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Multi-objective differential evolution with diversity enhancement*

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Abstract: Multi-objective differential evolution (MODE) is a powerful and efficient population-based stochastic search technique for solving multi-objective optimization problems in many scientific and engineering fields. However, premature convergence is the major drawback of MODE, especially when there are numerous local Pareto optimal solutions. To overcome this problem, we propose a MODE with a diversity enhancement (MODE-DE) mechanism to prevent the algorithm becoming trapped in a locally optimal Pareto front. The proposed algorithm combines the current population with a number of randomly generated parameter vectors to increase the diversity of the differential vectors and thereby the diversity of the newly generated offspring. The performance of the MODE-DE algorithm was evaluated on a set of 19 benchmark problem codes available from http://www3.ntu.edu.sg/home/epnsugan/. With the proposed method, the performances were either better than or equal to those of the MODE without the diversity enhancement.

Key words: Multi-objective evolutionary algorithm (MOEA), Multi-objective differential evolution (MODE), Diversity enhancement

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1 Introduction

Multi-objective optimization scenarios are encountered in fields such as engineering, finance, pharmaceutical, economics, and science (Coello and Lamont, 2004). As there is no single best solution for multi-objective optimization problems, the objective is to find Pareto optimal trade-off solutions representing the best compromises among the objectives. A multi-objective optimization problem (MOP) can be described as follows:

$$\text{Minimize } \boldsymbol{F}(x)=[f_1(x), f_2(x), ..., f_m(x)]^{\mathrm{T}}$$
$$\text{subject to } x_i^{\mathrm{L}} \le x_i \le x_i^{\mathrm{U}},$$

where x_i is the variable space, x_i^{L} and x_i^{U} are the lower and upper bounds, respectively, of the decision variable, and $\boldsymbol{F}$ is a vector of objectives to be optimized. Development of evolutionary algorithms to solve multi-objective optimization problems has attracted much interest. Consequently, several multi-objective evolutionary algorithms (MOEAs) have been developed (Zitzler and Thiele, 1999; Corne *et al.*, 2000; Deb *et al.*, 2002; Kim *et al.*, 2004; Du and Cai, 2007; Chen *et al*., 2008; Ghosh and Das, 2008; Qu and Suganthan, 2009; Huang *et al*., 2009; Zhao and Suganthan, 2010; Zhao and Suganthan, accepted).

The main advantage of MOEAs for solving MOPs is their ability to find multiple Pareto optimal solutions in one single run. Multi-objective differential evolution (MODE) is one such commonly used MOEA with this ability. However, when solving hard problems, loss of diversity is a major problem faced by the MODE and other MOEAs. To solve this problem, this paper proposes a multi-objective differential evolution with diversity enhancement (MODE-DE) to prevent the algorithm becoming trapped in a locally optimal Pareto front.

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2 Differential evolution

The differential evolution (DE) algorithm was proposed by Storn and Price (1995). It is a simple and powerful population-based stochastic global optimizer in continuous search spaces. The four main steps of the DE are the initialization, mutation, recombination, and selection of next generation parents from the combined parent-offspring population of the current generation. Mutation, recombination, and selection are applied to every solution member x_i in each generation to create a new solution. DE starts with a randomly initialized population of NP solutions. After initialization, DE employs the mutation operation to produce a mutant vector $\boldsymbol{v}_{\rm p}$ associated with each parent $\boldsymbol{x}_{\rm p}$ in each generation, using one of the following mutation strategies (Chen *et al.*, 2008; Qin *et al*., 2009):

DE/rand/1: $\boldsymbol{v}_{\rm p}=\boldsymbol{x}_{r_1}+F(\boldsymbol{x}_{r_2}-\boldsymbol{x}_{r_3})$;

DE/best/1: $\boldsymbol{v}_{\rm p}=\boldsymbol{x}_{\rm best}+F(\boldsymbol{x}_{r_1}-\boldsymbol{x}_{r_2})$;

DE/current-to-best/2:

$$\boldsymbol{v}_{\rm p}=\boldsymbol{x}_{\rm p}+F(\boldsymbol{x}_{\rm best}-\boldsymbol{x}_{\rm p})+F(\boldsymbol{x}_{r_1}-\boldsymbol{x}_{r_2});$$

DE/best/2: $\boldsymbol{v}_{\rm p}=\boldsymbol{x}_{\rm best}+F(\boldsymbol{x}_{r_1}-\boldsymbol{x}_{r_2})+F(\boldsymbol{x}_{r_3}-\boldsymbol{x}_{r_4})$;

DE/rand/2: $\boldsymbol{v}_{\rm p}=\boldsymbol{x}_{r_1}+F(\boldsymbol{x}_{r_2}-\boldsymbol{x}_{r_3})+F(\boldsymbol{x}_{r_4}-\boldsymbol{x}_{r_5})$.

Herein, r_1, r_2, r_3, r_4, and r_5 are mutually different integers randomly generated in the range [1, NP] (NP is the population size), which should also be different from the current parent vector. F is a scale factor in [0, 2] used to scale differential vectors. $\boldsymbol{x}_{\rm best}$ is the solution with the best fitness value in the population in the current generation.

After the mutation phase, a uniform crossover operation is applied to each pair of the generated mutant vectors and its corresponding parent vector. This process is represented as

$$u_{{\rm p},i}=\begin{cases} v_{{\rm p},i}, & \text{if rand}_i\le \text{CR or } j=\text{rand}_j,\\ x_{{\rm p},i}, & \text{otherwise,}\end{cases}$$

where $\boldsymbol{u}_{\rm p}$ is the offspring vector. The crossover rate CR is a user-specified constant within the range [0, 1), which determines which parameter dimensions of $\boldsymbol{u}_{\rm p}$ are copied from $\boldsymbol{v}_{\rm p}$ or $\boldsymbol{x}_{\rm p}$ respectively (Chen *et al.*, 2008; Das *et al*., 2009).

F and CR are important control parameters in the DE. These two parameters can significantly influence the optimization performance of the DE. Therefore, to successfully solve an optimization problem, it is necessary to perform a trial-and-error search for the most appropriate values for these parameters.

3 Multi-objective differential evolution

MODE is an extended version of the single-objective DE to solve multi-objective optimization problems (Babu and Jehan, 2003; Fan HY *et al.*, 2006; Zhang *et al.*, 2007; Zielinski and Laur, 2007b; Chen *et al.*, 2008; Fan J *et al.*, 2008; Zhang and Sanderson, 2008). The major difference between them is in the selection process. In single-objective optimization, comparison and selection are based on objective function values. However, in MODE, the selection process is based on the domination concept. A solution $\boldsymbol{x}_1$ is said to dominate another solution $\boldsymbol{x}_2$ if both conditions specified below are satisfied:

1. The solution $\boldsymbol{x}_1$ is no worse than $\boldsymbol{x}_2$ in all objectives.

2. The solution $\boldsymbol{x}_1$ is strictly better than $\boldsymbol{x}_2$ in at least one objective.

If either of the above conditions is violated, the solution $\boldsymbol{x}_1$ does not dominate the solution $\boldsymbol{x}_2$. In a typical MODE, the current non-dominated solutions are stored in an external archive with a pre-specified size. If the number of non-dominated solutions exceeds the archive size, the crowding distance (Deb *et al.*, 2002), which is based on the estimation of crowding density of solutions, is used to select diverse solutions. Fig. 1 shows the flowchart of a typical MODE.

4 Multi-objective differential evolution with diversity enhancement

The MODE-DE is very similar to the typical MODE. In the MODE, the r_1, r_2, r_3, r_4, and r_5 are randomly selected from the current population and external archive. However, as the search progresses, the parameter space differences between these vectors can become small, thereby resulting in offspring which are similar to the parent. This leads to premature convergence when solving hard problems with

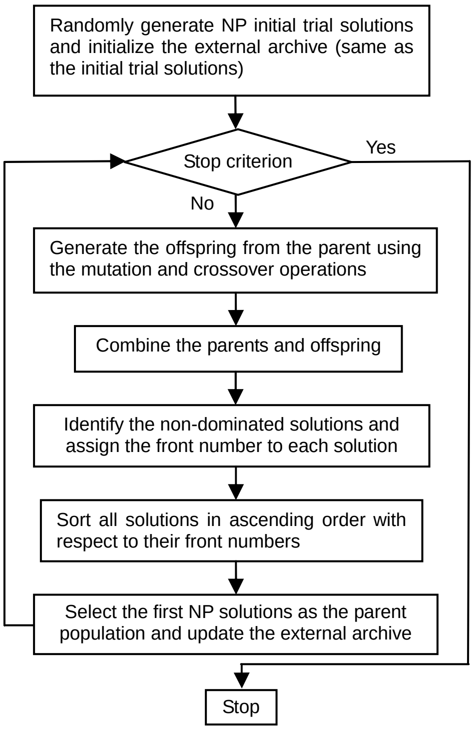

Fig. 1 Flowchart of a typical multi-objective differential evolution (MODE) algorithm

Table 1 Multi-objective differential evolution algorithm with diversity enhancement

Step	Description
1	Randomly generate NP initial parent solutions, evaluate their objective values, and initialize the external archive Set count=0, func_eval=NP
2	If func_eval/5000==0 // Every 5000 fitness // evaluations, randomly generate NP solutions Randomly generate NP parameter vectors without evaluating their objective values count=count+1 Endif

3	// Out of every 5000 function evaluations, diversity enhancement is used for 1000, or 20%
	If func_eval>=5000*count AND func_eval<= 5000*count+1000
	Population=parents+NP randomly generated parameters // 1000 or 20%
	Else
	Population=parents // The standard DE is // executed for 80% of the fitness evaluations
	Endif

4	For *p*=1 to NP Mutant vector is generated from the population using the 'DE/rand/1' mutation strategy Crossover operation is applied to the generated offspring // Refer to Section 2 Increase func_eval Endfor
5	Combine NP parents and NP offspring
6	Apply non-dominated sorting to select the best NP parents for the next generation Update external archive with the current set of non-dominated solutions. If the archive size exceeds the specified size, apply the crowding distance to truncate the non-dominated solutions
7	Stop if the termination criterion is satisfied; otherwise, go to Step 2

numerous local Pareto optimal solutions. To enhance the diversity, MODE-DE combines the current population with randomly generated parameter vectors of the same size in 20% of the function evaluations. As only the current parent population will generate offspring, we do not have to evaluate the fitness of these randomly generated points. The r_1, r_2, r_3, r_4, and r_5 are selected from the combined population to generate larger differential vectors, which may prevent the population becoming trapped in a locally optimal Pareto front. During the remaining 80% of the function evaluations, the mutant vector is generated using the current population only as in the typical MODE. The MODE-DE algorithm is shown in Table 1.

5 Experiments and results

5.1 Experimental setup

To test the MODE-DE algorithm, 19 benchmark problems (Huang *et al.*, 2007) were used. Among these 19 problems, the first 7 were two-objective problems and the next 6 were three-objective problems while the last 6 were five-objective problems. The MODE-DE algorithm was implemented in Matlab 7.1 using a Pentium IV computer with 2.99 GHz CPU, 2 GB RAM running the Microsoft Windows XP operating system. The maximal number of function evaluations was set as 500 000. The sizes of the external archive to hold the current set of non-dominated solutions were 100, 150, and 800 for two-, three-, and five-objective problems, respectively. The population size, NP, was 100. Two sets of DE parameters were tested with and without diversity enhancement: Set 1: F=0.35, CR=0.2; Set 2: F=0.9, CR=0.1.

5.2 Performance measures

To compare the performances of different algorithms quantitatively, some performance metrics are needed. There are two goals in a multi-objective optimization: (1) convergence to the Pareto optimal front and (2) diversity of solutions in the Pareto optimal set. We used the R indicator (I_{R2}) (Knowles *et al.*, 2006):

$$I_{R2}=\frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}u^*(\lambda, A)-u^*(\lambda, R),$$

where R is a reference set, and u^* is the maximum value reached by the utility function u with weight vector $\boldsymbol{\lambda}$ on an approximation set A, i.e., $u^*=\max_{z\in A}u_\lambda(z)$. We chose the augmented Tchebycheff's function (Knowles *et al.*, 2006) as the utility function.

5.3 Results

Each test problem was run 30 times. The performance metric R indicator values are presented in Tables 2 and 3. The performances of the MODE-DE were either comparable to or better than those of the MODE for the two sets of parameters. The t-test and Wilcoxon test were also applied to the R indicator and the results are shown in the last two columns of Tables 2 and 3. The numerical values −1, 0, 1 indicate that the MODE was statistically inferior to, equal to, or superior to the proposed MODE-DE. The better performance of the MODE-DE is due to its better global search ability as a result of the diversity enhancement process. Because the magnitude of the differential vector in the parameter space is increased by the diversity enhancement procedure, the algorithm is unlikely to become trapped in a locally optimal Pareto front. The final objective values of S_ZDT4 are also plotted in Fig. 2 (F=0.9, CR=0.1). We can see that the MOD-DE generated a much better result than the MODE.

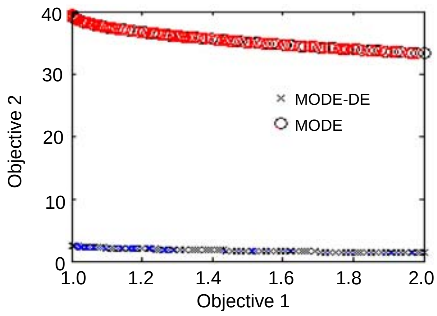

Fig. 2 Results on two-objective S_ZDT4 (F=0.9, CR=0.1)

Table 2 Comparison between MODE-DE and MODE (F=0.35, CR=0.2)

Problem	I_{R2}								h/p	
	MODE-DE				MODE				t-test	Wilcoxon test
	Best	Worst	Mean	Std.	Best	Worst	Mean	Std.		
OKA2	−1.07E-03	−1.07E-03	−1.07E-03	0	−1.07E-03	−1.07E-03	−1.07E-03	0	0/1	0/1
SYMPART	1.36E-06	2.11E-06	1.92E-06	2.89E-07	2.05E-06	2.59E-06	2.41E-06	1.95E-07	−1/1.66E-10	−1/2.04E-08
S_ZDT1	−1.05E-03	−1.05E-03	−1.05E-03	1.32E-07	−1.05E-03	−1.05E-03	−1.05E-03	2.25E-07	0/2.55E-01	0/5.48E-01
S_ZDT2	−1.14E-04	−1.14E-04	−1.14E-04	5.51E-20	−1.14E-04	4.03E-02	2.40E-02	2.00E-02	−1/1.37E-08	−1/1.77E-09
S_ZDT4	8.65E-04	2.40E-03	1.49E-03	5.21E-04	2.49E-03	3.66E-03	3.01E-03	4.07E-04	−1/3.56E-018	−1/2.42E-011
R_ZDT4	1.09E-03	2.03E-03	1.67E-03	3.27E-04	1.55E-03	3.97E-03	2.72E-03	1.01E-03	−1/1.24E-06	−1/3.34E-03
S_ZDT6	3.42E-06	2.36E-02	4.72E-03	9.59E-03	3.42E-06	6.65E-02	2.30E-02	2.92E-02	−1/1.89E-03	−1/2.18E-02
S_DTLZ2	5.26E-06	2.92E-05	1.37E-05	8.99E-06	5.24E-06	4.30E-05	2.28E-05	1.44E-05	−1/4.48E-03	0/6.22E-02
R_DTLZ2	4.24E-05	1.07E-04	6.59E-05	2.35E-05	3.91E-05	8.01E-05	6.31E-05	1.41E-05	0/5.65E-01	0/7.95E-01
S_DTLZ3	3.22E-06	5.73E-06	4.11E-06	8.87E-07	9.85E-06	2.07E-05	1.42E-05	3.88E-06	−1/4.47E-020	−1/2.42E-011
WFG1	−9.80E-05	−9.64E-05	−9.71E-05	6.32E-07	−9.77E-03	−9.76E-05	−9.77E-05	1.64E-08	0/1.18E-01	0/1.84E-01
WFG8	−2.89E-02	−2.88E-02	−2.89E-02	7.99E-05	−2.90E-02	−2.88E-02	−2.89E-02	9.56E-05	0/2.65E-01	0/3.09E-01
WFG9	−1.22E-02	−9.35E-03	−1.04E-03	1.38E-04	−9.45E-03	−9.40E-03	−9.42E-03	2.52E-05	0/1.51E-01	0/5.48E-01
S_DTLZ2	6.55E-06	1.09E-05	8.63E-06	1.42E-06	7.63E-06	1.03E-05	8.91E-06	8.73E-07	0/3.65E-01	0/6.22E-02
R_DTLZ2	3.43E-05	5.32E-05	4.44E-05	6.60E-06	3.76E-05	4.82E-05	4.39E-05	4.12E-06	0/7.33E-01	0/4.27E-01
S_DTLZ3	4.04E-06	4.80E-06	4.25E-06	2.85E-07	3.61E-06	4.88E-05	4.11E-06	4.60E-07	0/1.36E-01	0/6.22E-02
WFG1	2.78E-02	3.11E-02	2.96E-02	1.24E-03	2.98E-02	3.59E-02	3.25E-02	2.18E-03	−1/1.63E-08	−1/5.66E-08
WFG8	−1.17E-02	−1.15E-02	−1.16E-04	4.87E-05	−1.17E-02	−1.15E-02	−1.16E-02	5.19E-05	0/1.37E-01	0/1.84E-01
WFG9	4.81E-04	6.52E-04	5.81E-04	7.27E-05	5.66E-04	6.54E-04	6.08E-04	2.94E-05	0/5.64E-02	0/4.27E-01

The h value of −1, 0, or 1 indicates that the MODE was statistically inferior to, equal to, or superior to the proposed MODE-DE

Table 3 Comparison between MODE-DE and MODE (F=0.9, CR=0.1)

Problem	I_{R2}								h/p	
	MODE-DE				MODE				*t*-test	Wilcoxon test
	Best	Worst	Mean	Std.	Best	Worst	Mean	Std.		
OKA2	−1.07E-03	−1.07E-03	−1.07E-03	0	−1.07E-03	−1.07E-03	−1.07E-03	0	0/1	0/1
SYMPART	1.99E-06	2.72E-06	2.35E-06	2.35E-07	2.15E-06	1.019E-02	2.53E-03	3.93E-03	−1/8.41E-04	−1/6.23E-05
S_ZDT1	−1.05E-03	−9.75E-04	−1.04E-03	3.23E-05	−1.05E-03	−7.97E-04	−9.99E-04	1.03E-04	0/5.28E-02	0/6.22E-02
S_ZDT2	−1.14E-04	−7.58E-05	−1.10E-04	1.20E-05	−1.14E-04	3.99E-02	7.90E-03	1.69E-02	−1/1.42E-06	−1/2.19E-09
S_ZDT4	3.87E-05	2.22E-04	1.21E-04	6.87E-05	5.98E-03	1.31E-02	1.11E-02	2.66E-03	−1/2.44E-030	−1/2.42E-011
R_ZDT4	1.88E-03	4.01E-03	3.06E-03	8.49E-04	1.64E-03	9.01E-03	3.64E-03	2.76E-03	0/2.81E-01	0/1.84E-01
S_ZDT6	3.42E-06	3.06E-02	1.34E-02	1.31E-02	3.07E-02	8.37E-02	5.47E-02	1.80E-02	−1/1.83E-014	−1/2.42E-011
S_DTLZ2	2.36E-09	4.27E-05	2.40E-05	1.42E-05	1.04E-05	3.60E-05	2.24E-05	9.79E-06	0/6.01E-01	0/4.27E-01
R_DTLZ2	2.63E-05	3.41E-05	2.91E-05	2.69E-06	2.73E-05	3.89E-05	3.32E-05	4.38E-06	−1/4.73E-04	−1/6.23E-04
S_DTLZ3	4.54E-08	2.42E-06	1.49E-06	8.87E-07	1.89E-05	4.34E-05	2.98E-05	8.90E-06	−1/1.42E-024	−1/2.42E-011
WFG1	−9.16E-05	−8.21E-05	−8.78E-05	3.74E-06	−9.29E-05	−8.26E-05	−8.99E-05	4.02E-07	0/5.62E-01	0/6.22E-02
WFG8	−2.91E-02	−2.89E-02	−2.90E-02	8.67E-05	−2.92E-02	−2.88E-02	−2.90E-02	1.41E-04	0/6.98E-01	0/7.95E-01
WFG9	−1.19E-02	−9.30E-03	−9.89E-03	1.02E-03	−1.47E-02	−9.32E-03	−1.05E-02	2.38E-03	0/5.83E-01	0/3.10E-01
S_DTLZ2	4.31E-06	1.09E-05	8.40E-06	2.42E-06	5.49E-06	9.52E-05	8.24E-06	1.48E-06	0/7.62E-01	0/7.95E-01
R_DTLZ2	2.12E-05	3.07E-05	2.71E-05	3.24E-06	2.41E-05	4.66E-05	3.28E-05	8.09E-06	−1/8.08E-04	−1/1.64E-02
S_DTLZ3	3.06E-07	1.30E-06	7.22E-07	3.55E-07	2.39E-07	1.27E-06	7.68E-07	3.34E-07	0/6.04E-01	0/7.95E-01
WFG1	3.05E-02	3.23E-02	3.14E-02	5.70E-03	2.96E-02	3.23E-02	3.10E-02	1.15E-03	0/6.80E-02	0/4.27E-01
WFG8	−1.24E-02	−1.21E-02	−1.23E-02	9.59E-05	−1.22E-02	−1.20E-02	−1.21E-02	7.95E-05	−1/5.04E-04	−1/1.64E-02
WFG9	3.70E-04	5.55E-04	4.92E-04	6.59E-05	5.05E-04	5.90E-04	5.55E-04	3.23E-05	−1/1.42E-05	−1/5.66E-06

The h value of −1, 0, or 1 indicates that the MODE was statistically inferior to, equal to, or superior to the proposed MODE-DE

We also compared the MODE-DE with two state-of-the-art methods (Kumar *et al.*, 2007; Zielinski and Laur, 2007a). The proposed algorithm again showed the best performance among the three algorithms (Table 4).

6 Conclusions

This paper presents a multi-objective differential evolution algorithm with diversity enhancement procedure to avoid local convergence caused by the small magnitude of the differential vectors in the parameter space. The randomly generated parameter vectors provide large differential vectors which improve the global search ability and avoid the algorithm becoming trapped in an inferior local Pareto front. From the experimental results, we found that the MODE-DE gave either improved or equal performance when compared with the same MODE without the diversity enhancement procedure.

Table 4 Comparison of MODE-DE and other algorithms

Problem	I_{R2}		
	MODE-DE	MO_PSO	NSGA2_PCX
OKA2*	−1.07E-03 (2)	2.53E-03 (3)	−1.64E-03 (1)
SYMPART*	2.35E-06 (2)	1.64E-07 (1)	6.40E-05 (3)
S_ZDT1*	−1.04E-03 (1)	4.61E-03 (3)	7.98E-04 (2)
S_ZDT2*	−1.10E-04 (1)	6.50E-02 (3)	9.62E-04 (2)
S_ZDT4*	1.21E-04 (2)	2.50E-02 (3)	5.36E-06 (1)
R_ZDT4*	3.06E-03 (1)	6.56E-03 (2)	8.64E-04 (3)
S_ZDT6*	1.34E-02 (1)	1.03E-01 (3)	1.86E-02 (2)
S_DTLZ2**	2.40E-05 (1)	9.85E-05 (2)	1.01E-04 (3)
R_DTLZ2**	2.91E-05 (1)	3.07E-04 (3)	3.69E-05 (2)
S_DTLZ3**	1.49E-06 (1)	5.23E-04 (3)	3.03E-05 (2)
WFG1**	−8.78E-05 (1)	7.41E-02 (3)	5.21E-02 (2)
WFG8**	−2.90E-02 (1)	−1.68E-02 (3)	−2.70E-02 (2)
WFG9**	−9.89E-03 (2)	−1.04E-02 (1)	−9.42E-03 (3)
S_DTLZ2***	8.24E-06 (1)	1.27E-04 (3)	1.25E-04 (2)
R_DTLZ2***	3.28E-05 (1)	1.03E-04 (3)	7.08E-05 (2)
S_DTLZ3***	7.68E-07 (1)	2.36E-04 (2)	3.91E-04 (3)
WFG1***	3.01E-02 (1)	5.31E-02 (3)	4.46E-02 (2)
WFG8***	−1.21E-02 (1)	−3.40E-03 (2)	−1.66E-03 (3)
WFG9***	5.55E-04 (2)	2.57E-03 (3)	−8.43E-04 (1)
Total rank	24	49	41

* M=2; ** M=3; *** M=5. Within the brackets is the rank of the three algorithms for a specific problem

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