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Moments and Pasek's methods for parameter identification of a DC motor

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Abstract: Time moments have been introduced in automatic control because of the analogy between the impulse response of a linear system and a probability function. Pasek described a testing procedure for determining the DC parameters from the current response to a step in the armature voltage motor. In this paper, two identification algorithms developed based on the moments and Pasek's methods are introduced and applied to the parameter identification of a DC motor. The simulation and experimental results are presented and compared, showing that the moments method makes the model closer to reality, especially in a transient regime.

Key words: Identification, Moments method, Pasek's identification method, Separately excited DC motor
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1 Introduction

The DC motor is the obvious testing ground for advanced control algorithms in electric drives, due to its stable and straight forward characteristics (Rubaai and Kotaru, 2000; Basilio and Moreira, 2004). It is also ideally suited for trajectory control applications. The requirement for high performance speed control of DC motors has led to significant research efforts in the application of modern control theory (Weerasooriya and El-Sharkawi, 1991; Louis *et al*., 1998; Rigatos, 2009). System identification has important applications in many engineering fields such as model analysis, control system design, and condition monitoring. The main purpose of most identification techniques is to develop a mathematical model that fully describes a given system. This mathematical model can be used to explain the behavior of the system and to predict its response to various inputs at different conditions (Basilio and Moreira, 2004). The actual identification of models from data involves decision making on the part of the person in search of models, as well as fairly demanding computations to furnish bases for these decisions. There are situations where identification is necessary even though a relatively accurate mathematical model is available (Kara and Eker, 2004). For example, the DC motor parameters might be subject to some time variations, wherein a mathematical model that is accurate at the time of the design may not be accurate at a later time. In the literature there are many classical methods for identifying these parameters (Touhami *et al*., 1994; Coirault *et al*., 1995).

Time moments have been introduced in automatic control because of the analogy between the impulse response of a linear system and a probability function (Etien *et al*., 2000; Bentayeb *et al*., 2007). Thus, an impulse response is characterized by an infinity of moments. In a practical sense, only the first time moments are necessary for a probability density function. This basic idea has generated applications in identification, model order reduction, and controller design, known as the method of moments (Chiasson and Bodson, 1993; Goknar *et al*., 2001; Burridge and Qu, 2003). Pasek (1962) described a testing procedure for determining the parameters of a

DC motor linear model from the current response to a step in the armature voltage of the motor, and derived the Pasek equation for parameter determination (Jung *et al*., 1992).

2 DC motor model

The block diagram of the DC motor used in this study is shown in Fig. 1.

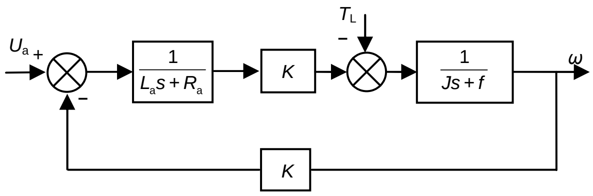

Fig. 1 Block diagram of the DC motor

In modeling a DC motor connected to a load via a shaft, the general approach is to neglect the nonlinear effects and build a linear transfer function representation for the input-output relationship of the DC motor and the load it drives. The dynamics of the separately excited DC motor may be expressed by the following equations:

$$K\omega(t) = -R_a i_a(t) - L_a \frac{\mathrm{d}i_a(t)}{\mathrm{d}t} + U_a(t), \tag{1}$$

$$Ki_a(t) = J\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} + f\omega(t) + T_L(t). \tag{2}$$

Herein, K, R_a, L_a, J, and f are the torque and back electromotive force (EMF) constant, the armature resistance, the armature inductance, the rotor mass moment of inertia, and the viscous friction coefficient, respectively. $\omega(t)$, $i_a(t)$, $U_a(t)$, and $T_L(t)$ denote the rotor angular speed, the armature current, the terminal voltage, and the load torque, respectively.

3 Pasek's method based model

Pasek (1962) proposed a simple method which requires one test for dynamical model identification of a separately excited DC motor, assuming the viscous friction coefficient null. Accordingly, we considered developing a model of identification based on the Pasek model by introducing the viscous friction coefficient. It is possible, from a terminal voltage step ΔU_a, to determine most of the DC motor parameters. This is also possible from the abrupt terminal voltage variation.

Eqs. (1) and (2) can be written as follows:

$$\Delta U_a = R_a \Delta i_a + L_a \frac{\mathrm{d}\Delta i_a}{\mathrm{d}t} + K\Delta\omega, \tag{3}$$

$$K\Delta i_a = J\frac{\mathrm{d}\Delta\omega}{\mathrm{d}t} + f\Delta\omega. \tag{4}$$

We record the initial and final currents, and speed values. At the steady state, we have

$$U_{a0} = R_a i_{a0} + K\omega_0, \tag{5}$$

$$U_{a1} = R_a i_{a1} + K\omega_1, \tag{6}$$

where U_{a0}, i_{a0}, and ω_0 denote the terminal voltage, armature current, and rotor speed in the initial mode respectively, and U_{a1}, i_{a1}, and ω_1 denote the terminal voltage, armature current, and rotor speed in the final mode, respectively.

According to Eqs. (5) and (6), the torque and back-EMF constant K can be written as

$$K = \left(U_{a1} - \frac{i_{a1}}{i_{a0}}U_{a0}\right) \Big/ \left(\omega_1 - \frac{i_{a1}}{i_{a0}}\omega_0\right). \tag{7}$$

The steady state check is

$$\Delta U_a = R_a \Delta i_a + K\Delta\omega, \tag{8}$$

where ΔU_a, Δi_a, and $\Delta\omega$ denote the terminal voltage variation, armature current variation, and rotor speed variation, respectively.

The armature resistance is given from Eq. (8) as

$$R_a = (\Delta U_a - K\Delta\omega)/\Delta i_a. \tag{9}$$

According to Eqs. (3) and (4), the armature current transfer function is given by

$$H_1(s) = \frac{\Delta i_a(s)}{\Delta U_a(s)} = \frac{\frac{f}{K^2 + R_a f}\left(1 + \frac{Js}{f}\right)}{1 + \tau_m \tau_e s^2 + (\tau_m + \mu\tau_e)s}, \tag{10}$$

and the rotor speed transfer function is given by

$$H_2(s) = \frac{\Delta\omega(s)}{\Delta U_a(s)} = \frac{K}{K^2 + R_a f}\frac{1}{1 + \tau_m \tau_e s^2 + (\tau_m + \mu\tau_e)s}, \tag{11}$$

where $\tau_e=L_a/R_a$ is the electrical time constant, $\tau_m=R_aJ/(K^2+R_af)$ is the mechanical time constant, and $\mu=R_af/(K^2+R_af)$ is usually a small coefficient.

Define

$$\lambda=\frac{\tau_m}{\tau_e}=\frac{R_a^2 J}{L_a(K^2+R_a f)}. \quad (12)$$

For a terminal voltage step ΔU_a, the current form is given by

$$\Delta i_a(t)=\frac{J\Delta U_a}{K^2+R_a f}\left[\frac{1}{T_2-T_1}(e^{-t/T_2}-e^{-t/T_1})+\frac{f}{J}\left(1+\frac{1}{T_2-T_1}(T_1e^{-t/T_1}-T_2e^{-t/T_2})\right)\right], \quad (13)$$

where $T_1=\frac{2\lambda\tau_e}{(\lambda+\mu)(-1+a)}$ and $T_2=\frac{2\lambda\tau_e}{(\lambda+\mu)(-1-a)}$ are the transfer function poles, with $a=\sqrt{1-4\lambda/(\lambda+\mu)^2}$.

We determine the moment t_1 when the current passes its maximum, noted $\Delta i_a(t_1)$, and have

$$t_1=\frac{\lambda\tau_e}{(\lambda+\mu)a}\ln\left(\frac{1+a-2\mu}{1-a+2\mu}\right). \quad (14)$$

Dividing the two terms of Eq. (14) by τ_e, we have

$$\frac{t_1}{\tau_e}=\frac{\lambda}{(\lambda+\mu)a}\ln\left(\frac{1+a-2\mu}{1-a+2\mu}\right). \quad (15)$$

Define

$$\delta=\Delta i_a(2t_1)/\Delta i_a(t_1). \quad (16)$$

Then

$$\delta=\frac{\frac{\lambda\Delta U_a}{aR_a(\lambda+\mu)}L_1+\frac{f\Delta U_a}{K^2+R_a f}(1+L_2)}{\frac{\lambda\Delta U_a}{aR_a(\lambda+\mu)}L_3+\frac{f\Delta U_a}{K^2+R_a f}(1+L_4)}, \quad (17)$$

where L_1, L_2, L_3, and L_4 are as shown in the Appendix.

From Eqs. (15) and (17), we deduce the abacuses for parameter identification (as shown later in Fig. 4), which give δ and t_1/τ_e according to λ.

Measurements of t_1, $\Delta i_a(t_1)$, and $\Delta i_a(2t_1)$ define the DC motor parameters by the following steps:

1. We calculate δ and an abacus gives λ.
2. The other abacus gives t_1/τ_e.
3. We deduce τ_e and $\lambda\tau_e=\tau_m$.
4. Finally, from τ_e and $\lambda\tau_e$ we deduce L_a and J, respectively.

The calculation of the output current gain gives

$$K_i=\Delta i_a(\infty)=\frac{f\Delta U_a}{K^2+R_a f}. \quad (18)$$

From Eq. (18) we calculate

$$f=\frac{K^2K_i}{\Delta U_a-R_aK_i}. \quad (19)$$

The static torque can be calculated from the steady state as

$$T_{st}=Ki_{a0}-f\omega_0. \quad (20)$$

4 Method of moments

The moments constitute the basis for a non-classical representation of linear systems. The characterization of an impulse response by its moments is equivalent to the moment characterization of a probability density function (Etien *et al*., 2000). Impulse response moments are system invariants. Like for a probability density function, it is not necessary to compute an infinity of moments to characterize, with good approximation, the shape of the impulse response. Only the first moments are necessary for performing this characterization.

4.1 Temporal moment of a function

Consider a stable linear system, characterized by its impulse $h(t)$. Then,

$$H(s)=B(s)/A(s). \quad (21)$$

$H(s)$ can be expanded in Taylor series in the vicinity of $s=j\omega_0$:

$$H(s)=\sum_{n=0}^{\infty}(-1)^n(s-j\omega_0)^n\bar{A}_{n,\omega_0}, \quad (22)$$

where $\bar{A}_{n,\omega_0}=\int_0^{\infty}\frac{t^n}{n!}h(t)e^{-j\omega_0 t}dt$ is the nth order frequency moment of $h(t)$ for $\omega=\omega_0$. Notice that $\bar{A}_{n,\omega_0}$ is complex. In the particular case $\omega_0=0$, frequency moments correspond to classical time moments:

$$A_n(h)=\int_0^{+\infty}\frac{t^n}{n!}h(t)\,\mathrm{d}t\,. \tag{23}$$

Frequency moments permit the characterization of $H(\mathrm{j}\omega)$ around ω_0=0, as well as that of the impulse response $h(t)$. $A_0(h)$ is the area of $h(t)$, $A_1(h)$ defines the mean time of $h(t)$, and $A_2(h)$ deals with the dispersion of $h(t)$ around its mean time (Etien *et al*., 2000; Bentayeb *et al*., 2007). Eq. (22) is rewritten as

$$H(s)=\sum_{n=0}^{\infty}(-1)^n s^n A_n(h)\,. \tag{24}$$

Let

$$H(s)=\sum_{n=0}^{\infty}\frac{S^n}{n!}\left[\frac{\mathrm{d}^n H(s)}{\mathrm{d}s^n}\right]_{s=0}.$$

Then, time moments can be expressed as

$$A_n(h)=\frac{(-1)^n}{n!}\left[\frac{\mathrm{d}^n H(s)}{\mathrm{d}s^n}\right]_{s=0}. \tag{25}$$

4.2 Moments and parameters of a transfer function

Let $y(t)$ be the step response of the studied system. We propose to identify the system by the model

$$H(s)=\frac{Y(s)}{E(s)}=K_1\frac{1+b_1s+b_2s^2+\ldots+b_ms^m}{1+a_1s+a_2s^2+\ldots+a_ns^n} \tag{26}$$

from the final value theorem, as time approaches infinity for a stable linear system. The system response approaches a steady state value K_1 given by

$$K_1=\lim_{t\to\infty}y(t)=y(\infty), \tag{27}$$

if a step input is applied to the system described in Eq. (26). Taking the Laplace transform of the normalized response gives

$$H(s)=s\cdot y(s). \tag{28}$$

Considering $\varepsilon(t)$ an error function with

$$\varepsilon(t)=K_1-y(t) \tag{29}$$

by introducing the Laplace transform in Eq. (29), Eq. (26) can be written as

$$\varepsilon(s)=\frac{K_1}{s}\left(1-\frac{1+b_1s+b_2s^2+\ldots+b_ms^m}{1+a_1s+a_2s^2+\ldots+a_ns^n}\right). \tag{30}$$

The development of Eq. (30) gives

$$\varepsilon(s)=K_1\frac{(a_1-b_1)+\ldots+(a_m-b_m)s^{m-1}+\ldots+a_ns^{n-1}}{1+a_1s+a_2s^2+\ldots+a_ns^n}. \tag{31}$$

Then using Eq. (24), we have

$$\varepsilon(s)=\sum_{n=0}^{\infty}(-1)^n s^n A_n(\varepsilon)\,. \tag{32}$$

According to Eqs. (24) and (32), we can deduce the coefficients of the transfer function $H(s)$ by solving the following matrix system:

$$\begin{bmatrix}K_1(a_1-b_1)\\K_1(a_2-b_2)\\\vdots\\K_1(a_{n+1}-b_{n+1})\end{bmatrix}=\begin{bmatrix}A_0(\varepsilon)&0&\cdots&0\\-A_1(\varepsilon)&A_0(\varepsilon)&\cdots&0\\\vdots&\vdots&&\vdots\\A_n(\varepsilon)&-A_{n-1}(\varepsilon)&\cdots&A_0(\varepsilon)\end{bmatrix}\begin{bmatrix}1\\a_1\\\vdots\\a_n\end{bmatrix}, \tag{33}$$

where $A_n(\varepsilon)$ is the nth order temporal moment.

4.3 DC motor transfer function and its moments

For our cases, when n=2 and m=1, the transfer function Eq. (8) becomes

$$H(s)=K_1\frac{1+b_1s}{1+a_1s+a_2s^2}. \tag{34}$$

The system (33) is reduced to the following matrix system:

$$\begin{bmatrix}K_1(a_1-b_1)\\K_1a_2\\0\end{bmatrix}=\begin{bmatrix}A_0&0&0\\-A_1&A_0&0\\A_2&-A_1&A_0\end{bmatrix}\begin{bmatrix}1\\a_1\\a_2\end{bmatrix}. \tag{35}$$

The resolution of this matrix system (35) gives the following coefficients:

$$a_1=\frac{A_1A_0-K_1A_2}{A_0^2-K_1A_1},\ a_2=\frac{-A_1+a_1A_0}{K_1},\ b_1=a_1-\frac{A_0}{K_1}. \tag{36}$$

4.4 Parametric identification

After having deduced the mathematical forms which are used for the calculation of the transfer function coefficients, and which allow us at the same time to calculate the electric and mechanical motor parameters, we present here the stages to be followed at the time of the determination of these parameters (Hadef *et al.*, 2008). The calculation of K_i and K_ω gains of the two outputs $i_a(t)$ and $\omega(t)$, respectively, by taking into account Eqs. (3) and (4), gives

$$K_i = \Delta i_a(\infty) = f\Delta U_a / (K^2 + R_a f), \quad (37)$$

$$K_\omega = \Delta\omega(\infty) = K\Delta U_a / (K^2 + R_a f). \quad (38)$$

According to Eqs. (37) and (38), we deduce f and μ:

$$f = K\Delta i_a(\infty)/\Delta\omega(\infty), \quad (39)$$

$$\mu = R_a f / (K^2 + R_a f). \quad (40)$$

By identification of $H_1(s)$ and $H_2(s)$ denominators with the $H(s)$ denominator, we obtain

$$a_1 = \tau_m + \mu\tau_e, \quad (41)$$

$$a_2 = \tau_m\tau_e. \quad (42)$$

According to Eqs. (41) and (42), we can obtain a second-order equation:

$$\mu\tau_e^2 - a_1\tau_e + a_2 = 0. \quad (43)$$

The resolution of Eq. (43) gives two roots: one is positive, and the other is negative (rejected). According to Eqs. (41) and (42) we deduce τ_m. The deduction of τ_m and τ_e gives L_a and J. The static torque can be calculated from the steady state as

$$T_{st} = Ki_{a0} - f\omega_0. \quad (44)$$

We recorded the initial and final speed and current values (Figs. 2 and 3).

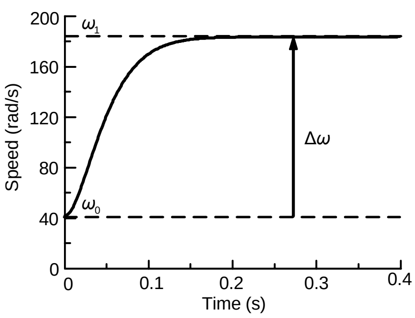

Fig. 2 Rotor speed angular response

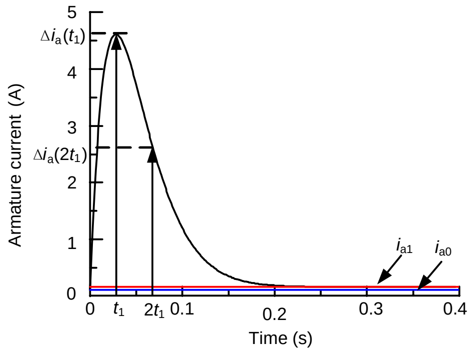

Fig. 3 Armature current response

5 Results and discussion

The separately excited DC motor used for experimental tests has the nominal characteristics as shown in Table 1. The dynamic test allows us to determine most of DC motor parameters. The first experiment to be carried out is to identify the DC motor parameters according to a step amplitude ΔU_a of terminal voltage applied to the armature circuit of the DC motor. The initial and final values of the armature current and the angular speed obtained from this test are shown in Table 2. The recording of $i_a(t)$ gives t_1=0.026 s where the current passes its maximum. For the two identification methods (Pasek's and moments methods), the back-EMF constant K, armature resistance R_a, and static torque T_{st} can be determined using Eqs. (7), (9), and (20), respectively. The viscous friction coefficient f can be determined using Eq. (19) for Pasek's method and Eq. (39) for the moments method. Using the first method and knowing $\Delta i_a(t_1)$ and $\Delta i_a(2t_1)$, δ is then calculated from Eq. (16) (δ=0.79). From Fig. 4, we deduce λ=4.5 and t_1/τ_e=1.82. Knowing λ and τ_e, we deduce τ_m and consequently J and L_a.

Table 1 Specification of the DC motor used for the experiment

Parameter	Value
Rated power (W)	180
Rated speed (r/min)	1500
Armature voltage (V)	270
Field voltage (V)	220
Armature current (A)	1.1
Field current (A)	0.4

Table 2 Dynamic test for parameter identification

Parameter	Value	
	Initial mode	Final mode
U_a (V)	60	248
i_a (A)	0.113	0.167
ω (r/min)	400	1745

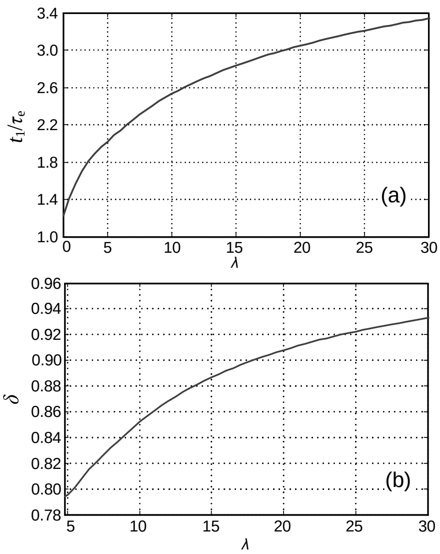

Fig. 4 Determination of DC motor parameters
(a) $t_1/\tau_e=f(\lambda)$; (b) $\delta=f(\lambda)$

Table 3 shows the first-, second-, and third-order moments values, as well as the transfer function coefficients values successively calculated using the trapezoids method.

Table 3 First-, second-, and third-order moments and transfer function coefficients values

	$\omega(t)$ (V)	$i_a(t)$ (A)
A_0	6.121 818	−0.296 570
A_1	0.140 349	−0.152 533
A_2	−0.000 940	−0.000 504
a_1	0.055 600	0.470 565
a_2	0.001 440	0.240 323
b_1	0.012 524	−5.962 602

Assuming we know a_1, a_2, and μ, then the resolution of the second-order equation $0.008\,74\tau_e^2-0.0556\tau_e+0.001\,44=0$ gives τ_{e1}=0.026 s and τ_{e2}=6.34 s (τ_{e2} is a rather large time constant, and is thus rejected). The deduction of τ_e and τ_m enables one to calculate J and L_a. Table 4 summarizes the values of the parameters calculated from the two identification methods. It is clear from Table 4 that the motor inductance value came out to be very different for the two methods because Pasek's method tries to identify the parameters with real transfer function poles. In our cases, the transfer function poles are complex. The moments method is shown to be more robust than the Pasek method, especially in the transient regime.

Table 4 Comparison between Pasek's method and the moments method for parameter identification of the DC motor

Parameter	Value	
	Pasek's method	Moments ethod
R_a (Ω)	30.9	30.9
L_a (H)	0.438	0.803
K (N·m/A)	1.323	1.323
J (kg·m^2)	0.0036	0.0031
f (N·m·s/rad)	0.0005	0.0005
T_{st} (N·m)	0.128	0.128

Finally, to check the precision of each method, we conducted the dynamic test, applying a step amplitude of terminal voltage ΔU_a=188 V to the armature circuit of the DC motor (Fig. 5). Also, we conducted the deceleration test (Fig. 6) and mechanical characteristic test (Fig. 7). According to Figs. 5a, 6a, and 7a, the curves simulated from the dynamic test parameters with the moment method are close to the experimental curves simulated from the Pasek method. With regard to the steady state (Fig. 8), the curves simulated from the dynamic test with the two proposed techniques parameters are almost identical or close to real measurements.

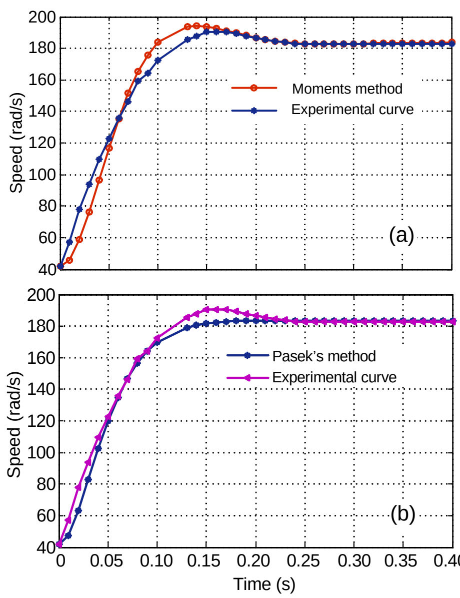

Fig. 5 Rotor speed angular response: (a) moments method; (b) Pasek's method

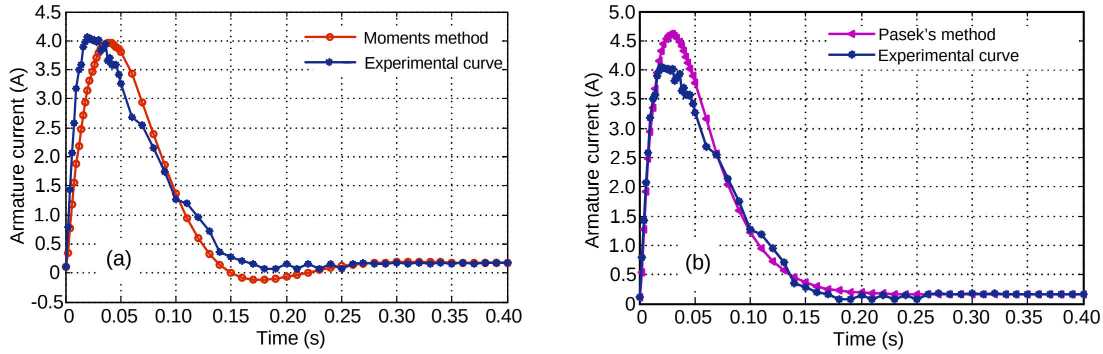

Fig. 6 Armature current response: (a) moments method; (b) Pasek's method

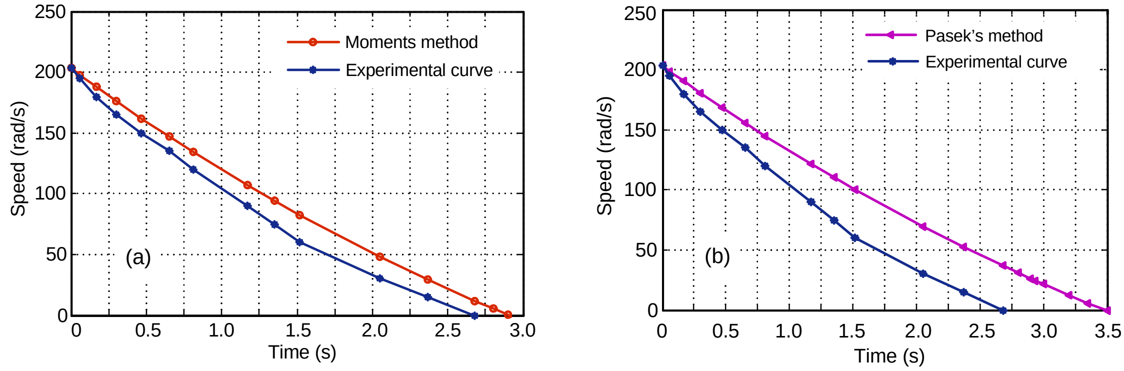

Fig. 7 Deceleration test: (a) moments method; (b) Pasek's method

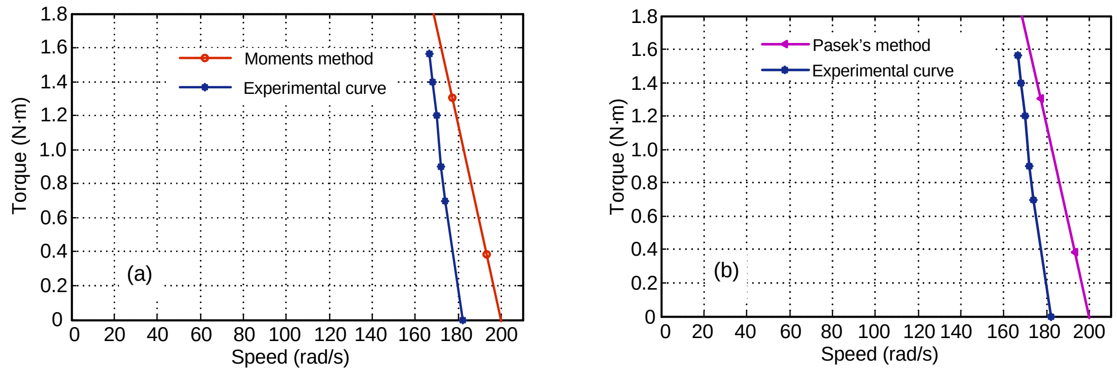

Fig. 8 Mechanical characteristic: (a) moments method; (b) Pasek's method

6 Conclusions

In this paper, two methods have been proposed to identify the parameters of a separately excited DC motor. The first model is based on Pasek's method, and the second based on the moments method, both of which can be used to identify all the motor parameters. The second method makes the model closer to reality, especially in a transient regime. In future work, the algorithms will be tested on different types of electric motors such as the permanent magnet synchronous motor and the induction motor.

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Appendix

The constants L_1, L_2, L_3, and L_4 are given by

$$L_1=\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-\frac{1}{a}}\left[\left(\frac{1+a-2\mu}{1-a+2\mu}\right)-\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-1}\right],$$

$$L_2=\frac{1}{2a}\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-\frac{1}{a}}\left[(1-a)\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-1}-(1+a)\left(\frac{1+a-2\mu}{1-a+2\mu}\right)\right],$$

$$L_3=\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-\frac{1}{2a}}\left[\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{\frac{1}{2}}-\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-\frac{1}{2}}\right],$$

$$L_4=\frac{1}{2a}\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-\frac{1}{2a}}\left[(1-a)\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{-\frac{1}{2}}-(1+a)\left(\frac{1+a-2\mu}{1-a+2\mu}\right)^{\frac{1}{2}}\right].$$