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Control of cascading failures in coupled map lattices based on adaptive predictive pinning control^{*}

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Abstract: An adaptive predictive pinning control is proposed to suppress the cascade in coupled map lattices (CMLs). Two monitoring strategies are applied: (1) A specific fraction of nodes with the highest degree or betweenness are chosen to constitute the set of monitored nodes; (2) During the cascade, an adaptive pinning control is implemented, in which only the nodes in the monitored set whose current state is normal but whose predictive state is abnormal, are pinned with the predictive controller. Simulations show that for the scale-free (SF) CML the degree-based monitoring strategy is advantageous over the betweenness-based strategy, while for the small-world (SW) CML the situation is the opposite. With the adaptive predictive pinning control, the fewer local controllers can effectively suppress the cascade throughout the whole network.

Key words:Coupled map lattices, Cascade, Pinning, Predictive control, Degree, Betweennessdoi:10.1631/jzus.C1000369Document code: ACLC number: TP27

1 Introduction

Cascading failures triggered by initial shocks are common in many real large-scale complex systems, such as the electrical power grid, the Internet, the transportation networks, and the human society (Crucitti et al., 2004; Wang and Chen, 2008). For instance, on August 14, 2003, an initial disturbance in Ohio triggered the largest blackout in U.S. history in which millions of people remained without electricity for as long as 15 hours (Ash and Newth, 2007). In another example, the ILOVEYOU computer virus, first detected in Hong Kong in May 2000, spread over the Internet and caused a loss of nearly seven billion dollars in facility damage and computer down-time (Wang and Chen, 2003). In light of the wide occurrence and catastrophic impact of the cascade in many real large infrastructures, the cascading failures in

complex networks have become one of the most concerned issues today (Motter and Lai, 2002; Motter, 2004; Lai *et al.*, 2005; Li and Wang, 2006; 2007; Zheng *et al.*, 2007; Bao *et al.*, 2008a; 2008b; Huang *et al.*, 2008).

Coupled map lattices (CMLs) have been widely investigated over the past decades to model the rich space-time dynamic behaviors of complex systems (Gade and Hu, 2000; Jost and Joy, 2001). In comparison to partial differential equations (PDEs), CMLs are more suitable for computational studies because of the discrete nature of time and space. In CML, the dynamic elements are situated at discrete points in space, and time is discrete. Each spatial unit is coupled to its neighbors by various coupling schemes, including random coupling, global coupling, nearest neighbor coupling, small-world (SW) coupling proposed by Watts and Strogatz (1998), and scale-free (SF) coupling proposed by Barabási and Albert (1999), each of which is motivated by different physical systems and achieves varying degree of success in modeling those systems. Recently, studies on dynamics of CML with different connections have

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been performed. Some researchers have studied the synchronization in CML. Wang *et al.* have made great achievements in the cascading failure model based on CML and the mechanism of the cascading failure in the model (Wang and Xu, 2004; Xu and Wang, 2005), and they found that a sufficiently large perturbation on a single node can lead to cascading failures of all the other nodes in the network. Bao *et al.* (2008b) further pointed out that the macroscopic properties of SF CML during the failure propagation are governed by the general laws of synergetics.

The existing research on the cascading failures in CML focused only on the mechanism and characteristics of cascade, and the relationship between the cascade and the coupling topology. In this paper, we investigate how to suppress the cascading failures in CML with SW and SF coupling topologies based on the adaptive predictive pinning control. The control strategy is illustrated as follows: choose a fraction of nodes with the highest degree or betweenness to constitute the monitored node set; during the cascade, only the nodes in the monitored node set, whose current state is normal but whose predictive state is abnormal, are adaptively pinned with the predictive control. The adaptive pinning strategy is different from the random and specific pinning scheme previously investigated in terms of the stabilization of complex networks (Li and Chen, 2004; Li et al., 2004; Xiang et al., 2007).

2 Formulation of the predictive control algorithm

Predictive control is the advanced control technique with a significant and widespread impact on industrial process control, which now can be found in a wide variety of manufacturing environments including chemicals, power plants, and aerospace (Raimondo *et al.*, 2007; Rawlings and Mayne, 2009). The basic concept of predictive control is to solve an optimization problem for a finite future interval at the current time (Raimondo *et al.*, 2007; Rawlings and Mayne, 2009). The index to be optimized is normally the expectation of a function measuring the discrepancy between the predicted output and the desired output of the system over the prediction horizon plus a function measuring the control increment over the control horizon. Only the first calculated control input in the control horizon is actually implemented. The horizons are moved by one sample period towards the future and the optimization process is repeated at each subsequent instant.

Consider a single-input-single-output system governed by

$$y(t+1) = f(y(t), y(t-1), \cdots, y(t-n), u(t), u(t-1), \cdots, u(t-m)),$$
(1)

where parameters *m* and *n* are the delay in input and output, respectively. At time *t*, the future system outputs $y_p(t+i|t)$ for $i=1,2,...,H_p$, can be predicted by using Eq. (1). These predicted values depend on the previous outputs and inputs, the current system output y(t), and the future control inputs u(t+i) for i=0, $1,...,H_u$, where $H_u \leq H_p$ is the control horizon. The control variable is manipulated only within the control horizon and remains constant afterwards, i.e., $u(t+i)=u(t+H_u-1)$ for $i=H_u,...,H_p$. The sequence of future control inputs u(t+i) for $i=0,1,...,H_u-1$ is computed by optimizing a given objective function. A general objective function is defined as the following quadratic form (Rawlings and Mayne, 2009):

$$\min_{\Delta u(t+k)} \left(J = \sum_{k=1}^{H_p} [y_p(t+k \mid t) - y_s(t+k)]^2 p_k + \sum_{k=0}^{H_u-1} (\Delta u(t+k))^2 q_k \right),$$
(2)

where p_k and q_k are the weighting values. In the expression of *J*, the first term accounts for minimizing the variance between the predictive output and the desired output of the system, while the second term, in which $\Delta u(t+k)=u(t+k)-u(t+k-1)$, represents a penalty on the control cost (related, for instance, to energy).

3 Scale-free and small-world network models

This study mainly investigates how to suppress the cascading failures in CML with SF and SW coupling topologies. In this section, we briefly describe the generation algorithm of SF and SW network models used in this work.

The SF network model proposed by Barabási

and Albert (1999) is adopted, in which two basic features of most real-life complex networks, growth and preferential attachment, are represented. The SF model starts with m_0 nodes. At every time step, a new node is introduced until the desired number of nodes N is reached. The new node connects to m already-existing nodes by a preferential attachment with probability $\Pi(k(i))=k(i)/\sum_i k(j)$, where k(i) is the degree of node i. The SF model develops the power-law degree distribution.

The SW topology is generated using the famous Watts-Strogatz model (Watts and Strogatz, 1998). Starting from a nearest-neighbor coupled network with N nodes arranged on a ring and K edges per node, each edge is rewired at random with a probability $0 \le p \le 1$. The SW model exhibits large clustering as in the regular network and small average path between nodes as in the random network. Because the SW model may lead to the formulation of isolated clusters, we hold the assumption in the whole paper that the SW network before the attack is always connected without any isolated clusters.

4 Cascading failure model based on CML

We consider a CML consisting of *N* nodes, described as follows (Wang and Xu, 2004):

$$x_i(t+1) = \left| (1-\varepsilon)f(x_i(t)) + \varepsilon \sum_{j=1, j \neq i}^N \frac{a_{ij}}{k(i)} f(x_j(t)) \right|, \quad (3)$$

where $x_i(t)$ (*i*=1,2,...,*N*) is the state variable of node *i* at time *t*. The connection information among the *N* nodes is given by the adjacency matrix $A=(a_{ij})_{N\times N}$. If there is an edge between node *i* and node *j*, then $a_{ij}=a_{ji}=1$; otherwise, $a_{ij}=a_{ji}=0$. It is assumed that no two different nodes can have more than one edge in between, and no node can have an edge with itself. Parameter k(i) is the degree of node *i*, and $\varepsilon \in (0, 1)$ represents the coupling strength. The function *f*, which defines the local dynamics, is chosen in this work as the chaotic logistic map f(x)=4x(1-x).

Node *i* is said to be in a normal state at time *m* if $0 \le x_i(t) \le 1$ when $t \le m$. On the other hand, if $0 \le x_i(t) \le 1$ when $t \le m$ and $x_i(m) \ge 1$, node *i* is said to fail at time *m* and it is assumed in this case that $x_i(t) \equiv 0$ when $t \ge m$. If the initial states of the nodes in a CML described in

Eq. (3) all lie in the interval (0, 1) and there is not any external perturbation, then *N* nodes in the network will be in normal states forever.

To trigger the cascading failure by attacking a single node, an external perturbation $R \ge 1$ is added to node *c* at time *m*,

$$x_{c}(m) = \left| (1-\varepsilon)f(x_{c}(m-1)) + \varepsilon \sum_{j=1, j\neq c}^{N} \frac{a_{cj}}{k(c)}f(x_{j}(m-1)) \right| + R.$$
(4)

In this case, node *c* will fail at time *m* and we have $x_c(t)\equiv 0$ for all t>m. At time m+1, the states of these nodes that are directly connected with node *c* will be affected by $x_c(m)\geq 1$ according to Eq. (4), and the states of these nodes may also be larger than 1 and thus may lead to a new round of node failures. The evolution of cascade is represented by the parameter I(t), which is the cumulative proportion of the nodes that fail in the network at time *t*.

5 Suppression of the cascading failure in CML based on adaptive predictive pinning control

A strategy, integrating the advanced predictive control into the adaptive pinning scheme, is proposed to greatly suppress the cascade in CML with the SW and SF coupling topologies. In the following simulations, we take N=2000, $m_0=m=3$ for SF CML, and K=6, p=0.1 for SW CML. Then, the CML contains 2000 nodes and about 6000 connections.

Firstly, we explore which nodes play an important role in accelerating the propagation. For the SF and SW CML with different parameters, simulations are implemented to find the key nodes. Figs. 1a and 1b illustrate the dynamics of cascade in SF and SW CML caused by a deliberate attack and a random attack occurring at time 10, respectively. In the deliberate attack, the attacked node has the largest degree, and in the random attack, the attacked node is randomly selected.

Figs. 2a and 2b present the degree and betweenness of the nodes that fail during the cascade in SF CML, respectively. From Figs. 1a and 2 we find that, once the nodes with the highest degree or betweeness fail (for example, at time 12 for deliberate attack caused cascade and at time 13 for random attack caused cascade), the proportion of the nodes that fail immediately increases sharply at the following time. Therefore, in SF CML, the nodes with the largest degree or betweenness can significantly speed up the cascade.

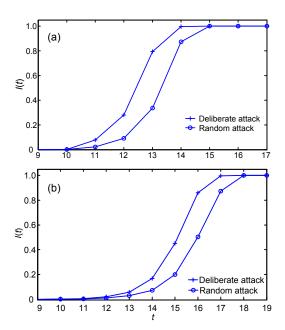


Fig. 1 Cascade in CML (ε=0.6) attacked by a perturbation R=4 to a signal node at time 10: (a) scale-free CML; (b) small-world CML

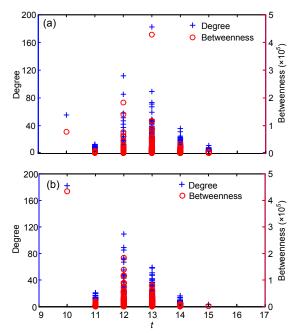


Fig. 2 Degree and betweenness of the nodes that fail during the cascade in scale-free CML under a random (a) and a deliberate (b) attack as shown in Fig. 1a

Figs. 3a and 3b present the degree and betweenness of the nodes that fail during the cascade in SW CML under random and deliberate attacks, respectively.

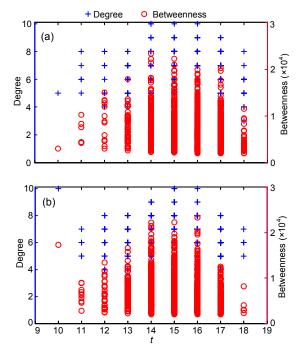


Fig. 3 Degree and betweenness of the nodes that fail during the cascade in small-world CML under a random (a) and a deliberate (b) attack as shown in Fig. 1b

As shown in Figs. 2 and 3, the distribution of the nodes with the highest degree across the course of propagation in SW CML is more even than that in SF CML, and then the influence of those nodes on the propagation is not obvious; however, the impact of the nodes with the highest betweenness on the propagation is distinct because the nodes with the higher degree, which fail in the early stage of cascade, usually have the smaller betweenness. Consequently, in SW CML the nodes with the highest betweenness are important to the propagation. We consider that fewer local intelligent controllers can suppress the cascade by pinning those nodes.

Secondly, a fraction δ_m of the nodes with the highest degree or betweenness are chosen to constitute the monitored node set M. Monitoring and adaptive pinning are applied only to the nodes in M. The pinned node i is represented as

$$x_{i}(t+1) = \left| (1-\varepsilon)f(x_{i}(t)) + \varepsilon \sum_{j=1, j \neq i}^{N} \frac{a_{ij}}{k(i)} f(x_{j}(t)) \right| + u_{i}(t),$$
(5)

where $u_i(t)$ is the predictive control law to node *i* at time *t*. Considering the equation of node state described in Eq. (5), the one-step-ahead predictive control is implemented to each of the pinned nodes to keep it in normal state. Based on the monitored node set *M*, the adaptive pinning scheme is carried out; i.e., only the nodes in *M* whose current states are normal but whose one-step-ahead predictive states are abnormal, form the adaptively pinned node set *P* and are actually pinned with the predictive controllers. During the control process, when failures spread, the adaptively pinned node set *P* varies while satisfying $P \subset M$. δ_p is the adaptive pinning fraction of the nodes.

Finally, we illustrate the algorithm of one-stepahead predictive control applied to the pinned nodes in CML.

Since one-step-ahead predictive control is used, the parameters in the objective function shown in Eq. (2) are chosen as $H_p=H_u=1$ and then only $\Delta u_i(t)$ needs to be optimized.

As $\Delta u_i(t) = u_i(t) - u_i(t-1)$, Eq. (5) can be rewritten as

$$x_i(t+1) = \left| (1-\varepsilon)f(x_i(t)) + \varepsilon \sum_{j=1, j\neq i}^N \frac{a_{ij}}{k(i)}f(x_j(t)) \right|$$

$$+ \Delta u_i(t) + u_i(t-1).$$
(6)

Substituting Eq. (6) into the objective function, we have

$$\min_{\Delta u_{i}(t)} \left\{ J = p_{1} \left\{ \left| (1 - \varepsilon) f(x_{i}(t)) + \varepsilon \sum_{j=1, j \neq i}^{N} \frac{a_{ij}}{k(i)} f(x_{j}(t)) \right| + \Delta u_{i}(t) + u_{i}(t-1) - x_{is}(t+1) \right\}^{2} + \left(\Delta u_{i}(t) \right)^{2} q_{0} \right\}.$$
(7)

Considering the necessary condition for the *J* in expression (7) to have a minimum is to set $\frac{\partial J}{\partial \Delta u_i(t)} = 0$,

we derive that the optimal increment of control at time *t* is

$$\Delta u_{i}(t) = \frac{p_{1}}{q_{0} + p_{1}} \left\{ x_{is}(t+1) - u_{i}(t-1) - \left| (1-\varepsilon)f(x_{i}(t)) + \varepsilon \sum_{j=1, j \neq i}^{N} \frac{a_{ij}}{k(i)} f(x_{j}(t)) \right| \right\}.$$
(8)

Since our goal is to keep the pinned node in normal state (0, 1), in the simulations the set-point value $x_{is}(t+1)$ of node *i* at time t+1 is chosen as 0.9. The set-point value is selected by balancing the control cost and performance.

The procedure of adaptive predictive pinning control is illustrated as follows:

Step 1: Choose the values of parameters p_1 , q_0 , $x_{is}(t+1)$ and determine the monitored node set M according to the degree- or betweenness-based monitoring strategy.

Step 2: At time k, examine the current state and one-step-ahead predictive state of each node in M, and then decide the adaptive pinning node set P.

Step 3: For each pinned node *i*, achieve the optimal control increment $\Delta u_i(t)$ according to Eq. (8).

Step 4: Add $\Delta u_i(t)$ to $u_i(k-1)$ to generate the control law $u_i(k)$ at the current time k, and apply $u_i(k)$ to the pinned node.

Step 5: $k+1 \rightarrow k$; go to step 2 if the control process is not complete.

6 Simulation results

In SF CML, under the max-degree based attack the cascade propagates rapidly throughout the network in only four time periods, as shown in Fig. 1a. For the deliberate attack, the cascade can be greatly suppressed when the adaptive predictive pinning control is employed. The controlled cascade dynamics and the adaptive pinning fraction (δ_m =30%) are depicted in Figs. 4a and 4b with the degree- and betweenness-based monitoring strategies applied, respectively. Figs. 4a and 4b show that in the worst case when the SF CML is deliberately attacked, the degree-based monitoring strategy is more effective than the betweenness-based monitoring strategy, and that fewer local predictive controllers can hold back the cascade, with the largest fraction δ_p of pinning control being only 5.6%. As shown in Fig. 1a, the SF CML is still vulnerable to a random attack although

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the speed of propagation is slower than that in a deliberate attack.

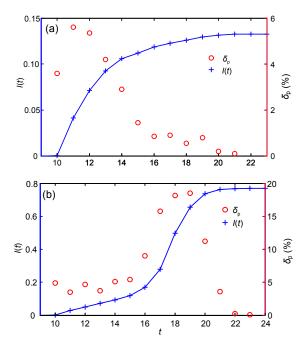


Fig. 4 Suppressed cascades and adaptive pinning fraction under degree-based (a) and betweenness-based (b) monitoring strategies when adaptive predictive pinning control is applied to the deliberate attack caused cascade in scalefree CML shown in Fig. 1a (δ_m =30%)

For the random attack, the cascade dynamics and the corresponding adaptive pinning fraction are illustrated in Figs. 5a and 5b under the degree- and betweenness-based monitoring strategies, respectively. For the random attack in SF CML both of the two monitoring strategies prove effective, with the largest pinning fraction δ_p being below 2%. Consequently, the adaptive predictive pinning control can prevent the further propagation of random or deliberate attack caused cascade in SF CML with fewer local controllers applied.

The adaptive predictive pinning control with the different monitoring strategies is applied to suppress the deliberate or random attack caused cascade in SW CML. For the deliberate attack in SW CML, the suppressed cascades and adaptive pinning fraction are shown in Figs. 6a and 6b for the degree- and betweenness-based monitoring schemes respectively (δ_m =40%). We can see that the performance of the betweenness-based monitoring scheme is more ef-

fective than that of the degree-based monitoring strategy.

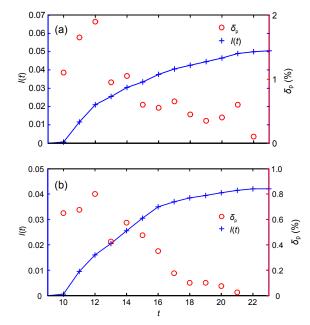


Fig. 5 Suppressed cascades and adaptive pinning fraction under degree-based (a) and betweenness-based (b) monitoring strategies when adaptive predictive pinning control is applied to the random attack caused cascade in scalefree CML shown in Fig. 1a (δ_m =30%)

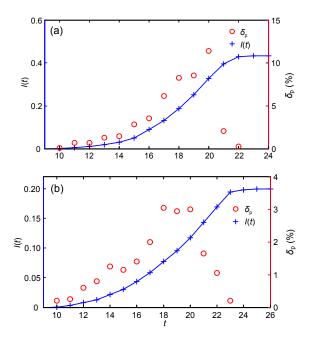


Fig. 6 Suppressed cascades and adaptive pinning fraction under degree-based (a) and betweenness-based (b) monitoring strategies when adaptive predictive pinning control is applied to the deliberate attack caused cascade in small-world CML shown in Fig. 1b (δ_m =40%)

For the random attack in SW CML, Figs. 7a and 7b depict the control effect and adaptive pinning fraction under the two monitoring strategies (δ_m = 40%), and the betweenness-based monitoring scheme is remarkably advantageous over the degree-based one.

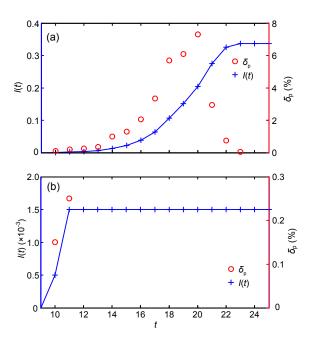


Fig. 7 Suppressed cascades and adaptive pinning fraction under degree-based (a) and betweenness-based (b) monitoring strategies when adaptive predictive pinning control is applied to the random attack caused cascade in small-world CML shown in Fig. 1b (δ_m =40%)

For the different values of monitored fraction δ_{m} , we further investigate the control performance of the two monitoring strategies. For the cascades of SF and SW CML shown in Figs. 1a and 1b, we try different $\delta_{\rm m}$ to examine the controlled cascade with the degreeand betweenness-based monitoring strategies applied. Fig. 8 illustrates the relationship between the final fraction I_{∞} of the nodes that fail and the monitored fraction δ_m under the different monitoring schemes. For the random attack in SF CML, these two monitoring strategies have similar performance, while for the deliberate attack in SF CML the degree-based monitoring shows better performance than the betweenness-based one (Fig. 8a). Fig. 8b shows that for the random and deliberate attacks in SW CML, the betweenness-based monitoring strategy is more effective than the degree-based one.

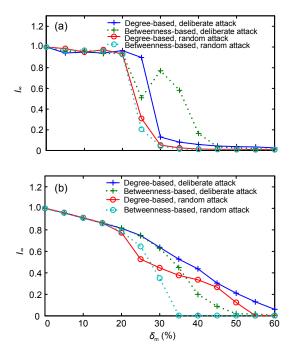


Fig. 8 Relationship between the final failure fraction I_{∞} and the monitored fraction $\delta_{\rm m}$ in scale-free (a) and smallworld (b) CML

7 Conclusions

This paper investigates how to control the cascade in the CML with SW and SF topologies. In the CML, the nodes with the highest degree or betweenness can play an important role in accelerating the cascading failures. By integrating the advanced predictive control into the pinning scheme, an adaptive predictive pinning control strategy for suppressing the cascade in the CML is proposed. A specific fraction of nodes that have the highest degree or betweenness are chosen to constitute the monitored node set. During the cascade, the adaptive pinning strategy is applied; i.e., only the nodes in the monitored node set, whose current state is normal but whose predictive state is abnormal, are pinned with the one-step-ahead predictive control. Simulation results show that for the SF CML the degree-based monitoring strategy is advantageous over the betweenness-based monitoring strategy, while for the SW CML the situation is the opposite. The fewer local predictive pinning controllers can effectively suppress the cascading failures throughout the whole CML. This work might shed some light on the analysis and control of cascading failures in real-world complex networks. For example, in a large-scale electric network, some important generators and transmission lines are monitored, and a blackout can be avoided by pinning them.

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