\}} g(H,\lambda)\}$ (Fig. 3b). Otherwise, the dual gap is zero (Fig. 3a). According to this observation, we derive the following theorem:

Theorem 2 If $\mathrm{SNR}_{n,n'} \gg 1 \quad \forall n, n' = 1, 2, \cdots, N$, where $\mathrm{SNR}_{n,n'} = \dfrac{p_S^n p_k^{n'} H_{SR}^n H_{RD}^{n'}}{N_0 (p_S^n H_{SR}^n + p_k^{n'} H_{SR}^n)}$, the optimal value of parameter λ^* is constant, $\lambda^* = N / (2P \ln 2)$, and the dual gap is zero.

Proof See Appendix D.

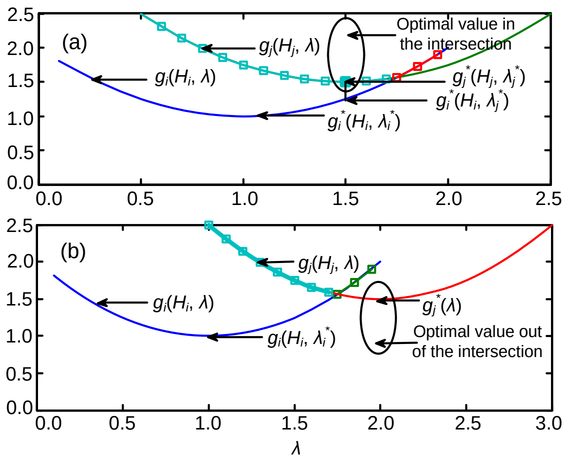

Fig. 3 Lagrange dual function
(a) Optimal λ is in the intersection; (b) Optimal λ is out of the intersection

6 Distributed algorithm

The two optimal algorithms (OAS, OAM) presented in Sections 3 and 4 are centralized. Deriving corresponding distributed optimal algorithms is straightforward. In the proposed centralized algorithms, each relay needs to know whether it is selected by the source and if so, which subchannel pair is assigned to it. Furthermore, to update the power allocation, each relay has to know the value of parameter λ. Actually, the Lagrange dual decomposition has a natural economic interpretation (Boyd and Vandenberghe, 2004). The relay network can be regarded as a market, where the source is the supplier of the goods (transmission power). The dual variable λ can be interpreted as the price of the power supplied by the source. The bisection search method can be regarded as the price adjustment rule, and is indeed consistent with the law of supply and demand: if the demand $\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} I_{n,n'}^k (p_S^n + p_k^{n'})$ for power exceeds the total power constraint P, the system will raise its price λ; otherwise, it will lower price λ. A higher value of λ tells relays that present subchannel pair and power are 'expensive' and provides incentive to find the best scheme. After several iterations, the price will converge to the optimal price λ^*, and the system will converge to the stable state.

Obviously, the centralized algorithms could be modified to this negotiated secondary market approach. The source needs to update the parameter λ and broadcast it in the system so that each relay can know the 'shadow price' of the current power and subchannel pair scheme. In addition, the source needs to know which subchannel pair is assigned to each relay and the total transmission power allocation at each subchannel pair to update the parameter λ. Fortunately, the subchannel pairing at relays can be implemented in a distributed way by introducing virtual clock as in Bletsas *et al.* (2006). The initial timer value is

$$V_{n,n'}^k = l / g_{n,n'}^k(\lambda), \tag{29}$$

where l is a constant (in ms) and $g_{n,n'}^k(\lambda)$ is presented in Eqs. (19) and (20). Each relay node k generates a virtual clock matrix such as

$$\begin{bmatrix} \dfrac{l}{g_{1,1}(\lambda)} & \cdots & \dfrac{l}{g_{1,n'}(\lambda)} & \cdots & \dfrac{l}{g_{1,N}(\lambda)} \\ \vdots & & \vdots & & \vdots \\ \dfrac{l}{g_{n,1}(\lambda)} & \cdots & \dfrac{l}{g_{n,n'}(\lambda)} & \cdots & \dfrac{l}{g_{n,N}(\lambda)} \\ \vdots & & \vdots & & \vdots \\ \dfrac{l}{g_{N,1}(\lambda)} & \cdots & \dfrac{l}{g_{N,n'}(\lambda)} & \cdots & \dfrac{l}{g_{N,N}(\lambda)} \end{bmatrix}. \tag{30}$$

The virtual clock method fails only when two or more relays' timers on the same subchannel pair expire during the same time interval and the flag packets collide. To assure that the best 'relay' can always be selected in the 'scheduling slot' for each subchannel pair, the carrier sense multiple access (CSMA) protocol in IEEE 802.11 is adopted; i.e., each of the collided relay generates a random back-off interval before retransmitting until the best relay is selected. Clearly, the cost of distributed implementation is the increased latency overhead caused by the scheduling and contention resolution protocol.

Finally, the proposed distributed algorithm DOAM is summarized as follows:

Step 1: Set $\lambda_{\min}$=0, $\lambda_{\max}$ to some big value. Then it broadcasts the value $\lambda=(\lambda_{\max}+\lambda_{\min})/2$ in the system.

Step 2: According to Eqs. (20) and (29), each relay k generates a virtual timer matrix like (30) in which each item corresponds to the time value of the corresponding subchannel pair. Furthermore, each node also needs to generate power matrices of hop 1 and hop 2 for each subchannel pair in terms of Eqs. (10) and (11), such as

$$\begin{bmatrix} (p_S^1, p_k^1) & \cdots & (p_S^1, p_k^n) & \cdots & (p_S^1, p_k^N) \\ \vdots & & \vdots & & \vdots \\ (p_S^n, p_k^1) & \cdots & (p_S^n, p_k^n) & \cdots & (p_S^n, p_k^N) \\ \vdots & & \vdots & & \vdots \\ (p_S^N, p_k^1) & \cdots & (p_S^N, p_k^n) & \cdots & (p_S^N, p_k^N) \end{bmatrix}. \quad (31)$$

At the beginning of the 'scheduling slot', each relay k starts the virtual timer on each subchannel pair with initial time $l / g_{n,n'}^{k}(\lambda)$ in its virtual clock matrix (30).

Step 3: The relay which expires first on subchannel pair (n, n') transmits a flag packet including the subchannel pair ID, its own ID, and the corresponding transmission power pair $(p_S^n, p_k^{n'})$ to signal its presence. Hearing the flag packet, other relays stop their timers on this subchannel pair and back off.

Step 4: After collecting all the information about the

$$g_{n,\pi(n)}^{k(n)}(\lambda) = \max_{p_S^n, p_{k(n)}^{\pi(n)}} [R(p_S^n, p_{k(n)}^{\pi(n)}) - \lambda(p_S^n + p_{k(n)}^{\pi(n)})] \quad (32)$$

on each subchannel pair (n, n') from all relays, the source will use the Hungarian method to find the best relay selection and subchannel matching scheme under current parameter λ and according to Step 3 in the OAM algorithm to update λ. Then the source broadcasts the data packet and decision packet including the newly updated parameter λ, the decision of relay selection, and the corresponding assigned subchannel pairs in the system.

Step 5: After relays receive the packets from the source, they will check whether their IDs are in the packet. If so, the selected relays obtain the opportunity to retransmit the data from the source to the destination on the assigned subchannel pairs decided by the source, and they start to transmit by the subchannel pairs with corresponding transmission power in matrix (31). And then, all relays will update their virtual time value matrix (30) and transmission power matrix (31).

Compared with the centralized algorithm, there are two advantages of this distributed method:

1. The source does not need to know any CSI. Only relays collect their own local CSI from themselves to the source and destination. Hence, the feedback overhead is reduced.

2. The computational burden is evenly allocated to each node in the system except the destination. In contrast, in the centralized algorithm, all computational burdens are taken by the source node.

However, DOAM fits only the slow CSI fluctuation environment (quasi-static channel or block fading channel) to guarantee the convergence.

7 Simulation results

In this section, we evaluate the performance of the proposed algorithms by computer simulations. We assume that there is a single pair source-destination and the source S is located at (0, 0), the destination D is at (d_0, 0). When only a single relay is used, the distance between S and R is d_r. The three-path Rayleigh fading channel model is considered between the two terminals, and the power of each delayed path is exponentially attenuated. The path loss is given as d^{-3} (d indicates the distance between the transmitter and the receiver), and the shadowing effects are not considered in the simulations. The total system bandwidth is 1.25 MHz. The variance of noise on each subchannel is 1. For comparison, Shi *et al*. (2005)'s algorithm is also simulated in a single-relay scenario. Centralized and distributed algorithms are considered in the simulation, respectively.

7.1 Performance of centralized optimal algorithms

In the first scenario, there are N=16 subchannels with a single relay. The relative distance between the source and the relay is d_r=0.5d_0. The average spectrum efficiency per subchannel is shown in Fig. 4 with the transmission power per subchannel raised from 5 dB to 15 dB. We observe that both OAS and ESM achieve the maximum value. Although Shi *et al.* (2005)'s scheme is near the optimal schemes on performance, its complexity is much higher and it does not always converge. Obviously the equal power allocation with the ordered subchannel pairing scheme (EOS) (subchannel 1 in hop 1 matched with subchannel 1 in hop 2, number 2 in hop 1 matched with number 2 in hop 2, and so on) is the worst scheme in improving the system performance. To further verify the optimality of OAS, we achieve the system performance under different numbers of subchannels using ESM and OAS (Fig. 5). The average transmission power per subchannel is 10 dB and the number of subchannels is changed from 2 to 7. We see that the curve of the system spectrum efficiency achieved by OAS perfectly matches that achieved by ESM.

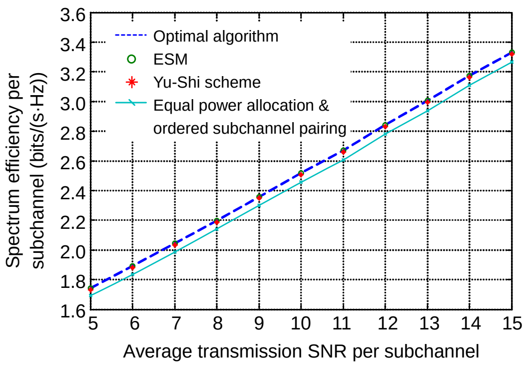

Fig. 4 The average spectrum efficiency per subchannel by different algorithms

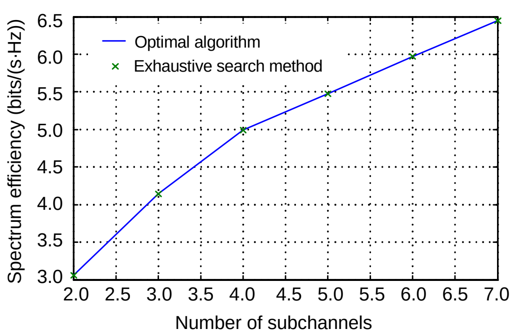

Fig. 5 System spectrum efficiency under different numbers of subchannels using ESM and OAS algorithms

The scenario is also simulated when the relative distance between the source and the relay, d_r, changes from 0.1 to 0.9 (Fig. 6). The total number of subchannels is N=16, and the average transmission SNR per subchannel is 15 dB. Four algorithms are examined in the simulation: the OAS algorithm, equal power allocation with optimal subchannel pairing, optimal power allocation with ordered subchannel pairing, and EOS. Fig. 6 shows that OAS is the best one all the time, because the power allocation and subchannel pairing are considered jointly to improve the spectrum efficiency. In contrast, the algorithm considering neither power allocation nor subchannel pairing is the worst on performance. Interestingly, it is found that when d_r/d_0≤0.3 or d_r/d_0≥0.7, the power allocation is more efficient in improving the spectrum efficiency than the subchannel pairing scheme since the difference on channel attenuation between the source and the relay (H_{SR}) and between the relay and the destination (H_{RD}) is large. On the other hand, when 0.4<d_r/d_0<0.6 or the difference on channel attenuation between *S*-*R* and *R*-*D* is small, the subchannel pairing is more efficient than the power allocation in improving the system performance.

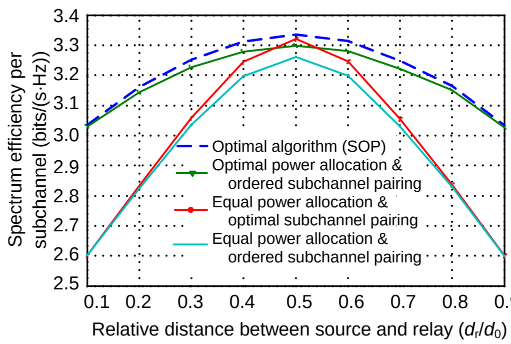

Fig. 6 System spectrum efficiency by different algorithms with different relative distances between source and relay

In another set of simulations, it is examined whether the algorithm is optimal in the presence of multiple relays. Two relays are uniformly distributed in the circle area with a semi-diameter of 10 m in the middle of the source and the destination and there are 16 subchannels in the system. Fig. 7 shows the average spectrum efficiency per subchannel achieved using the OAM scheme, ESM, and single best relay selection with the OAS scheme. The performance curve of the OAM algorithm perfectly matches the curve obtained using ESM. Furthermore, although a single 'best' relay is selected and the optimal algorithm OAS is used, the performance achieved using this scheme is still lower than that achieved using the OAM scheme. This is because when only a single relay is used in the multi-relay system, the diversity on the frequency domain is not fully exploited. Hence, when multiple relays are presented in OFDM based networks, the power allocation and subchannel pairing algorithm should be based on multi-relay selection. Similar to the first set of simulations, the optimal nature of the OAM algorithm is further proved in Fig. 8. As the number of subchannels is raised from 2 to 6 in the system with 2 relays and an average transmit power per subchannel of 10 dB, the spectrum efficiencies achieved using both optimal algorithms OAM and ESM perfectly matches. Hence, with Theorem 2, the algorithm derived using the Lagrange dual method is optimal in a multi-relay scenario.

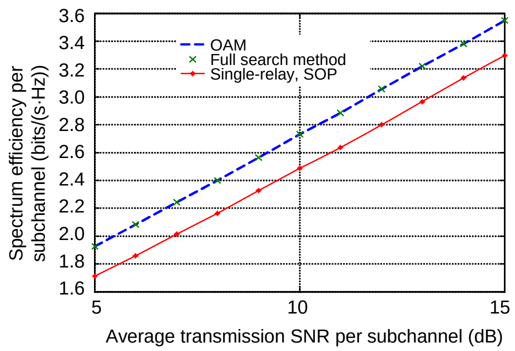

Fig. 7 The average spectrum efficiency on each subchannel achieved using different algorithms

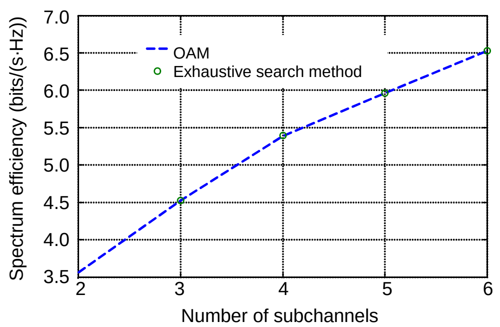

Fig. 8 System spectrum efficiency under different numbers of subchannels using ESM and OAM methods

7.2 Performance of the distributed optimal algorithm

In this subsection, the performance of the distributed optimal algorithm (DOAM) is examined in a multi-relay scenario. In the system, assuming there are 16 subchannels (N=16) and similar to Section 7.1, two relays (K=2) are uniformly distributed in the circle area with a semi-diameter of 10 m in the middle of the source. The average transmission SNR per subchannel is 15 dB. Figs. 9 and 10 show the convergence process of the value of parameter λ and the system spectrum efficiency, respectively. It can be observed that the DOAM algorithm converges to the optimal solution after around 20 iterations. Hence, the optimality condition of the proposed DOAM algorithm in practical multi-relay systems is not as strict. Therefore, DOAM is a proper distributed algorithm for OFDM based multi-relay systems.

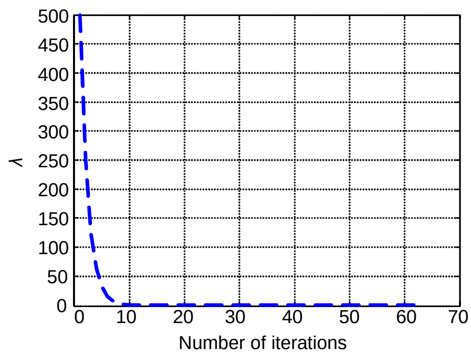

Fig. 9 Parameter λ convergence in the DMOP method

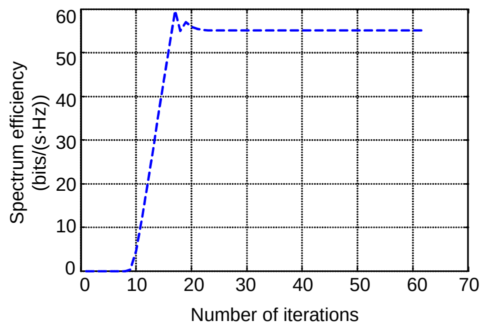

Fig. 10 Performance convergence in the DMOP method

8 Conclusions

In this study, we present an analytical framework in which transmission power, relay selection, and subchannel pairing can be jointly optimized under the aggregate transmit power constraint in multi-relay systems. Both centralized and distributed algorithms are derived in this framework using the Lagrange dual method. According to the analysis and computer simulation, they prove to be optimal. ESM is also discussed for comparison. Compared with ESM, the computational complexity of our proposed algorithm is reduced from non-polynomial to polynomial time.

The OAM algorithm we propose still has some deficiencies. First, the overall channel state information (CSI) is required. Although the computational complexity is reduced tremendously compared with ESM, it is still too high to implement in practical systems. In the future, we will extend our work by designing more practical resource allocation schemes with less complexity and limited CSI overhead.

References

Bletsas, A., Khisti, A., Reed, D.P., Lippman, A., 2006. A simple cooperative diversity method based on network path selection. *IEEE J. Sel. Areas Commun.*, **24**(3):659-672. [doi:10.1109/JSAC.2005.862417]

Boyd, S., Vandenberghe, L., 2004. Convex Optimization. Cambridge University Press, London, UK, p.215-273.

Cendrillon, R., Yu, W., Moonen, M., Verlinden, J., Bostoen, T., 2006. Optimal multiuser spectrum balancing for digital subscriber lines. *IEEE Trans. Commun.*, **54**(5):922-933. [doi:10.1109/TCOMM.2006.873096]

Chow, P.S., Cioffi, J.M., Bingham, J.A.C., 1995. A practical discrete multi-tone transceiver loading algorithm for data transmission over spectrally shaped channels. *IEEE Trans. Commun.*, **43**(2-4):773-775. [doi:10.1109/26.380108]

Hammerstroem, I., Wittneben, A., 2006. On the Optimal Power Allocation for Nonregenerative OFDM Relay Links. IEEE Int. Conf. on Communications, **10**:4463-4468. [doi:10.1109/ICC.2006.255341]

Han, Z., Himsoon, T., Siriwongpairat, W.P., Ray Liu, K.J., 2005. Energy-Efficient Cooperative Transmission Over Multiuser OFDM Networks: Who Helps Whom and How to Cooperate. Wireless Communications and Networking Conf., **2**:1030-1035. [doi:10.1109/WCNC.2005.1424650]

Kivance, D., Liu, H., 2000. Subcarrier Allocation and Power Control for OFDMA. 34th Asilomar Conf. on Signals, Systems and Computers, **1**:147-151. [doi:10.1109/ACSSC.2000.910933]

Li, G.Q., Liu, H., 2006. Resource allocation for OFDMA relay networks with fairness constraints. *IEEE J. Sel. Areas Commun.*, **24**(11):2061-2069. [doi:10.1109/JSAC.2006.881627]

Li, H.X., Luo, H.W., Wang, X.B., Lin, C.Y., Li, C.S., 2009. Fairness-Aware Resource Allocation in OFDMA Cooperative Relaying Network. IEEE Int. Conf. on Communications, p.1-5. [doi:10.1109/ICC.2009.5199025]

Li, Y., Wang, W.B., Kong, J., Hong, W., Zhang, X., Pen, M.G., 2008. Power Allocation and Subcarrier Pairing in OFDM-Based Relaying Networks. IEEE Int. Conf. on Communications, **6**:2602-2606. [doi:10.1109/ICC.2008.493]

Mesbah, W., Davidson, T., 2008. Joint power and channel resource allocation for two-user orthogonal amplify-and-forward cooperation. *IEEE Trans. Wirel. Commun.*, **7**(11):4681-4691. [doi:10.1109/T-WC.2008.070748]

Müller, C., Klein, A., Wegner, F., Kuipers, M., Raaf, B., 2007. Dynamic Subcarrier Bit and Power Allocation in OFDMA-Based Relay Networks. Proc. 12th Int. OFDM Workshop, p.1-5.

Shi, J., Zhao, Z.Y., Qiu, P.L., Yu, G.D., 2005. Subcarrier and Power Allocation for OFDMA Based Regenerative Multi-Hop Links. IEEE Int. Conf. on Wireless Communications, Networking and Mobile Computing, **1**:207-210. [doi:10.1109/WCNM.2005.1544018]

Wong, C.Y., Cheng, R.S., Letaief, K.B., Murch, R.D., 1999. Multiuser OFDM with adaptive subcarrier, bit, and power allocation. *IEEE J. Sel. Areas Commun.*, **17**(10):1747-1758. [doi:10.1109/49.793310]

Yu, G.D., Zhang, Z.Y., Chen, Y., Chen, S., Qiu, P.L., 2005. Power Allocation for Non-Regenerative OFDM Relaying Channels. IEEE Int. Conf. on Wireless Communications, Networking and Mobile Computing, **1**:185-188. [doi:10.1109/WCNM.2005.1544013]

Yu, W., Lui, R., 2005. Dual methods for nonconvex spectrum optimization of multicarrier systems. *IEEE Trans. Commun.*, **54**(7):1310-1322. [doi:10.1109/TCOMM.2006.877962]

Zhao, J., Hammerstroem, I., Kuhn, M., Wittneben, A., Herdin, M., Bauch, G., 2007. Coverage Analysis for Cellular Systems with Multiple Antennas Using Decode-and-Forward Relays. IEEE 65th Vehicular Technology Conf., p.944-948. [doi:10.1109/VETECS.2007.203]

Appendix A: proof of Proposition 1

It is known that if a matrix is positive definite, then all eigenvalues of the matrix are positive. The Hessian of matrix $\boldsymbol{R}$ is

$$\text{Hessian}(\boldsymbol{R})=\frac{1}{2\ln 2}\begin{bmatrix}\dfrac{\partial^2 R(p_S^n,p_R^{\pi(n)})}{\partial(p_S^n)^2} & \dfrac{\partial^2 R(p_S^n,p_R^{\pi(n)})}{\partial p_S^n\partial p_R^{\pi(n)}}\\ \dfrac{\partial^2 R(p_S^n,p_R^{\pi(n)})}{\partial p_R^{\pi(n)}\partial p_S^n} & \dfrac{\partial^2 R(p_S^n,p_R^{\pi(n)})}{\partial(p_R^{\pi(n)})^2}\end{bmatrix}.$$

Let the eigenvalues of Hessian(***R***) be λ_1 and λ_2. From Hessian(***R***), we have

$$\lambda_1\lambda_2=\frac{(H_{SR}^n)^2(H_{RD}^{\pi(n)})^2}{4(\ln 2)^2}\left(\frac{2H_{SR}^n p_S^n H_{RD}^{\pi(n)} p_R^{\pi(n)}}{N_0}-1\right)$$
$$\cdot(1+H_{SR}^n p_S^n/N_0+H_{RD}^{\pi(n)}p_R^{\pi(n)}+H_{SR}^n p_S^n H_{RD}^{\pi(n)}p_R^{\pi(n)}/N_0)^{-2}$$
$$\cdot(1+H_{SR}^n p_S^n/N_0+H_{RD}^{\pi(n)}p_R^{\pi(n)})^{-2}.$$

If $H_{SR}^n p_S^n H_{RD}^{\pi(n)} p_R^{\pi(n)} \le N_0/2$, then $\lambda_1\lambda_2 \le 0$. Hence Hessian(***R***) is not positive semi-definite.

Appendix B: proof of Proposition 2

By definition,

$$g(\lambda)=\max_{p_S^n,p_R^{\pi(n)}}\sum_{n=1}^N R(p_S^n,p_R^{\pi(n)})+\lambda\left(P-\sum_{n=1}^N(p_S^n+p_R^{\pi(n)})\right).$$

Let $p_S^{n^*}$ and $p_R^{\pi(n)^*}$ be the optimal power allocation for $g(\lambda)$. Then,

$$\begin{aligned}g(\lambda')&=\max_{p_S^n,p_R^{\pi(n)}}\sum_{n=1}^N R(p_S^n,p_R^{\pi(n)})+\lambda'\left(P-\sum_{n=1}^N(p_S^n+p_R^{\pi(n)})\right)\\&\ge\sum_{n=1}^N R(p_S^{n^*},p_R^{\pi(n)^*})+\lambda'\left(P-\sum_{n=1}^N(p_S^{n^*}+p_R^{\pi(n)^*})\right)\\&=\sum_{n=1}^N R(p_S^{n^*},p_R^{\pi(n)^*})+\lambda\left(P-\sum_{n=1}^N(p_S^{n^*}+p_R^{\pi(n)^*})\right)\\&\quad+(\lambda'-\lambda)\left(P-\sum_{n=1}^N(p_S^{n^*}+p_R^{\pi(n)^*})\right)\\&=g(\lambda)+(\lambda'-\lambda)\left(P-\sum_{n=1}^N(p_S^{n^*}+p_R^{\pi(n)^*})\right).\end{aligned}$$

Proposition 2 is hence proven using the definition of subgradient in Boyd and Vandenberghe (2004).

Appendix C: proof of Theorem 1

Assume $(I_{n,n'}^{k*},p_S^{n*},p_k^{n'*})$ is the optimal primary solution of the primary problem, and $R(\lambda^*)=g(\lambda^*)=\sum_{k=1}^K\sum_{n=1}^N\sum_{n'=1}^N I_{n,n'}^{k*}R(p_S^{n*},p_k^{n'*})$ the optimal primary objective value, where $\lambda^*=\arg\min_\lambda g(\lambda)$.

Since $R(\lambda_1)$ is Lipschitz:

$$G|\lambda_1-\lambda_2|\ge|R(\lambda_1)-R(\lambda_2)|\quad\forall\lambda_1>0,\lambda_2>0,$$

where G is a constant, using the bisection search method when selecting the initial lowest and highest values of $\lambda_l^1=0$, $\lambda_u^1=$ some big value, $\lambda_l^1\le\lambda^*\le\lambda_u^1$, then at the ($t$−1)th iteration step,

$$\lambda_l^{t-1}\le\lambda^*\le\lambda_u^{t-1}\quad\forall t\ge 1,$$

and λ is updated to $\lambda^t=(\lambda_l^{t-1}+\lambda_u^{t-1})/2$ at the tth iteration step. Hence,

$$|R(\lambda^t)-R(\lambda^*)|\le G|\lambda^t-\lambda^*|\le\frac{G|\lambda_u^1-\lambda_l^1|}{2^t}=\frac{G\lambda_u^1}{2^t}.$$

As $t\to\infty$, $|R(\lambda^t)-R(\lambda^*)|=G\lambda_u^1/2^t\to 0.$

Appendix D: proof of Theorem 2

First we derive the optimal λ for any pair of subchannels. Assuming subchannel n at hop 1 is paired with subchannel n' at hop 2 at relay k. Then let $p_S^n=x_n$, $p_k^{n'}=y_{n'}$, $H_{Sk}^n/N_0=a_n$, $H_{kD}^{n'}/N_0=b_{n'}$. Hence,

$$\begin{aligned}R(x_n,y_{n'})&=\frac{1}{2}\log_2\left(1+\frac{a_nb_{n'}x_ny_{n'}}{a_nx_n+b_{n'}y_{n'}}\right)\\&\approx\frac{1}{2}\log_2\left(\frac{a_nb_{n'}x_ny_{n'}}{a_nx_n+b_{n'}y_{n'}}\right).\end{aligned}\tag{D1}$$

The approximation in Eq. (D1) is because the SNR of the subchannel pair is greater than 1, i.e., $\mathrm{SNR}_{n,n'}\gg 1$. Then $g(\lambda)$ can be written as

$$\begin{aligned}g(\lambda)&=\sum_{n=1}^N\{R(x_n,y_{n'})-\lambda x_n-\lambda y_{n'}\}+\lambda P\\&=\sum_{n=1}^N g_n(x_n,y_{n'})+\lambda P,\end{aligned}\tag{D2}$$

where

$$g_n(x_n, y_{n'}) = R(x_n, y_{n'}) - \lambda x_n - \lambda. \quad \text{(D3)}$$

To obtain the optimal x_n and $y_{n'}$, we have

$$\frac{\partial g_n(\lambda)}{\partial x_n} = 0, \quad \frac{\partial g_n(\lambda)}{\partial y_{n'}} = 0. \quad \text{(D4)}$$

By solving Eq. (D4), we can achieve optimal x_n and $y_{n'}$:

$$x_n = \frac{\sqrt{b_{n'}}}{\left(\sqrt{a_n} + \sqrt{b_{n'}}\right)\lambda'}, \quad y_{n'} = \frac{\sqrt{a_n}}{\left(\sqrt{a_n} + \sqrt{b_{n'}}\right)\lambda'}, \quad \text{(D5)}$$

where $\lambda' = 2\ln 2$. Putting Eq. (D5) back to Eq. (D2), we have

$$g(\lambda) = \sum_{n=1}^{N}\left(\frac{1}{2}\log_2 \frac{a_n b_{n'} x_n y_{n'}}{a_n x_n + b_{n'} y_{n'}} - \lambda x_n - \lambda y_{n'}\right) + \lambda P$$

$$= \frac{1}{2}\left(N \log_2 \frac{1}{\lambda'} + \log_2 \prod_{n=1}^{N} \frac{a_n b_{n'}}{\left(\sqrt{a_n} + \sqrt{b_{n'}}\right)^2}\right) + \lambda\left(P - \sum_{n=1}^{N} \frac{1}{\lambda'}\right). \quad \text{(D6)}$$

Hence, an optimal λ is derived by the derivative of Eq. (D6):

$$\frac{\partial g(\lambda)}{\partial \lambda} = \left(\frac{N}{2}\log_2\left(\frac{1}{2\ln 2}\frac{1}{\lambda}\right)\right)' + P - \left(\lambda \frac{N}{2\ln 2\lambda}\right)' = 0$$

$$\Rightarrow -\frac{N}{2\ln 2}\cdot\frac{1}{\lambda} + P = 0 \Rightarrow \lambda = \frac{N}{2\ln 2 \cdot P}.$$

Because the optimal λ is a constant for each possible subchannel pairing and relay selection scheme, we declare that, when $\mathrm{SNR}_{n,n'} \gg 1$, there is no dual gap between Eqs. (2) and (4) or between Eqs. (16) and (23).